Unit 1: FACTORS and PRODUCTS

1.1 Factors and Multiples of Whole Numbers

Factor: a whole number that a value is evenly divisible by (e.g. 4 & 5 are factors of 20)

Product: the result when you multiply 2 or more numbers (18 is the product of 2x9)

Multiple: the product of a # and any natural # (4, 8, 16 are multiples of 2)

Prime Number: a # that is only divisible by 1 and itself (e.g. 7)

Composite Number: a # that is divisible by 3 or more factors (not prime)

Circle all the prime numbers and put an X through all the composite numbers in the 100 chart below.

1	2	3	~~4~~	5	6	7	8	9	10
11	~~12~~	13	~~14~~	15	16	17	18	19	20
21	~~22~~	23	~~24~~	25	26	27	28	29	30
31	~~32~~	33	~~34~~	35	36	37	38	39	40
41	~~42~~	43	~~44~~	45	46	47	48	49	50
51	~~52~~	53	~~54~~	55	56	57	58	59	60
61	~~62~~	63	~~64~~	65	66	67	68	69	70
71	~~72~~	73	~~74~~	75	76	77	78	79	80
81	~~82~~	83	~~84~~	85	86	87	88	89	90
91	~~92~~	93	~~94~~	95	96	97	98	99	100

What about 0 and 1?

Not prime or composite

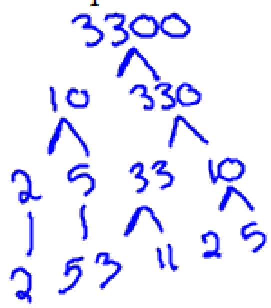
Prime Factorization:

A number written as the product of its prime factors

Greatest Common Factor: greatest factor that 2 #s have in common

Least Common Multiple: the least # that is divisible by 2 or more #s

Example 1: Determining the Prime Factors of a Whole Number
Write the prime factorization of 3300.



$$\therefore 3300 = 2 \times 5 \times 3 \times 11 \times 2 \times 5$$
$$= 2 \times 2 \times 3 \times 5 \times 5 \times 11$$
$$= 2^2 \times 3 \times 5^2 \times 11$$

Example 2: Determining the Greatest Common Factor
Determine the GCF of 138 and 198.

$\therefore$ common factors 2, 3 + 6

GCF = 6

Example 3: Determining the Least Common Multiple
Determine the LCM of 18, 20, and 30.

18 → 36, 54, 72, 90, 108, 126, 144, 162, 180

180 is a multiple of 18, 20 & 30

∴ 180 is the LCM

Example 4: Problems involving GCF and LCM

a. What is the side length of the smallest square that could be tiled with rectangles that measure 16 cm by 40 cm? Assume the rectangles cannot be cut.

∴ find the LCM of 16 & 40

16 → 32, 48, 64, 80

∴ 80 is the LCM of 16 & 40.

b. What is the side length of the largest square that could be used to tile a rectangle that measures 16 cm by 40 cm? Assume that the squares cannot be cut.

∴ find the GCF of 16 & 40

2 4 16 4 8   2 4 5 40 8 10 20

∴ GCF is 8.

1.2 Perfect Squares, Perfect Cubes, and Their Roots

Perfect Square Number: a number written as a power with an integer base and an exponent of 2 ($4^2=16$, $5^2=25$)

Perfect Cube Number: a # written as a power with an integer base and an exponent of 3 ($1^3=1$, $2^3=8$)

Which numbers from 1 to 100 are perfect squares and which are perfect cubes? In the hundred chart below, but a circle around the perfect squares and an X through the perfect cubes.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

D. Continue this investigation for the numbers from 101 to 200. Which numbers are perfect squares and which are perfect cubes?

Positive Integers	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Perfect Square Numbers	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225
Perfect Cube Numbers	1	8	27	64	125	216	343	512	729	1000	1331	1728	2197	2744	3375

Example 1: Determining Perfect Squares

Is 40 a perfect square? Explain.

No.

$6\times6=36$
$7\times7=49$ } No # between 6 & 7 ∴ 40 isn't a perfect square

OR

$\sqrt{40} = 6.32..$

This is a decimal, not an integer.

Example 2: Determining Perfect Cubes

Is 1728 a perfect cube number? Explain

$1728 = 12^3$ ∴ it is a perfect cube.

or

$\sqrt[3]{1728} = 12$

Square Root: $\sqrt[2]{n}$, a number whose square is n.

Cube Root: $\sqrt[3]{n}$, a number whose cube is n.

Example 3: Determining the Square Root of a Whole Number

Determine the square root of 1296.

$\sqrt{1296} = 36$

Example 4: Determining the Cube Root of a Whole Number
Determine the cube root of ~~1728.~~
4210.

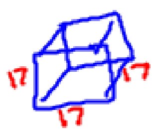
$$\sqrt[3]{4210} = 16.147\ldots$$

$\therefore$ 4210 is not a perfect cube.

Example 5: Using Roots to Solve a Problem
A cube has a volume of 4913 cubic inches. What is the surface area of the cube?

Volume $= l \times l \times l$
$= 4913 \text{ in}^3$
$l^3 = 4913 \text{ in}^3$
$l = \sqrt[3]{4913}$
$l = 17$ inches

Side length of cube is 17 inches.



6 sides of a cube.

S.A. of one side $= 17 \times 17$
$= 289$

S.A of whole cube
$= 289 \times 6$
$= 1734 \text{ in}^2$

pg. 146
#4, 5, 7, 8, 12

1.3 Multiplying Polynomials

Polynomial: one term or the sum of terms containing variables & whole number exponents (e.g. $3x^2+7$)

Monomial: a polynomial w/ one term

Binomial: a polynomial w/ 2 terms

Trinomial: a polynomial w/ 3 terms

Constant: a # that doesn't change & is not attached to a variable

Descending Order of Powers: $7x^{\boxed{4}}+6x^{\boxed{3}}-2x^{\boxed{2}}+x^{\boxed{1}}+8$

Naming Polynomials:

- **Degree 1:** x^1 or x → Linear
- **Degree 2:** x^2 → Quadratic
- **Degree 3:** x^3 → Cubic
- **Degree 4:** x^4 → Quartic
- **Degree 5:** x^5 → Quintic

} Most important!

Polynomial	# of Terms	Polynomial Name	Degree	Degree Name	Variables	Coefficients	Constants
$1x^{\boxed{2}}$	1	monomial	2	quadratic	x	1	0
$-4y^3$	1	monomial	3	cubic	y	−4	0
$5x^{\boxed{2}}-1$	2	binomial	2	quadratic	x	5	−1
$8r^2-4r+2$	3	trinomial	2	quadratic	r	8, −4	2
$9m^4+1m^2-3$	3	trinomial	4	quartic	m	9, 1	−3
$-6r^5+2r^3-k-10$	4	polynomial	5	quintic	r, k	−6, 2, −1	−10

Example 1: Polynomial Review: Adding and Subtracting
Simplify.

a. $3y + \boxed{2x} + 7y \boxed{-x} =$
$10y + x$

b. $-3x^2 + 2x^2 + 7x - 2x =$
$-x^2 + 5x$

c. $(3x + 2) + (2x + 2) =$
$3x + 2x + 2 + 2 = 5x + 4$

d. $(-3x - 2) - 1(2x + 7y) =$
$-3x - 2 - 2x - 7y = -5x - 7y - 2$

Remember, when you multiply two powers that have the same base, you add the exponents.
$(x^2)(x^6) = x^8$

Example 2: Polynomial Review: Multiplying
Simplify.

a. $(2x)(7y) = 14xy$

b. $(-3a)(2bc) = -6abc$

c. $(3x^2)(4x^3) = 12x^5$

d. $(-8xy^4)(3xy) = -24x^2y^5$

e. $(x^2)^3 = x^6$

f. $(3x^2)^3 = 3^3x^6$
$= 27x^6$

g. $2x(3x + 1) = 6x^2 + 2x$

h. $-3x^2y(2x + 7xy - 2) = -6x^3y - 21x^3y^2 + 6x^2y$

i. $5m(m^2 \; - 4)-\; 2m^2(m \; + \; 1) =$

$5m^3 - 20m - 2m^3 - 2m^2$

$3m^3 - 2m^2 - 20m$

Example 3: Polynomial Review: Dividing

Simplify.

$$\frac{4x^2+2x}{2x}=$$

$$\frac{-18r^5t^2+12r^3t+3rt}{6rt}=$$

The Distributive Property: Recall: $2(x-4) = 2x-8$

The Distributive Property for Binomials: $(x+2)(2x-1)$

$$(ax+b)(cx+d) =$$

$(x+3)(2x-1)$

The BOX Method:

	x	$+3$
$2x$	$2x^2$	$6x$
-1	$-x$	-3

$2x^2+6x-x-3$

$=2x^2+5x-3$

FOIL: First Outside Inside Last

$(x+3)(2x-1)$

$2x^2 - x + 6x - 3$

$= 2x^2+5x-3$

Example 4: The Distributive Property

Expand and simplify.

a. $(x + 2)(x + 3) =$

$x^2+3x+2x+6$

$= x^2+5x+6$

b. $(2x + 1)(x + 8) =$

$2x^2+16x+x+8$

$= 2x^2+17x+8$

c. $(x + 7)^2 =$ $(x+7)(x+7)$

$= x^2+7x+7x+49$

$= x^2+14x+49$

d. $(x - 5)(x + 5) =$

$x^2+5x-5x-25$

$= x^2-25$

e. $(x + 4)(x^2 + 4x + 2) =$

$x^3+4x^2+2x+4x^2+16x+8$

$x^3+8x^2+18x+8$

f. $(x^2 + 3x - 4)(2x^2 - 2x + 5) =$

$2x^4-2x^3+5x^2+6x^3-6x^2+15x-8x^2+8x-20$

$2x^4+4x^3-9x^2+23x-20$

g. $(x + 2)^3 =$

Example 4: The Distributive Property

Expand and simplify.

a. $(x + 2)(x + 3) =$

b. $(2x + 1)(x + 8) =$

c. $(x + 7)^2 =$

d. $(x - 5)(x + 5) =$

e. $(x + 4)(x^2 + 4x + 2) =$

f. $(x^2 + 3x - 4)(2x^2 - 2x + 5) =$

g. $(x + 2)^3 = \underbrace{(x+2)(x+2)}(x+2)$

$(x^2+2x+2x+4)(x+2)$

$(x^2+4x+4)(x+2)$

$(x^3+4x^2+4x+2x^2+8x+8)$

$x^3+6x^2+12x+8$

Example 5: Simplifying Sums and Differences of Polynomial Products
Expand and simplify.

a. $(2c - 3)(c + 5) + 3(c - 3)(-3c + 1)$

$(2c^2 + 10c - 3c - 15) + 3(-3c^2 + c + 9c - 3)$

$(2c^2 + 7c - 15) + 3(-3c^2 + 10c - 3)$

$(2c^2 + 7c - 15) + (-9c^2 + 30c - 9)$

$\boxed{-7c^2 + 37c - 24}$

b. $(3x + y - 1)(2x - 4) - (3x + 2y)^2$

$(3x + y - 1)(2x - 4) - (3x + 2y)(3x + 2y)$

$(6x^2 + 2xy - 2x - 12x - 4y + 4) - (9x^2 + 6xy + 6xy + 4y^2)$

$(6x^2 + 2xy - 14x - 4y + 4) - (9x^2 + 12xy + 4y^2)$

$6x^2 + 2xy - 14x - 4y + 4 - 9x^2 - 12xy - 4y^2$

$\boxed{-3x^2 - 10xy - 14x - 4y - 4y^2 + 4}$

1.4 Common Factors of a Polynomial

Multiplying vs. Factoring:

Factoring is the "opposite" of multiplying

Recall G.C.F: Find the GCF of each following set of numbers.

a. 72 and 54

2, 3, 6, 9 GCF=9

b. $48x^2y^3$ and $60x^3y$

$12x^2y$

Common Factor: The expression that 2 or more terms have in common, and they are evenly divisible by that expression

Example 1: Binomial GCF Factoring

Factor each binomial.

a. $6n + 9$

$3(2n+3)$

$3(2n+3)$

$6n+9$

b. $6c + 4c^2$

$2c(3+2c)$

c. $72x^2 - 8x$

$8x(9x-1)$

d. $24x^2y - 12x^3y^3 + 36xyw$

$12xy(2x-1x^2y^2+3w)$

e. $2(x - y) + 7x(x - y)$

Let $w=(x-y)$

$2w+7xw$

$w(2+7x)$

$(x-y)(2+7x)$

f. $3x^2 - 12x^3 + 15xy$

$3x(x-4x^2+5y)$

GCF Factoring

1. $8xy + 4xy^2$

$4xy(2+y)$

2. $9m^3 - 9m^2$

$(m)(m)(m)$ $(m)(m)$

$9m^2(m-1)$

3. $20a^2b^4c^2 - 5ab^3c^2$

$5ab^3c^2(4ab-1)$

4. $12w^3t^2 - 9wt^2 + 15w^2t^3$

$3wt^2(4w^2-3+5wt)$

5. $9y^2z^2 - 81y^3z^2 - 90y^2z^4$

$9y^2z^2(1-9y-10z^2)$

6. $2(x+1) + y(x+1)$ Let $w=x+1$

$2w+yw$

$w(2+y) = (x+1)(2+y)$

7. $3a(2+y) + 4b(y+2)$

$w=y+2$

$3a(y+2)+4b(y+2)$

$3aw + 4bw$

$w(3a+4b)$ $(y+2)(3a+4b)$

1.5 Polynomial Factoring

Factoring Trinomials ($ax^2 + bx + c$... where $a = 1$)
In this case, you are looking for a pair of numbers that multiply to ___C___ and add to ___b___. "Break it up" in brackets and you are good to go!

Example 1: Factoring Trinomials.
Factor.

a. $x^2 + 8x + 16$

Two #s that multiply to 16 and add to 8.

$(x+4)(x+4)$

$x^2+4x+4x+16$

b. $x^2 - 1x - 6$

Two #s that multiply to -6, and add to -1.

$(x-3)(x+2)$

c. $x^2 - 5x - 24$

Two #s that multiply to -24 & add -5.

$(x-8)(x+3)$

$x^2-8x+3x-24$

$x^2-5x-24$

d. $x^2 - 7x + 12$

Two #s that multiply to 12 & add to -7.

$(x-3)(x-4)$

$-3x-4=12$

$-3+(-4)=-7$

e. $4x^2 - 20x - 24$

Two #s that multiply to -6 and add to -5.

$4(x^2-5x-6)$

$4(x-6)(x+1)$

~~$-2+-3=-5$~~

~~$(-2)(-3)=6$~~

$(-6)(1)=-6$ ✓

$-6+1=-5$

f. $-12 - 10x + x^2$

Two #s that multiply to -12 & add to -10.

$x^2-10x-12$

Can't factor.

Factoring Trinomials ($ax^2 + bx + c$... where $a \neq 1$)

In this case, look for a pair of numbers that multiply to ac and add to b. Rewrite the middle term using the two numbers you found. Now factor by grouping.

Example 2: Factoring Trinomials.

Factor.

a. $2x^2 + 7x + 3$

$2(3)=6$

$\underbrace{2x^2+6x}+\underbrace{1x+3}$

$2x(x+3)+1(x+3)$

$(x+3)(2x+1)$

b. $8x^2 - 2x - 1$

$8(-1)=-8 \rightarrow$ Use -4 & 2

$\underbrace{8x^2-4x}+\underbrace{2x-1}$

$4x(2x-1)+1(2x-1)$

$(2x-1)(4x+1)$

c. $3x^2 - 5x - 8$

$3(-8)=-24$

Use -8 & 3

$\underbrace{3x^2-8x}+\underbrace{3x-8}$

$x(3x-8)+1(3x-8)$

$(3x-8)(x+1)$

d. $6x^2 + 8x - 8$

$3(-4)=-12$

Use 6 & -2

$2(3x^2+4x-4)$

$2(\underbrace{3x^2+6x}\underbrace{-2x-4})$

$-2x-4$

$-2(x+2)$

$2(3x(x+2)-2(x+2))$

$2(x+2)(3x-2)$

e. $12x^2 + 1x - 6$

$12(-6)=-72$

Use 9 & -8.

$\underbrace{12x^2+9x}\underbrace{-8x-6}$

$3x(4x+3)-2(4x+3)$

$(4x+3)(3x-2)$

f. $-12 + 8a + 15a^2$

$(15)(-12)=-180$

Use 18 & -10

$15a^2+8a-12$

$\underbrace{15a^2+18a}\underbrace{-10a-12}$

$3a(5a+6)-2(5a+6) = (5a+6)(3a-2)$

1.6 Special Polynomial Factoring

A trinomial that is a difference of squares will be in the form:

$x^2 - y^2 = (x - y)(x + y)$

- Both terms are perfect squares
- A minus sign means a difference

Investigation:

Multiply the following binomials. What do you notice?

a. $(x + 3)(x - 3)$

$x^2 - 9$

b. $(x - 6)(x + 6)$

$x^2 + 6x - 6x - 36 = x^2 - 36$

Example 1: Factor a Difference of Squares

Factor. $(x)^2 - (4)^2$

a. $x^2 - 16$

Think: both terms are perfect squares. What value do you need to "square" to get each term. *

$(x + 4)(x - 4)$

b. $x^2 - 100$

$(x)^2 - (10)^2$

$(x + 10)(x - 10)$

c. $36m^2 - 81n^2$

$(6m)^2 - (9n)^2$

$(6m + 9n)(6m - 9n)$

$36m^2 - 54mn + 54mn - 81n^2$

d. $16 - x^2$

$(4)^2 - (x)^2$

$(4 + x)(4 - x)$

e. $m^{10} - x^8$

$(m^5)^2 - (x^4)^2$

$(m^5 + x^4)(m^5 - x^4)$

f. $x^4 - 1$

$(x^2)^2 - (1)^2$

$(x^2 + 1)(x^2 - 1)$

$(x^2 + 1)(x + 1)(x - 1)$

Example 2: Factoring a Perfect Square Trinomial

Factor each trinomial. Verify by multiplying the factors.

$4(9)=36$

a. $4x^2 + 12x + 9$

$(2x + 3)(2x + 3)$

$4x^2 + 6x + 6x + 9$

$4x^2 + 12x + 9$

$\underbrace{4x^2 + 6x}+\underbrace{6x + 9}$

$2x(2x+3) + 3(2x+3)$

$(2x+3)(2x+3)$

$(25)(4)=100$

b. $4 - 20x + 25x^2$

$25x^2 - 20x + 4$

$(5x - 2)(5x - 2)$

$25x^2 - 20x + 4$

$\underbrace{25x^2 - 10x}\underbrace{-10x + 4}$

$5x(5x-2) - 2(5x-2)$

$(5x-2)(5x-2)$

Example 3: Factoring Trinomials in Two Variables

Factor each trinomial. Verify by multiplying the factors.

a. $x^2 + 6xy + 5y^2$

Think: What two numbers multiply to 5 and add to 6? (Ignore the y's for now!)

$x^2 + 6x + 5$

$(x+5)(x+1)$

$(x + 5y)(x + 1y)$

$x^2 + xy + 5xy + 5y^2$

$x^2 + 6xy + 5y^2$

b. $4x^2 + 4xy + y^2$

Think: What two numbers multiply to 4(1) = 4 and add to 4? (Ignore the y's for now!)

$\underbrace{4x^2 + 2xy}+\underbrace{2xy + y^2}$

$2x(2x + y) + y(2x + y)$

$(2x+y)(2x+y)$

c. $2a^2 - 7ab + 3b^2$ $-6+-1$

Think: What two numbers multiply to 2(3) = 6 ar add to -7? (Ignore the b's for now!)

$\underbrace{2a^2 - 6ab}\underbrace{-1ab + 3b^2}$

$2a(a-3b) - b(a-3b)$

$(2a-b)(a-3b)$

d. $10c^2 - cd - 2d^2$ $-5+4$

Think: What two numbers multiply to 10(-2) = -2 and add to -1? (Ignore the d's for now!)

$\underbrace{10c^2 - 5cd}+\underbrace{4cd - 2d^2}$

$5c(2c-d) + 2d(2c-d)$

$(5c+2d)(2c-d)$