Unit 3: Measurement

Modern Day Measuring Units

Metric:

Kilometer km $1 \text{ km} = 1000 \text{ m}$

Meter m $1 \text{ m} = 100 \text{ cm}$

Centimeter cm $1 \text{ cm} = 10 \text{ mm}$

Millimeter mm

Imperial

Mile mi. $1 \text{ mi} = 1\,760 \text{ yd}$

Yard yd. $1 \text{ yd} = 3$ feet

Foot ft $1 \text{ ft} = 12 \text{ in.}$

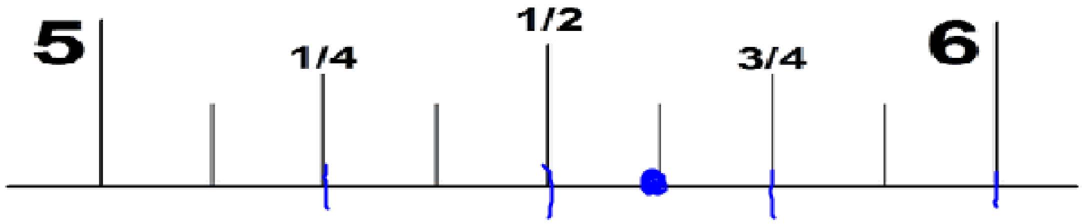
Inch in.

Converting between Imperial and Metric Units

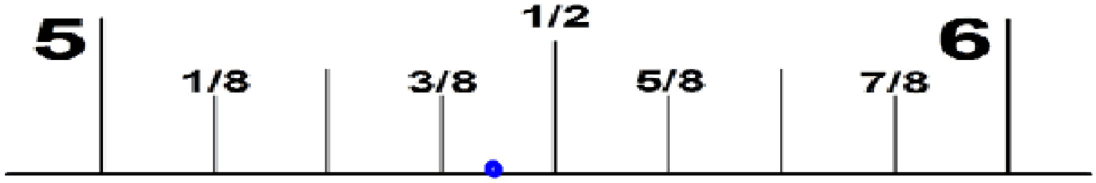
SI Units to Imperial Units	Imperial Units to SI Units
$1 \text{ mm} \doteq \frac{4}{100} \text{ in.}$	$1 \text{ in.} \doteq 2.5 \text{ cm}$
$1 \text{ cm} \doteq \frac{4}{10} \text{ in.}$	$1 \text{ ft.} \doteq 30 \text{ cm}$ $1 \text{ ft.} \doteq 0.3 \text{ m}$
$1 \text{ m} \doteq 39 \text{ in.}$ $1 \text{ m} \doteq 3\frac{1}{4} \text{ ft.}$	$1 \text{ yd.} \doteq 90 \text{ cm}$ $1 \text{ yd.} \doteq 0.9 \text{ m}$
$1 \text{ km} \doteq \frac{6}{10} \text{ mi.}$	$1 \text{ mi.} \doteq 1.6 \text{ km}$

A closer look at the imperial unit:

Look at the section between the "5" and the "6". The lengths of these lines differ and indicate different fractions or parts of an inch.

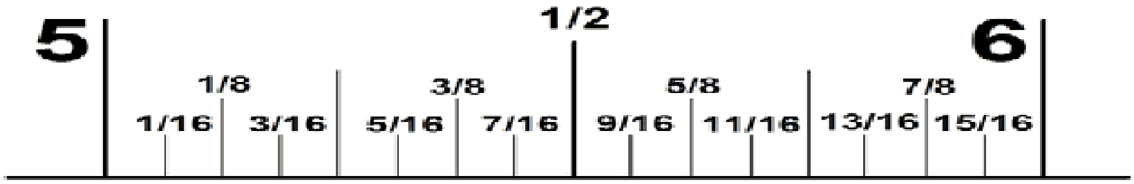


$5\frac{5}{8}$



$5\frac{7}{16}$

Imperial units can be divided in half again (and again).



Appropriate Units of Measurement

Which units of measurement are appropriate for each of the following (do not measure)

		Metric	Imperial
(a)	Your calculator	cm.	in.
(b)	Your height	m.	ft.
(c)	Length of classroom	m.	ft./yd.
(d)	Distance from Wpg. To Brandon	km.	mi.
(e)	A computer screen	cm.	in.
(f)	Thickness of math text book	mm.	in.
(g)	Distance from home to school	km.	mi.
(h)	A paper clip	mm	in.

A Golden Ratio Activity

A GOLDEN GREEK FACE

Toolbox: Calculator; metric ruler (measures to mm)

Statues of human bodies considered most perfect by the Greeks had many Golden Ratios. It turns out that the "perfect" (to the Greeks) human face has a whole flock of Golden Ratios as well.

You'll be measuring lengths on the face of a famous Greek statue (with a broken nose) by using the instructions on this page. Before you start, notice that near the face on the second page are names for either a location on the face or a length between two places on the face. Lines mark those lengths or locations exactly.

Using your cm/mm ruler and the face picture on the next page, find each measurement below to the nearest millimeter, that is tenth of a cm or .1 cm. Remember, you are measuring the **distance** or **length** between the **two locations** mentioned. You can use the marking lines to place the ruler for your measurements. Fill in this table.

a = Top-of-head to chin	= ___.__ cm
b = Top-of-head to pupil	= ___.__ cm
c = Pupil to nosetip	= ___.__ cm
d = Pupil to lip	= ___.__ cm
e = Width of nose	= ___.__ cm
f = Outside distance between eyes	= ___.__ cm
g = Width of head	= ___.__ cm
h = Hairline to pupil	= ___.__ cm
i = Nosetip to chin	= ___.__ cm
j = Lips to chin	= ___.__ cm
k = Length of lips	= ___.__ cm
I = Nosetip to lips	= ___.__ cm

Now **use these letters** and go on to the next page to compute **ratios** with them with your calculator. Remember: $\frac{a}{g}$, the first proportion, means divide measurement ***a*** by measurement ***g***. Round your answer to **three decimal places.**

Now, find these ratios to three decimal places,
using your calculator:

$$\frac{a}{g} = \frac{7.4 \text{ cm}}{5 \text{ cm}} = 1.48$$

$$\frac{b}{d} = \frac{3.7 \text{ cm}}{2.2 \text{ cm}} = 1.68$$

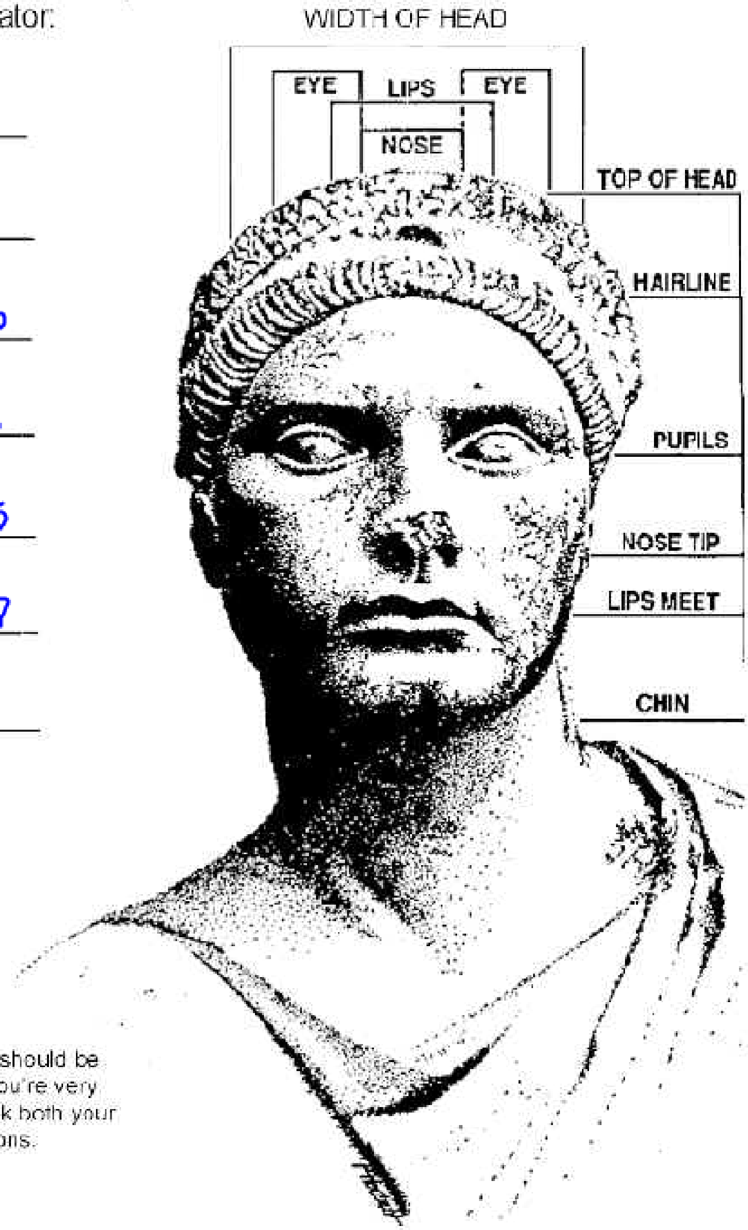
$$\frac{i}{j} = \frac{2.3 \text{ cm}}{1.5 \text{ cm}} = 1.53$$

$$\frac{i}{c} = \frac{2.3 \text{ cm}}{1.4 \text{ cm}} = 1.64$$

$$\frac{e}{l} = \frac{1.4 \text{ cm}}{0.8 \text{ cm}} = 1.75$$

$$\frac{f}{h} = \frac{3.9 \text{ cm}}{2.2 \text{ cm}} = 1.77$$

$$\frac{k}{e} = \frac{2.3 \text{ cm}}{1.4 \text{ cm}} = 1.64$$



Your answers to the above ratios should be near the Golden Ratio, 1.618. If you're very far off on any one of them, recheck both your measurements and your calculations.

Measurement Conversions

A. Metric System

1 km = 1000 m

1 m = 1000 mm

1 m = 100 cm

1 cm = 10 mm

Examples:

a) 30 cm = 300 mm $30(10)=300$

b) 240 cm = 2.4 m $\frac{240}{100}=2.4$

c) 960 m = 0.96 km $\frac{960}{1000}=0.96$

d) 3.4 km = 3400 m $3.4(1000)=3400$

e) 0.5 km = 500,000 mm $(0.5\times1000)\times1000=500{,}000$

B. Imperial System

1 foot = 12 inches
1 yard = 3 feet
1 mile = 1760 yards

*note: we cannot use decimals as readily as the metric system

Examples:

a) 4 ft. = 48 in. $4(12)=48$

b) 12 ft. = 4 yd $\frac{12}{3}=4$

c) 2.5 yd. = 90 in $2.5\times3\times12=90$

d) 9 in. = $\frac{3}{4}$ ft. $\frac{9}{12}=0.75$

e) 2 ft. 7 in = 31 in $2(12)+7=24+7=31$

f) 252 in = 7 yd $\frac{252}{12}=21$ $\frac{21}{3}=7$

C. Conversions from metric to imperial and imperial to metric

Examples:

a) 1 mile = 1.6 km

b) 1 yd. = 0.9 m

c) 1 ft. = 0.3 m

d) 1 in. = 2.5 cm

D. Area and Volume Conversions

Examples:

a) $1\ m^2 =$ ______ cm^2

b) $2.6\ m^2 =$ ______ mm^2

c) $3.2\ cm^2 =$ ______ mm^2

d) $2345\ mm^2 =$ ______ cm^2

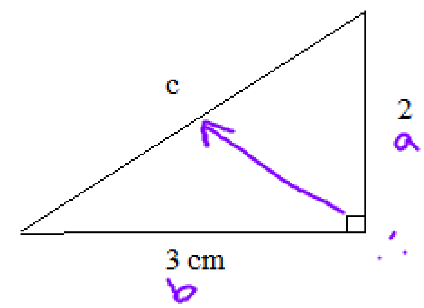
e) $1\ m^3 =$ ______ mm^3

f) $3.2\ m^3 =$ ______ cm^3

Perimeter and Area Review

<u>**PERIMETER**</u> of a polygon: <u>ADD ALL THE SIDE LENGTHS TOGETHER</u>

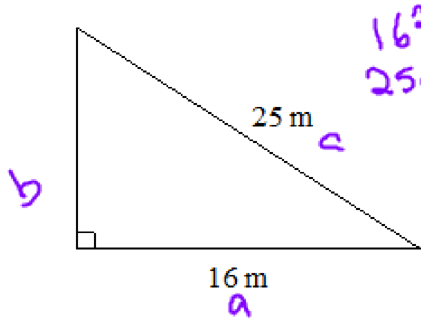
Example 1: Right triangle missing the hypotenuse.
Find the perimeter.



$a^2+b^2=c^2$
$2^2+3^2=c^2$
$4+9=c^2$
$\sqrt{13}=\sqrt{c^2}$
$c=3.6$

$\therefore P=2+3+3.6$
$=8.6$ cm

Example 2: Right triangle missing a side.
Find the perimeter.

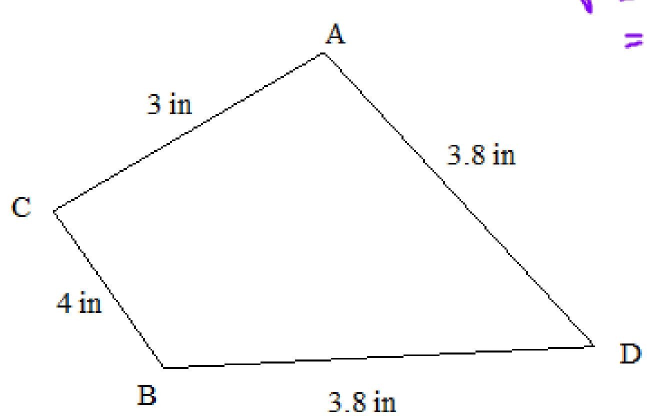


$16^2+b^2=25^2$
$256+b^2=625$
$b^2=625-256$
$b^2=369$
$b=\sqrt{369}$
$b=19.2$

$\therefore P=25+16+19.2$
$=60.2$ m

Example 3: Find the perimeter of quadrilateral ADBC.

$P=3+3.8+3.8+4$
$=14.6$ in.



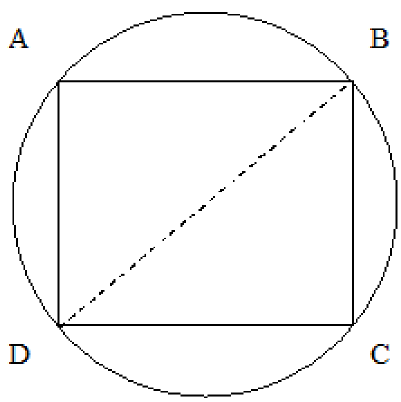
Recall: Perimeter of a circle is called the **CIRCUMFERENCE.**

NOTE: Use the π key on your calculator, not 3.14.

$C = 2\pi r$

Example 4: Perimeter of circles

a. Find the circumference of the circle with center E. Given radius = 4 m.

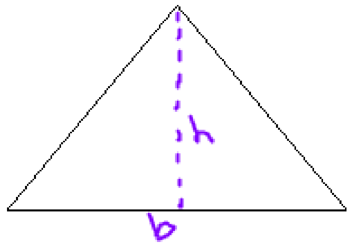


$C = 2\pi(4)$
$= 8\pi$
$= 25.13\text{m}$

b. Find the perimeter of ABCD given that ABCD is a square.

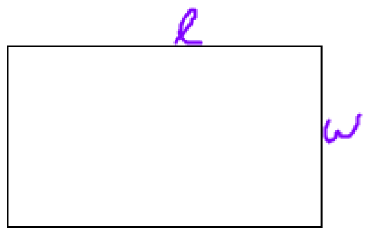
AREA

TRIANGLE:



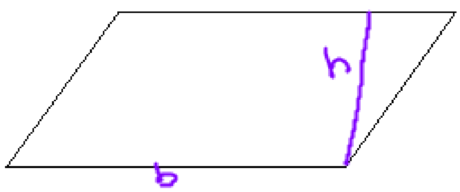
$A = \frac{bh}{2}$

RECTANGLE:



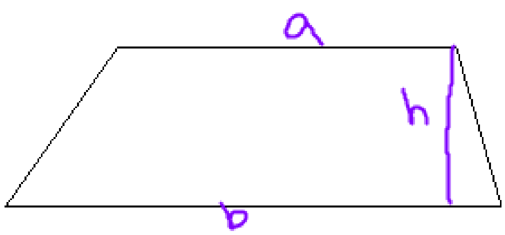
$A = lw$

PARALLELOGRAM:



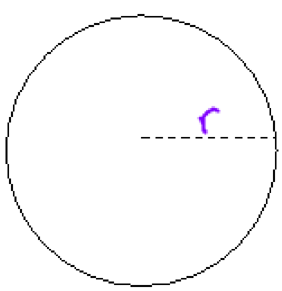
$A = bh$

TRAPEZOID:



$A = \frac{(a+b)h}{2}$

CIRCLE:

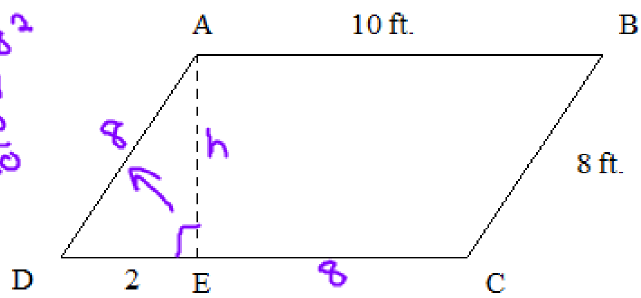


$A = \pi r^2$

Example 5: Finding area.

a. Find the area of parallelogram ABCD.

$a^2+b^2=c^2$
$2^2+h^2=8^2$
$4+h^2=64$
$h^2=60$
$h=\sqrt{60}$
$h=7.7$

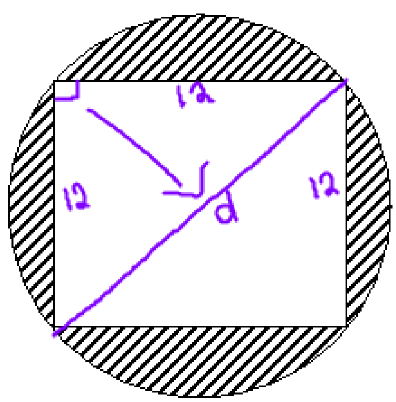


$A=bh$
$A=(10)(7.7)$
$A=77ft^2$

b. Find the area of triangle ADE.

$A=\frac{bh}{2}=\frac{2(7.7)}{2}=7.7ft^2$

Example 6: Given a square in a circle with sides 12 cm. Find the area that is shaded.

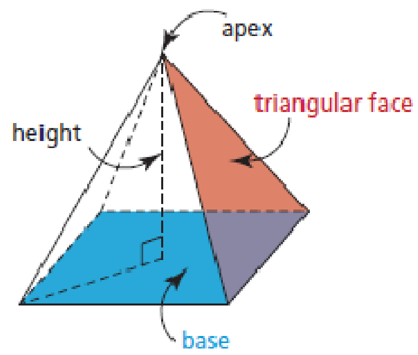


$A=\pi r^2-lw$
$=\pi(8.5)^2-(12)(12)$
$=227-144$
$=\boxed{83\ cm^2}$

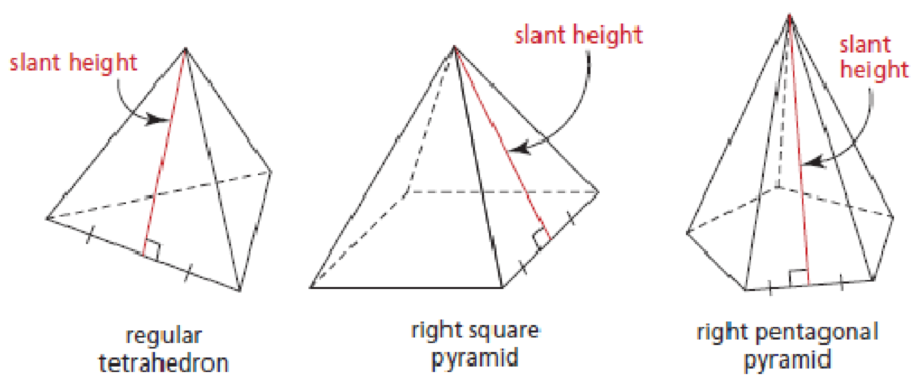
$12^2+12^2=d^2$
$144+144=d^2$
$288=d^2$
$\sqrt{288}=d$
$17=d$ $\therefore r=\frac{17}{2}=8.5$

Surface Area of Right Pyramids

A right pyramid is a 3-D object that has triangular faces and a base that is a polygon. The shape of the base determines the name of the pyramid. The triangular faces meet at a point called the **apex.** The *height* of the pyramid is the perpendicular distance from the apex to the center of the base.



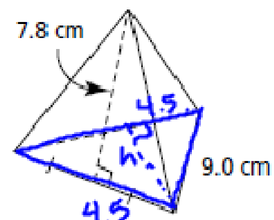
When the base of a right pyramid is a regular polygon, the triangular faces are congruent. Then the **slant height** of the right pyramid is the height of a triangular face.



To find the surface area of a right pyramid, add the areas of the triangular faces and the base.

Example 1: Determining the Surface Area of a Regular Tetrahedron Given Its Slant Height

Determine the surface area of the regular tetrahedron below.



$SA = \text{base} + 3 \text{ sides}$

$= \frac{9(7.8)}{2} + 3\left(\frac{9(7.8)}{2}\right)$

$= 35.1 + 105.3$

$= 140.4 \text{ cm}^2$

$a^2+b^2=c^2$

$4.5^2+h^2=9^2$

$20.25+h^2=81$

$h^2=60.75$

$h=7.8$ cm

Example 2: Determining the Surface Area of a Right Rectangular Pyramid

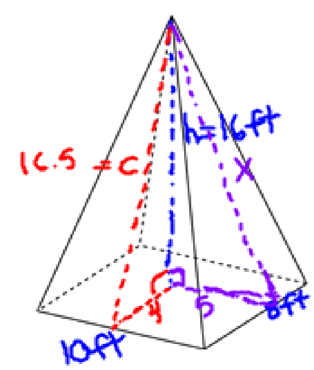
A right rectangular pyramid has base dimensions 8 ft. by 10 ft., and a height of 16ft. Calculate the surface area of the pyramid to the nearest foot.

$SA = \text{base} + \underset{\text{front + back}}{\Delta} + \underset{\text{Left + Right}}{\Delta}$

$= (10)(8) + 2\left(\frac{10(16.5)}{2}\right) + 2\left(\frac{8(16.8)}{2}\right)$

$= 80 + 165 + 134.4$

$= \boxed{379.4 \text{ ft}^2}$



Front Δ

$4^2+16^2=c^2$

$16+256=c^2$

$272=c^2$

$16.5=c$

Right Δ

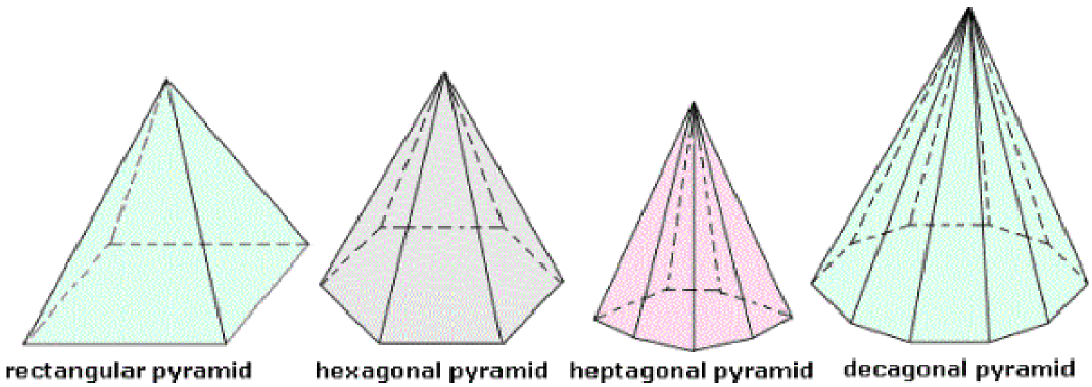
$16^2+5^2=x^2$

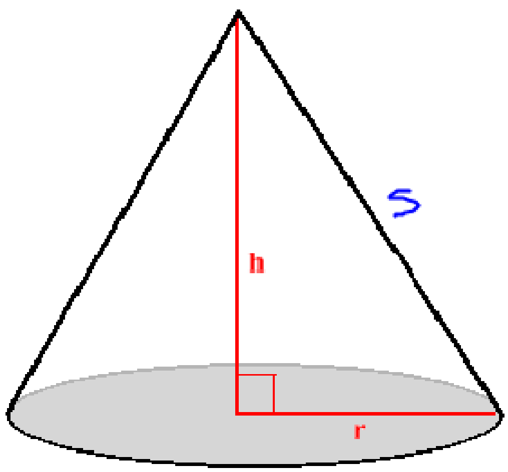
$256+25=x^2$

$281=x^2$

$16.8=x$

Surface Area of Right Cones



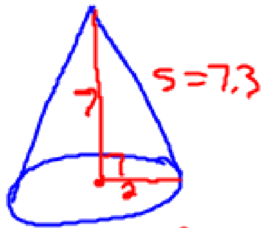


$$\boxed{S.A. = \pi r s + \pi r^2}$$

Find s using $a^2+b^2=c^2$
or: $h^2+r^2=s^2$

Example 3: Determining the Surface Area of a Right Cone

A right cone has a base radius of 2 ft. and a height of 7 ft. Calculate the surface area of this cone to the nearest square foot.



$SA = \pi r s + \pi r^2$

$= \pi(2)(7.3) + \pi(2)^2$

$= 45.87 + 12.57$

$= 58.44 \text{ ft}^2$

$\boxed{= 58 \text{ ft}^2}$

$7^2 + 2^2 = s^2$

$49 + 4 = s^2$

$53 = s^2$

$7.3 = s$

Example 4: Determining an Unknown Measurement

$r = 5$

The lateral area of a cone is 220 cm^2. The diameter of the cone is 10 cm. Determine the height of the cone to the nearest tenth of a centimeter.

area w/o base

$SA = \underbrace{\pi r s}_{\text{lateral area}} + \underbrace{\pi r^2}_{\text{circle base}}$

$\pi r s = 220$

$\pi(5)s = 220$

$s = \dfrac{220}{5\pi}$

$s = 14 \text{ cm}$

Find h.

$14^2 = h^2 + 5^2$

$196 = h^2 + 25$

$171 = h^2$

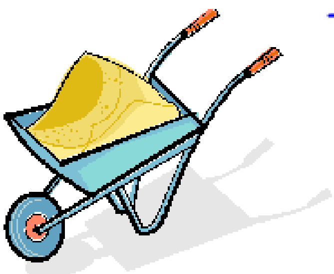
$\boxed{h = 13.1 \text{ cm}}$

Volumes of Right Pyramids, Right Prisms and Right Cones

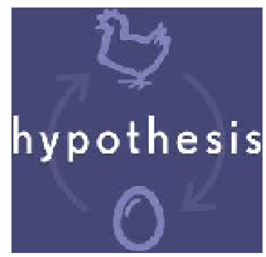
Sandbox Investigation

Fermi Problem: How much sand would it take to fill Grand Beach?

$\frac{1}{2}$ km^3



PREDICTIONS



1. Do you believe there are enough bags of sand to fill the sandbox? Why or why not?

   Yes

   a. If you believe there are not enough bags of sand to fill the sandbox, how many more bags do you think you will need?

   b. If you believe there are more than enough bags of sand to fill the sandbox, how many bags do you believe will be left over?

2. What information do you need to find out how much sand is needed to fill the sandbox? Record this information below.

   Length & Width & Height of Sandbox

   40" x 6" x 42"

   37" x 6" x 42"

   How much sand is in the bag. → 0.55 ft^3

CALCULATIONS

3. Calculate how much sand is needed to fill the sandbox.

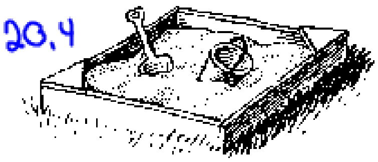
$3\frac{1}{3} \times \frac{1}{2} \times 3\frac{1}{2} = 5.8 \text{ ft}^3$

$3\frac{1}{10} \times \frac{1}{2} \times 3\frac{1}{2} = 5.4 \text{ ft}^3$

$\overline{\quad 11.2 \text{ ft}^3}$

$11.2 \div 0.55 = 20.4$

≈ 21 bags

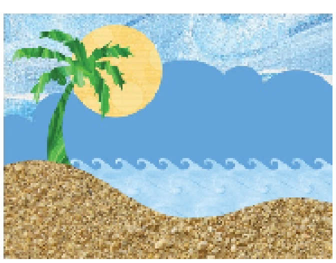


4. Are there enough bags of sand to fill the sandbox? Explain.

   a. If there are more than enough bags to fill the sandbox, record how many.

   b. If you need more bags of sand to fill the sandbox, record how many.

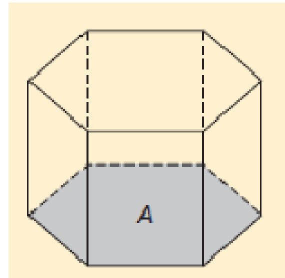
5. After seeing the conclusion to the video, how accurate were your calculations? What could have contributed to your calculations being inaccurate?



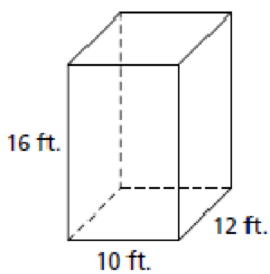
The volume of a **right prism** is:

Volume = (base area)(height)

Volume = A(h)



Example 1: Determining the Volume of a Right Rectangular Prism
Determine the volume of the following prism.

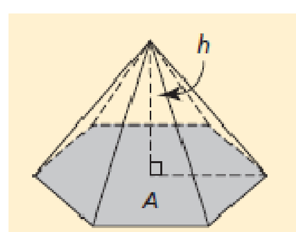


$V = lwh$
$= 16(10)(12)$
$= 1920 \text{ ft}^3$

The volume of a **right pyramid** is:

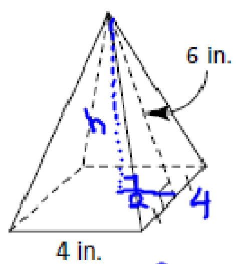
Volume = $\frac{1}{3}$(base area)(height)

Volume = $\frac{1}{3}$A(h)



Example 2: Determining the Volume of a Right Square Pyramid Given Its Slant Height

Calculate the volume of this right square pyramid to the nearest cubic inch.



$V = \frac{1}{3}(\text{base})(\text{height})$

$V = \frac{1}{3}(4 \times 4)(5.7)$

$V = 30.4 \text{ in}^3$

$6^2 = h^2 + 2^2$

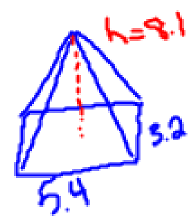
$36 = h^2 + 4$

$32 = h^2$

$\boxed{5.7 = h}$

Example 3: Determining the Volume of a Right Rectangular Pyramid

Determine the volume of a right rectangular pyramid with base dimensions 5.4 cm by 3.2 cm and height 8.1 cm. Answer to the nearest tenth of a cubic centimeter.



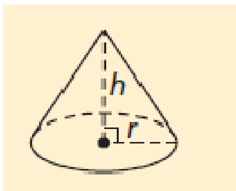
$V = \frac{1}{3}(\text{base})(\text{height})$

$= \frac{1}{3}(5.4 \times 3.2)(8.1)$

$= 46.7 \text{ cm}^3$

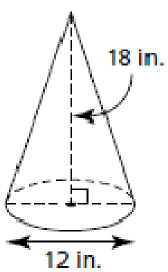
The volume of a right cone with base radius r and height h has volume:

$$\boxed{V = \frac{1}{3}\pi r^2 \text{h}}$$



Example 4: Determining the Volume of a Cone

Determine the volume of this cone to the nearest cubic inch.



$V = \frac{1}{3}\pi r^2 h$ $\quad r = 6$ in, $h = 18$

$= \frac{1}{3}\pi (6^2)(18)$

$= 679 \text{ in}^3$

Example 5: Determining an Unknown Measurement

A cone has a height of 4 yd. and a volume of 205 cubic yards. Determine the radius of the base of the cone to the nearest yard.

$V = \frac{1}{3}\pi r^2 h$ $\quad V = 205 \quad h = 4 \quad r = ?$

$\frac{205}{4} = \frac{\frac{1}{3}\pi r^2 \cancel{(4)}}{\cancel{4}}$

$(3)\ 51.25 = \cancel{\frac{1}{3}}\pi r^2 \cancel{(3)}$

$\frac{153.75}{\pi} = \frac{\pi r^2}{\pi}$

$\sqrt{r^2} = \sqrt{48.94}$

$r = 6.99$

$\therefore r = 7$ yds.

Volumes of Right Cylinders

The formula for the volume of a right cylinder is: πr^2h.

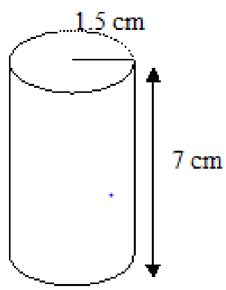
r = radius

h = height

Example 1: Volume of Cylinders

Find the volume of the following cylinder.

$V = \pi r^2 h$



$r = 1.5$ cm

$h = 7$ cm

$V = \pi (1.5)^2 (7)$

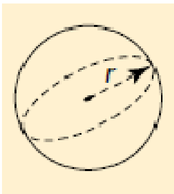
$= 49.5 \text{ cm}^3$

Surface Area and Volume of a Sphere

The surface area of a Sphere with radius *r* is:

$\boxed{SA = 4\pi r^2}$

r = radius



Example 1: Determining the Surface Area of a Sphere

The diameter of a baseball is approximately 3 in. Determine the surface area of a baseball to the nearest square inch.

$D = 3\text{ in}$ $\quad R = 1.5\text{ in}$

$SA = 4\pi r^2$

$= 4\pi(1.5)^2$

$= 28\text{ in.}^2$

Determining the Diameter of a Sphere

The surface area of a lacrosse ball is approximately 20 square inches. What is the diameter of the lacrosse ball to the nearest tenth of an inch?

$SA = 4\pi r^2$

$\frac{20}{4\pi} = \frac{\cancel{4\pi} r^2}{\cancel{4\pi}}$

$1.59 = r^2$

$\sqrt{1.59} = r$

$r = 1.3\text{ in}$

$\therefore$ diameter $= 1.3(2)$

$= 2.6\text{ in}$

The volume, V, of a sphere with radius r is:

$$V = \frac{4}{3}\pi r^3$$

Example 3: Determining the Volume of a Sphere

The sun approximates a sphere with diameter 870 000 miles. What is the approximate volume of the sun?

$V = \frac{4}{3}\pi r^3 \qquad r = 435{,}000$

$V = \frac{4}{3}\pi(435{,}000)^3$

$= 3.4 \times 10^{17}$ miles^3

Example 4: Determining the Surface Area and Volume of a Hemisphere

A hemisphere has radius 8.0 cm.

a. What is the surface area of the hemisphere to the nearest tenth of a square centimeter?

$SA = (4\pi r^2)\left(\frac{1}{2}\right) + \pi r^2$

$= 2\pi r^2 + \pi r^2$

$= 3\pi r^2$ ← SA of hemisphere

$= 3\pi(8)^2$

$= 603.2$ cm^2

*need half SA of a sphere plus area of a circle

b. What is the volume of the hemisphere to the nearest tenth of a cubic centimeter?

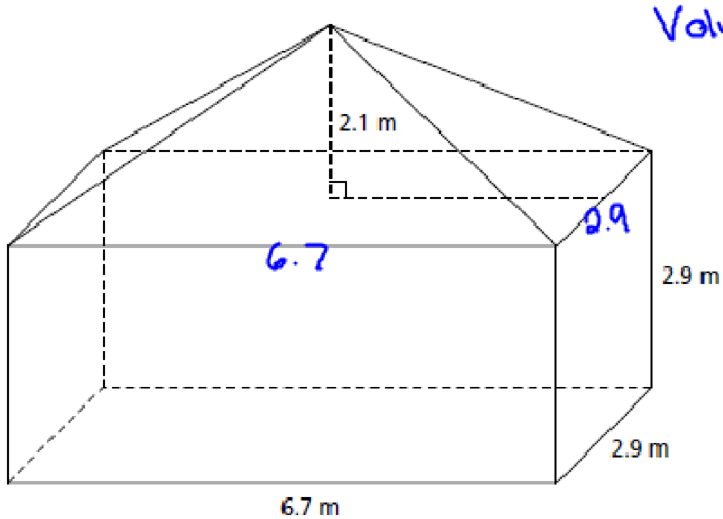
$V = \left(\frac{4}{3}\pi r^3\right)\left(\frac{1}{2}\right)$

$= \frac{4}{3}\pi(8)^3\left(\frac{1}{2}\right)$

$= 1072.3$ cm^3

Solving Problems with Surface Area and Volume

Example 1: Determining the Volume of a Composite Object
Determine the volume of this composite object to the nearest tenth of a cubic meter.



Volume = roof + box

$= \frac{1}{3}(\text{base})(\text{height}) + lwh$

$= \frac{1}{3}(6.7)(2.9)(2.1) + (6.7)(2.9)(2.9)$

$= 13.601 \quad + 56.347$

$= 69.948$

$\boxed{= 69.9 \text{ m}^3}$