Relations and Functions

5.1 Representing Relations

A **set** is a collection of distinct objects.

An **element** of a set is one object in the set.

A **relation** associates the elements of one set with the elements of another set.

One way to write a set is to list its elements inside brackets. For example, we can write the set of natural numbers from 1 to 5 as:

$\{1, 2, 3, 4, 5\}$ $= \{2, 4, 1, 3, 5\}$

Note: The order of the elements in the set does not matter.

You can represent relations using a table, an arrow diagram, as a set of ordered pairs.

Example 1: Representing a Relation Given as a Table

Fruit (x)	*Colour* (y)
Apple	Red
Apple	Green
Blueberry	Blue
Cherry	Red
Banana	Yellow

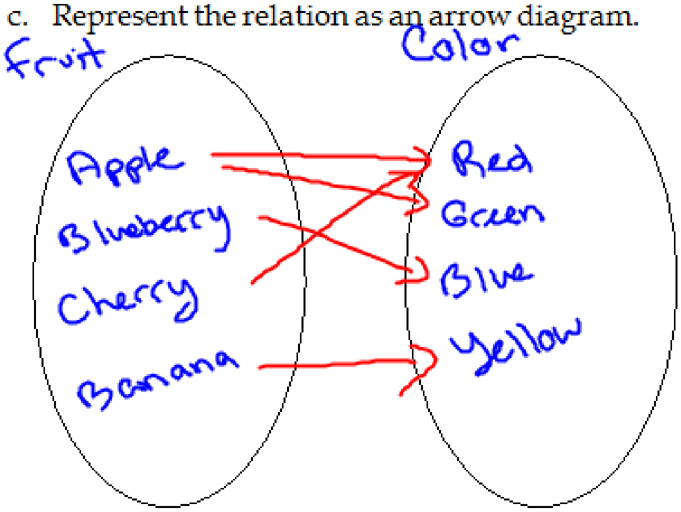
a. Describe the relation in words.

The relation is describing the color of each fruit.

b. Represent the relation as a set of ordered pairs.

{ (Apple, Red), (Apple, Green), (Blueberry, Blue), (Cherry, Red), (Banana, Yellow)}

c. Represent the relation as an arrow diagram.



Example 2: Representing a Numerical Relation Given as a Table

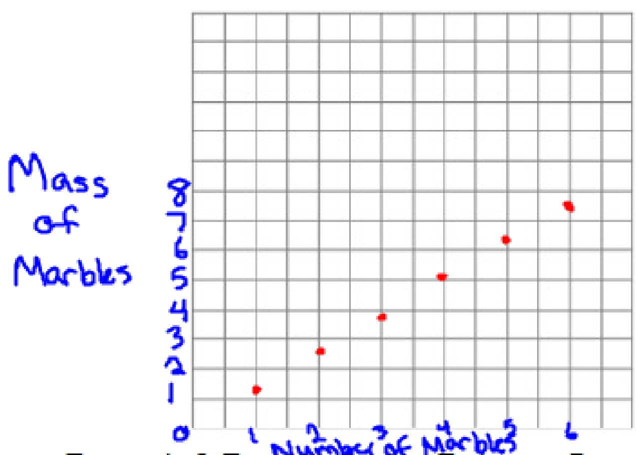
Number of Marbles (n) x	*Mass of Marbles in grams (m)* y
1	1.27
2	2.54
3	3.81
4	5.08
5	6.35
6	7.62

y x

a. Represent the relation as a set of ordered pairs.

{ (1, 1.27), (2, 2.54), (3, 3.81), (4, 5.08), (5, 6.35), (6, 7.62) }

b. Represent the relation in a graph.



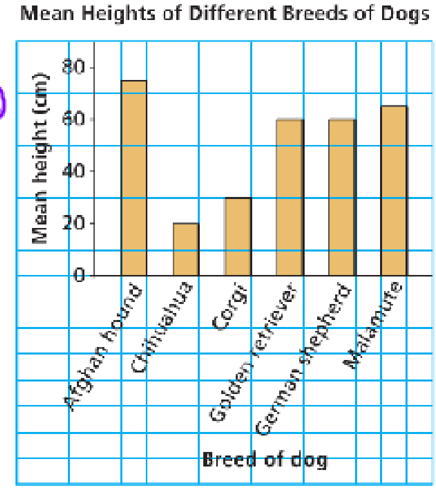
*Do not connect points b/c you cannot buy partial marbles.

Example 3: Representing a Relation Given as a Bar Graph

Represent this relation as a table

Breed of Dog	Mean Height (cm)
Afghan Hound	75
Chihuahua	20
Corgi	30
Golden Retriever	60
German Shepherd	60
Malamute	65

Mean Heights of Different Breeds of Dogs



Domain Range
Independent Dependent

5.2 Properties of Functions

IN	OUT
2	8
3	12
4	16
5 x4	20
6	24
7	28
Write the rule: Multiplying "IN" by 4.	

I	O
9	2
15	8
20	13
18	11
16	9
12	5
If I equals 24, what is O? $24-7=17$	

The **domain** (x) is the set of the first elements in a relation. It is also called the **input** value and is the **independent variable**.

The **range** (y) is the set of the second elements in a relation. It is also called the **output** value and is the **dependent variable**.

A **function** is a special type of relation where each element in the domain is associated with exactly one element in the range. In other words, if one value in the domain (x) gives two or more values in the range (y), then the relation is not a function.

When you are dating, you want to be in a "function"al relationship. Let the x-coordinate of the ordered pair by any person. Let the y-coordinate of the ordered pair be the x-coordinate's significant other. As you can see, Bob is dating both Jill and Sue. Therefore, this is not a "function"al relationship!

(Bob, Jill) Not a Function!
(Bob, Sue)

Example 1: Identifying Functions

For each relation below determine whether the relation is a function. If the relation is a function, identify the domain and range.

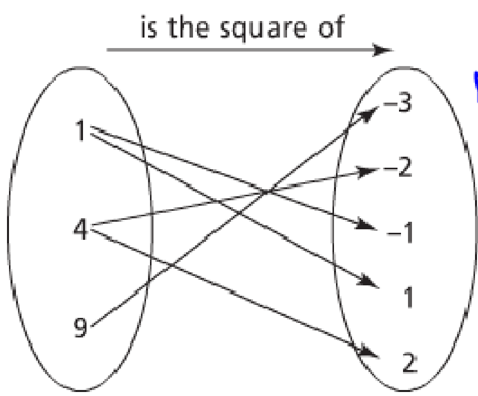
a. {(2, 0), (2, 5), (2, 1), (2, -5), (2, -3)}

Not a Function
2 is matched with multiple values.

b. {(5, 3), (4, 3), (-3, 3), (0, 5), (-3, -1)}

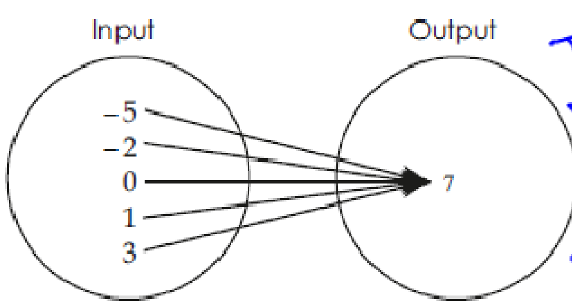
Not a function
-3 is matched with 2 different values

c.



Not a function.
Both 1 & 4 are matched w/ multiple values on the right.

d.



This is a function.
Domain:
{-5, -2, 0, 1, 3}
Range:
{ 7 }

Conclusion: All functions are relations but not all relations are functions.

Example 2: Identifying Functions from Tables of Values
For the table of values, write the values in the table as a set of ordered pairs. If the relation is a function, determine the domain and the range.

x	y
2	6
5	-7
-2	0
1	1
-8	5

$\{(2,6), (5,-7), (-2,0), (1,1), (-8,5)\}$
This is a function.
Domain: $\{2, 5, -2, 1, -8\}$
Range: $\{-7, 0, 1, 5, 6\}$

Example 3: Describing Functions

The table shows the masses, m, of different numbers of identical marbles, n.

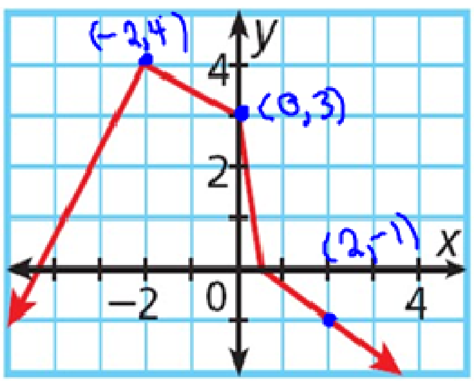
Number of Marbles (n)	*Mass of Marbles in grams (m)*
1	1.27
2	2.54
3	3.81
4	5.08
5	6.35
6	7.62

a. Why is this relation also a function?
All values of marbles have different masses.

b. Identify the independent variable and the dependent variable.
marbles mass

Puzzle Time!
Can you figure out what this notation means in relation to the graph below?

$f(-2) = 4$ $\quad\quad\quad$ $f(-1) = 3.5$
$f(0) = 3$ $\quad\quad\quad$ $f(2) = -1$



$f(x) = y$

Functional notation is used to represent a relation as a formula.

When the input of a function is x, the output value, y, is a function of x. In other words, the output value y depends on the input value x. Functional notation is another way of displaying this relationship.

x is the input value.

"f" is the name of the function.

$y = 3x - 1$

$f(x) = 3x - 1$

$f(x)$ is read as "f of x" and is equivalent to y.

$f(x)$ is the output value.

For example, the notation $f(2) = 5$ says that the point with coordinates $(2, 5)$ is on the graph of $f(x)$. Note: $f(2) = 5$ does not mean $2f = 5$, $f(2) = 5$ means $y = 5$.

Example 4: Using Function Notation
Evaluate each of the following algebraically.

a. $\boxed{f(x) = x^2}$ → This is your function.

$f(2) = 2^2 = 4$

$f(-1) = (-1)^2 = 1$

b. $\boxed{f(x) = 2x + 5}$ ← This is our new function.

$f(7) = 2(7) + 5$
$= 14 + 5 = 19$

$f(-3) = 2(-3) + 5$
$= -6 + 5$
$= -1$

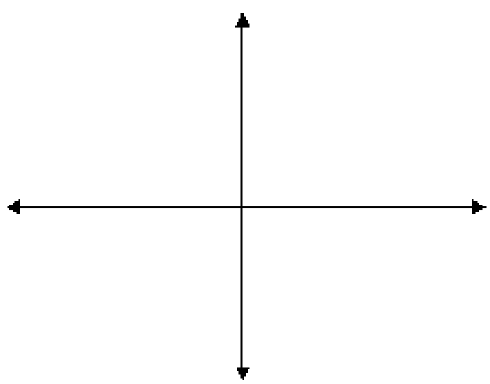
Determine the value that makes $f(x) = 17$.

We need to find x

$17 - 5 = 2x + 5 - 5$ $\qquad \frac{12}{2} = \frac{2x}{2}$ $\qquad \therefore \boxed{x = 6}$

Example 5: Graphing Using Function Notation
Using a graphing calculator, graph each of the following. Create a table of values for each and find the missing functional values using the calculator.

a. $f(x) = 2x + 5$



x	$f(x)$

x	$f(x)$
-2.5	
0.1	
3.3	

Example 6: Using Function Notation to Determine Values

The equation $V = -0.08d + 50$ represents the volume, V litres, of gas remaining in a vehicle's tank after travelling d kilometres. The gas tank is not refilled until it is empty.

a. Describe the function. Write the equation in function notation.

Volume depends upon distance ∴ Volume is a function of distance

$V(d) = -0.08d + 50$

b. Determine the value of $V(600)$. What does this number represent?

$V(600)$ means d is 600, or we travelled 600 km.

$V(600) = -0.08(600) + 50$
$= -48 + 50$
$= 2$

∴ After travelling 600 km, we have 2 L of gas left.

c. Determine the value of d when $V(d) = 26$. What does this number represent?

$V(d) = -0.08d + 50$

$V(d) = 26$

$26 = -0.08d + 50$
$-50 \qquad\qquad -50$

$\frac{-24}{-0.08} = \frac{-0.08d}{-0.08}$

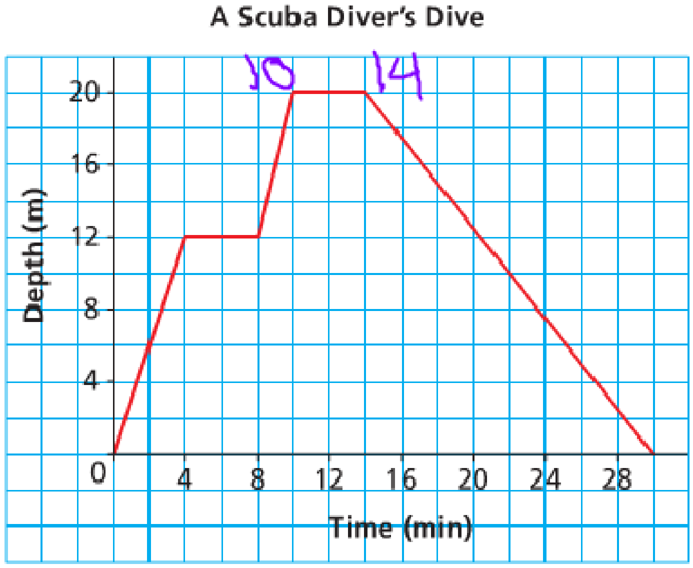
$\frac{-24}{-0.08} = d$

$\therefore d = 300$

You can travel 300 km & have 26 L of gas left.

Hwk
pg 270 # 4, 6a,b, 7c,d, 9b, 14b,c, 15b, 18

5.3 Interpreting and Sketching Graphs



How many minutes did the dive last?

30min

At what times did the diver stop her descent?

4min & 10min

What was the greatest depth the diver reached? For how many minutes was the diver at that depth?

20m for 4 min

What do the two horizontal lines in the graph mean?

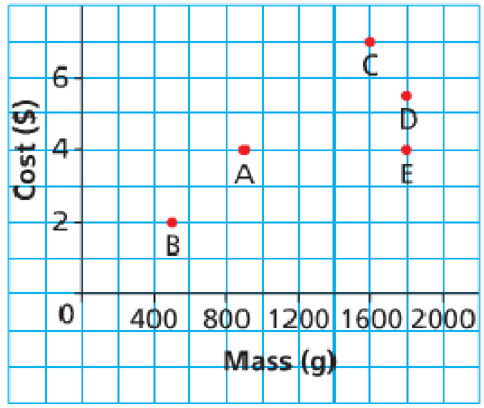
The diver stayed at a constant depth.

Note: Straight lines are used in graphs when the change in the variables are **constant**. Curves are used when the change in the independent and dependent variable is **not constant (e.g. acceleration).**

Example 1: Interpreting a Graph

Each point on this graph represents a bag of popping corn.

Costs and Masses of Various Bags of Popcorn



a. Which bag is the most expensive? What does it cost?

C → costs $7

b. Which bag has the least mass? What is this mass?

B → 500g

c. Which bags have the same mass? What is this mass?

D & E → 1800g

d. Which bags cost the same? How much do they cost?

A & E → $4

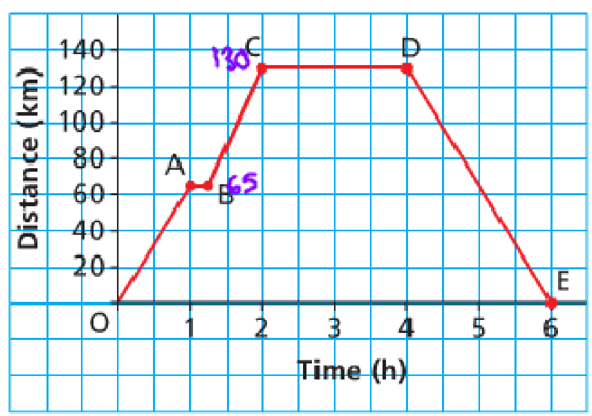
e. Which of bags C or D has the better value for money?

D → cheaper & more popcorn

Example 2: Describing a Possible Situation for a Graph

Describe the journey for each segment of the graph.

Day Trip from Winnipeg to Winkler, Manitoba



OA → Travel 65km to Winkler in 1 hour

AB → Stopped for 15 min to get gas & food.

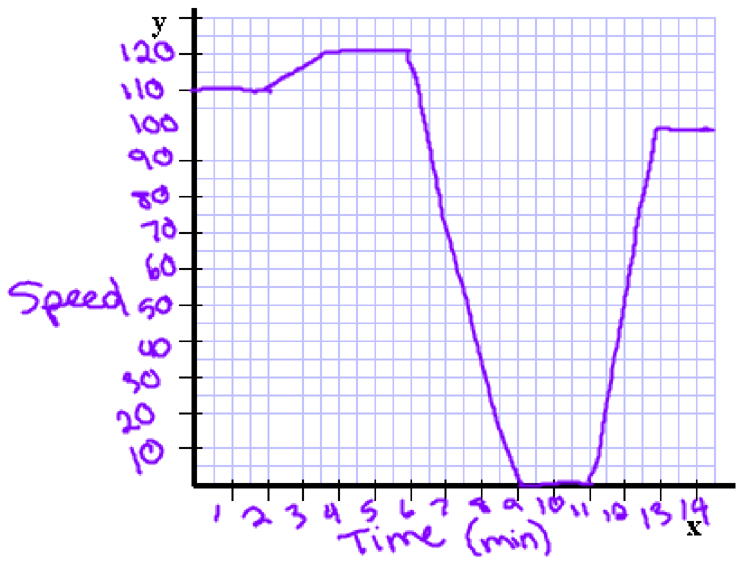
BC → Travel 65 km more to Winkler in 45 mins

CD → He stays & visits Winkler for 2 hours

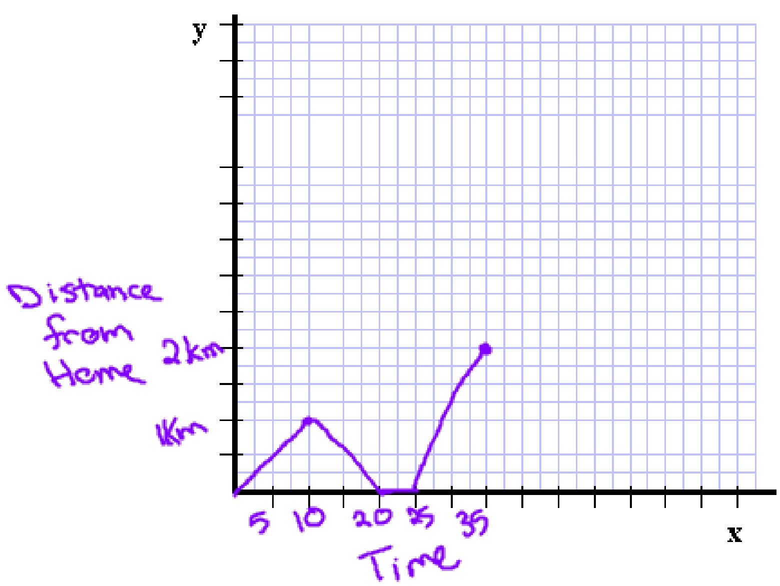
DE → He travels 130 km back home in 2 hours.

Example 3: Sketching a Graph for a Given Situation

a. Sketch a graph of the following situation. Use speed as a function of time.
A car is travelling along at a constant speed, then speeds up to pass a car only to maintain the new speed. The driver is then pulled over and given a ticket. He then continues his journey at a constant legal speed.
What are the independent and dependent variables?



b. Sketch a graph of the following situation. Use distance from home as a function of time.
A girl leave home for school and gets half way there but realizes that she forgot her cell phone. She returns home, and then goes to school.
What are the independent and dependent variables?



5.5 Part 2: Domain and Range

The **domain** (x) is the set of the first elements in a relation. It is also called the **input** value and is the **independent variable**.

The **range** (y) is the set of the second elements in a relation. It is also called the **output** value and is the **dependent variable**.

Domain and Range: Set Notation

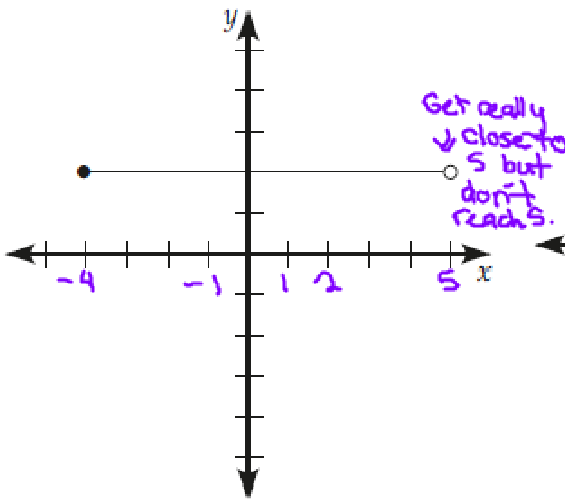
Set notation describes which values are included in the domain and range using the following symbols:

less than:	$<$
greater than:	$>$
less than or equal to:	$\leq$
greater than or equal to:	$\geq$
such that:	$\mid$
is an element of:	$\in$
the real number system:	$\Re$ → Whole, Natural, Integers, Fractions, Decimals

When points on a graph are indicated with solid dots (•), it implies the point is included. If a point is hollow (∘), the graph goes up to that point but the actual point is not included in the graph. A line with an arrow indicates the graph includes all points along the line, and it continues on indefinitely in the direction(s) indicated.

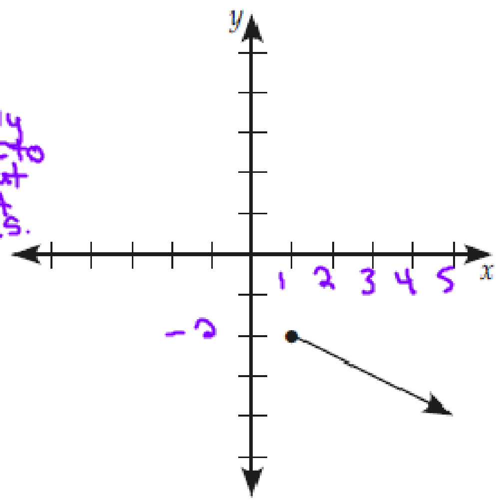
Example 2: Determining the Domain and Range of the Graph of a Function in Set Notation

Determine the domain and range of the following graphs in set notation.



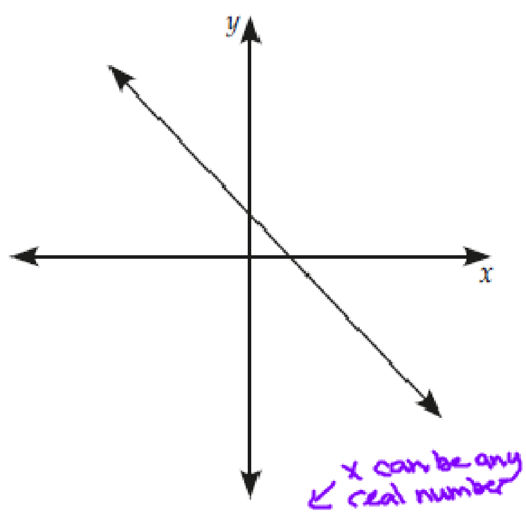
Domain: $-4 \le x < 5$

Range: $y = 2$



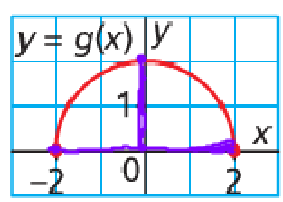
Domain: $x \ge 1$

Range: $y \le -2$



Domain: $x \in \mathbb{R}$

Range: $y \in \mathbb{R}$



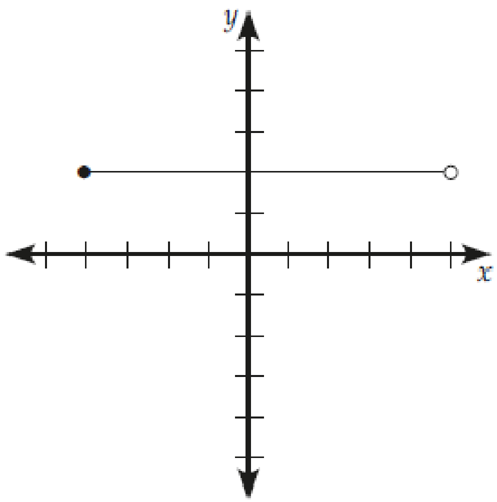
Domain: $-2 \le x \le 2$

Range: $0 \le y \le 2$

Note: In general, if you are describing the domain and range of a function, only use a list if the data is discrete or if you have unconnected data values on a graph.

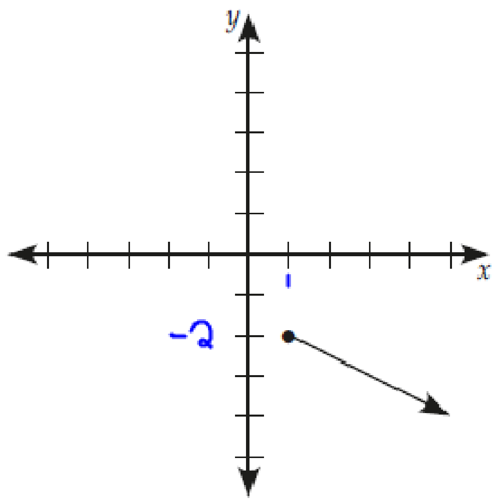
Example 2: Determining the Domain and Range of the Graph of a Function in Set Notation

Determine the domain and range of the following graphs in set notation.



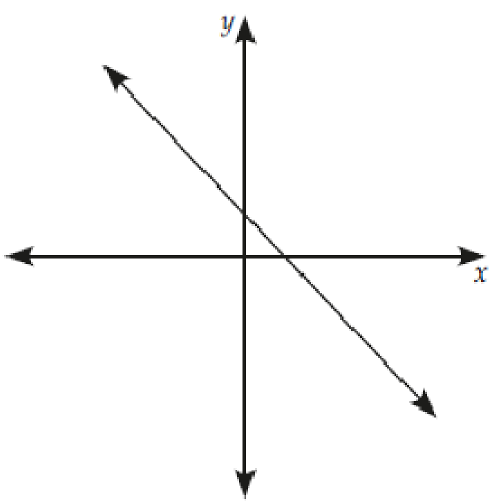
Domain: [−4, 5)

Range: [2]



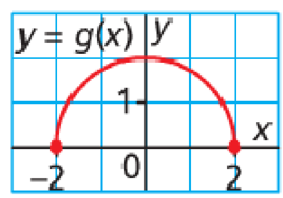
Domain: [1, ∞)

Range: ~~[−2, ∞)~~ (−∞, −2]



Domain: (−∞, ∞)

Range: (−∞, ∞)



Domain: [−2, 2]

Range: [0, 2]

Note: In general, if you are describing the domain and range of a function, only use a list if the data is discrete or if you have unconnected data values on a graph.

Domain and Range: Interval Notation

Interval notation describes the restrictions on the domain and range using different types of brackets:

starts at but does not include:	(
goes up to but does not include:	)
starts at and includes:	[
goes up to and includes:	]
the positive and negative infinity symbols:	∞ and $-\infty$

open hole $<$

closed hole $\leq$

$x \in \mathbb{R} \rightarrow (-\infty, \infty)$

Because infinity is not an actual value that you can include, the round brackets will always be used with the symbols ∞ and $-\infty$.

Example 3: Determining the Domain and Range of the Graph of a Function in Interval Notation

Rewrite each domain and range from Example 2 using interval notation.

Example 4: Determining Domain and Range Values from a Set of Ordered Pairs

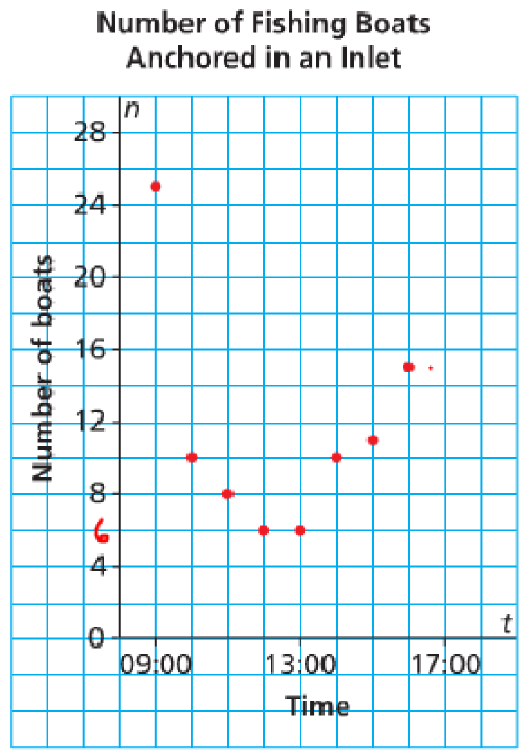
State the domain and range of the following relation.

A = {(1, 2), (3, 4), (5, 6), (7, 8)}

D: {1, 3, 5, 7}   R: {2, 4, 6, 8}

Example 5: Determining the Domain and Range of the Graph of a Situation

This graph shows the number of fishing boats, n, anchored in an inlet in the Queen Charlotte Islands as a function of time, t.



a. Identify the dependent variable and the independent variable.

I: time

D: # of boats

b. Why are the points on the graph not connected? Explain.

You cannot have partial boats in the inlet.

c. Determine the domain and range of the graph in set notation.

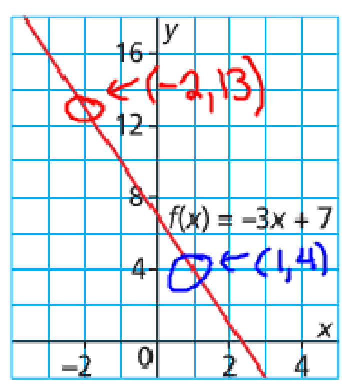
D: {9, 10, 11, 12, 13, 14, 15, 16}

R: {6, 8, 10, 11, 15, 25}

Example 6: Determining Domain and Range Values from the Graph of a Function

Here is a graph of the function $f(x) = -3x + 7$.

$y = -3x + 7$



a. Determine the range value when the domain value is -2.

x = -2 Find y.

∴ y = 13

b. Determine the domain value when the range value is 4.

y = 4 Find x.

∴ x = 1

5.6 Properties of Linear Relations

X	y
0	12
1	15
2	18
3	21
4	24
5	27
6	30

x	y
0	4
1	6
2	8
3	10
4	12
5	14
6	16

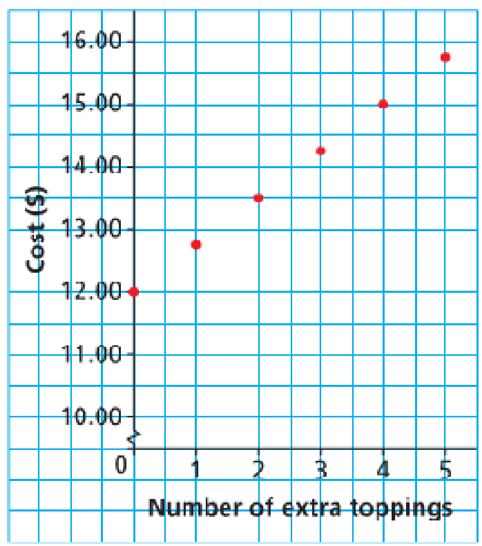
x	y
0	13
1	10
2	7
3	4
4	1
5	-2
6	-5

The table and graph below display the cost of a pizza related to the number of extra toppings on the pizza.

Number of Extra Toppings	Cost ($)
0	12.00
1	12.75
2	13.50
3	14.25
4	15.00
5	15.75

+0.75

Cost of a Pizza



What patterns do you see in the table?

Cost goes up \$0.75 for each topping.

Write a rule for the pattern that relates the cost of a pizza to the number of its toppings.

$C = 12 + 0.75t$

How are the patterns in the table shown in the graph?

Each dot is moved 1 to the right & 0.75 units up.

Conclusions: For a linear relation, a constant change in the independent variable results in a constant change in the dependent variable.

The graph of a linear relation is one of the form:

$$\boxed{y = mx + b}$$

where:
y is the dependent variable
x is the independent variable
b is the initial amount
m is the rate of change

The rate of change can be expressed as:

$$m = \frac{\text{change in the dependent variable}}{\text{change in the independent variable}} = \boxed{\frac{y_2 - y_1}{x_2 - x_1}}$$

The rate of change is constant, or stays the same, for a linear relation. The graph of a linear relation is a straight line.

Example 1: Determining whether a Table of Values Represents a Linear Relation

Which table of values represents a linear relation? Explain.

a. The relation between temperature in degrees Celsius, C, and temperature in degrees Fahrenheit, F.
b. The relation between the current I amps, and the power, P watts, in an electrical circuit.

a.

C	*F*
0	32
5	41
10	50
15	59
20	68

This is linear b/c each column is changing by a constant amount.

b.

I	*P*
0	0
5	75
10	300
15	675
20	1200

This is not linear b/c the 2nd column does not change by a constant amount.

Example 2: Determining Whether an Equation Represents a Linear Relation

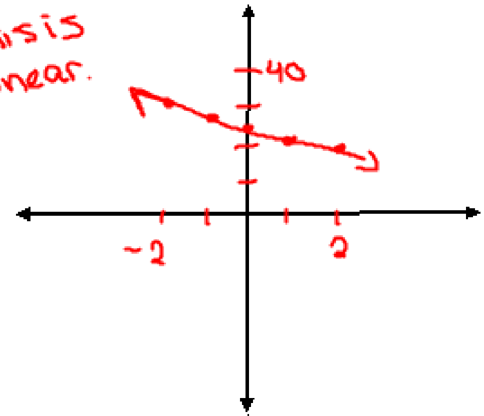
Graph each equation by creating a table of values. Determine if each equation represents a linear relation.

a. $y = -3x + 25$

x	y
-2	31
-1	28
0	25
1	22
2	19

$y=-3(-2)+25$
$y=-3(-1)+25$
$y=-3(0)+25$
$y=-3(1)+25$
$y=-3(2)+25$

This is linear.

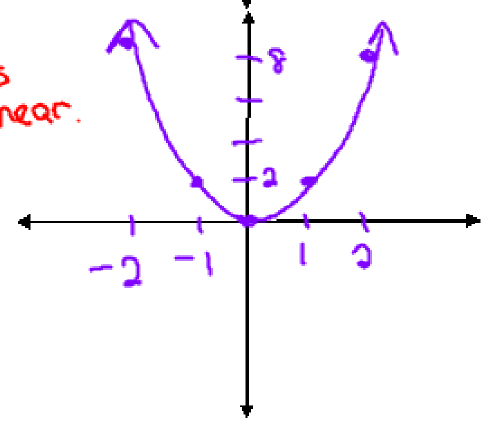


b. $y = 2x^2$

x	y
-2	8
-1	2
0	0
1	2
2	8

$y=2(-2)^2$
$y=2(-1)^2$
$y=2(0)^2$
$y=2(1)^2$
$y=2(2)^2$

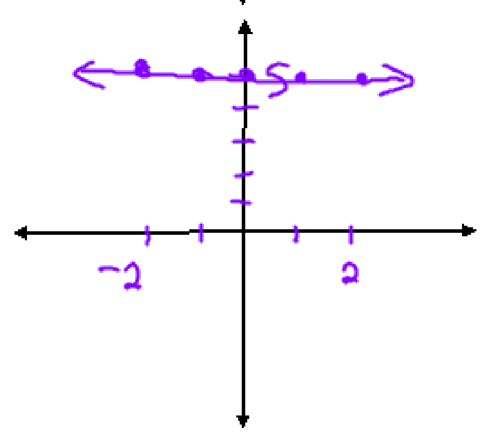
This is not linear.



c. $y = 5$

x	y
-2	5
-1	5
0	5
1	5
2	5

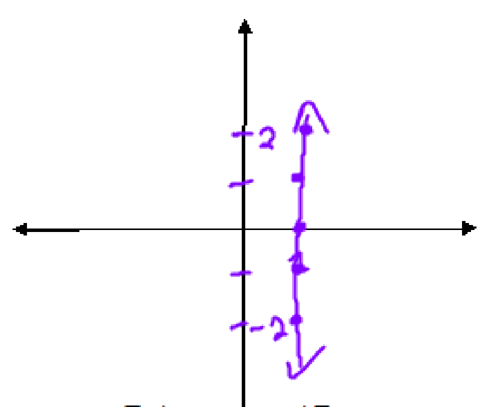
This is linear.



d. $x = 1$

x	Y
1	-2
1	-1
1	0
1	1
1	2

This is linear.



Example 3: Identifying a Linear Relation

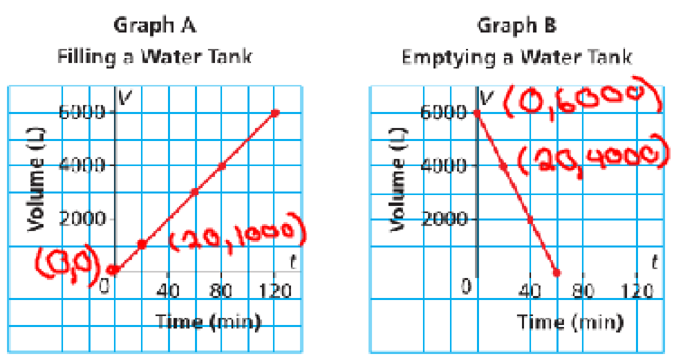
Which relation is linear? Explain.

a. A new car is purchased for $24 000. Every year, the car is worth 85% of what it was worth the previous year. The value is related to time.

b. For a service call, an electrician charges a $75 flat rate, plus $50 for each hour he works. The total cost for service is related to time.

Example 4 Determining the Rate of Change of a Linear Relation from its Graph

A water tank on a farm holds 6000 L. Graph A represents the tank being filled at a constant rate. Graph B represents the tank being emptied at a constant rate.



Determine the rate of change of each relation, then describe what it represents.

A Rate of Change = m

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{litres}}{\text{mins}}$

Find 2 points on the graph

(20,1000) & (0,0)

x_1, y_1 ; x_2, y_2

$m = \frac{0-1000}{0-20} = \frac{-1000}{-20}$

$m = 50$ L/min

B

$m = \frac{y_2 - y_1}{x_2 - x_1}$

1: (0,6000) 2: (20,4000)

x_1 y_1 ; x_2 y_2

$m = \frac{4000-6000}{20-0} = -\frac{2000}{20} = -100$

$m = -100$ L/min

$\boxed{\text{pg. 308 \# 3, 5, 6i, iii, v, 7, 12, 14, 15}}$

Example 3: Identifying a Linear Relation

Which relation is linear? Explain.

a. A new car is purchased for \$24 000. Every year, the car is worth 85% of what it was worth the previous year. The value is related to time.

1 → \$24000

2 → 24000(.85) = \$20400

3 → 20400(.85) = \$17340

∴ Not linear

b. For a service call, an electrician charges a \$75 flat rate, plus \$50 for each hour he works. The total cost for service is related to time.

0 → \$75

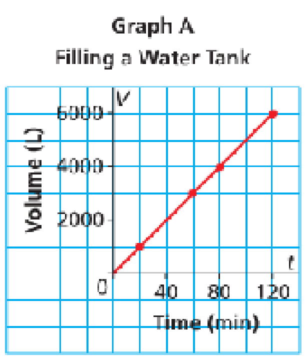
1 → 75+50 = \$125 +50

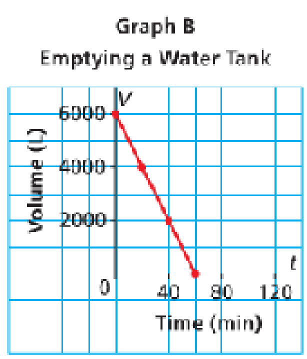
2 → 75+2(50) = \$175 +50

∴ Its linear.

Example 4 Determining the Rate of Change of a Linear Relation from its Graph

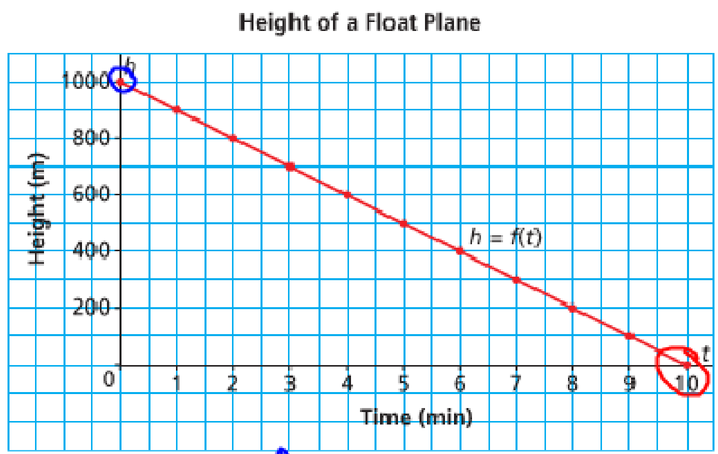
A water tank on a farm holds 6000 L. Graph A represents the tank being filled at a constant rate. Graph B represents the tank being emptied at a constant rate.





Determine the rate of change of each relation, then describe what it represents.

5.7 Interpreting Graphs of Linear Functions



Where goes graph intersect the vertical axis? What does this point represent?

@1000

The point represents 1000m @ t=0, aka where the plane starts

Where does the graph intersect the horizontal axis? What does this point represent?

@10

The point represents t=10 @ h=0, aka when the plane lands.

What is the rate of change for this graph? What does it represent?

$m=\frac{y_2-y_1}{x_2-x_1}$ $(\underset{x_1}{0},\underset{y_1}{1000})$ + $(\underset{x_2}{10},\underset{y_2}{0})$ $m=\frac{0-1000}{10-0}=\frac{-1000}{10}=-100$ m/min

Why is the rate of change positive or negative?

↳ The plane is headed towards the ground.

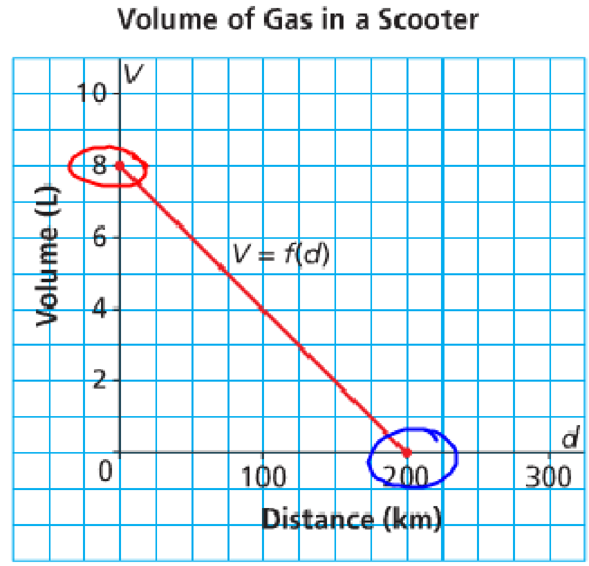
The point where the graph intersects the horizontal axis has coordinates (x,0). The horizontal intercept is (10,0). The horizontal intercept is also called the *x*-intercept.

The point where the graph intersects the vertical axis has coordinates (0,y). The vertical intercept is (0,1000). The vertical intercept is also called the *y*-intercept.

The domain is: [0, 10]

The range is: [0, 1000]

Example 1: This graph shows the fuel consumption of a scooter with a full tank of gas at the beginning of a journey.



a. Determine the following:

i. Domain

$[0, 200]$

ii. Range

$[0, 8]$

iii. Intercepts

Horizontal = x = 200

Vertical = y = 8

b. What are the restrictions on the domain and range?

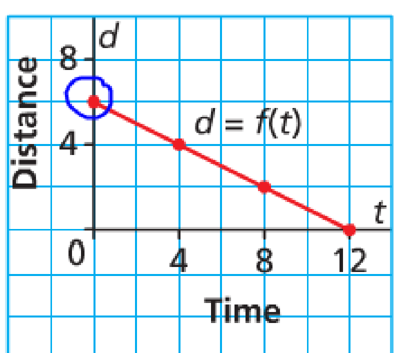
You cannot have negative distance.

You cannot have negative fuel & you cannot over fill your tank.

Example 2: Matching a Graph to a Given Rate of Change and Vertical Intercept

Which graph has a rate of change of $\frac{1}{2}$ and a vertical intercept of 6? Explain.

a)



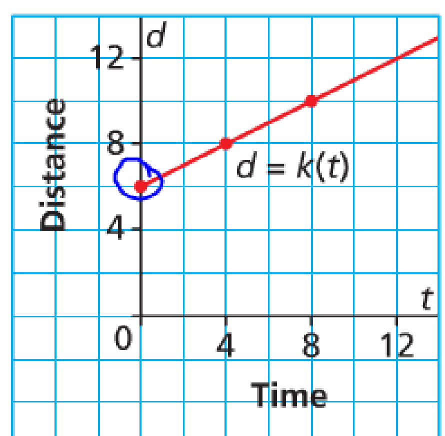
Vertical Intercept = 6

This graph has a negative rate of change

∴ Not the solution

$\boxed{\text{Hwk pg 319 \# 4, }\cancel{\text{6a, 6}}\text{, 7, 8, 10, 11, 17a, 19i}}$

b)



Vertical Intercept = 6

This graph has a positive rate of change = $\frac{1}{2}$.

∴ The solution

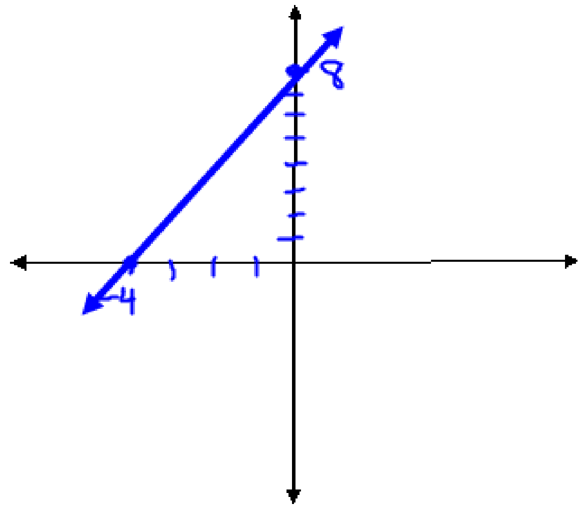
x-intercept: $y=0$
y-intercept: $x=0$

Example 3: Sketch the following graphs by finding the intercepts.

a. $y = 2x + 8$

x-int.
$y=0$
$0 = 2x + \cancel{8}$
$-8 \quad -\cancel{8}$
$-8 = 2x$
$\frac{-8}{2} = \frac{2x}{2}$
$\boxed{-4 = x}$
$\therefore (-4, 0)$

y-int.
$x=0$
$y = 2(0) + 8$
$y = 0 + 8$
$\boxed{y = 8}$
$\therefore (0, 8)$



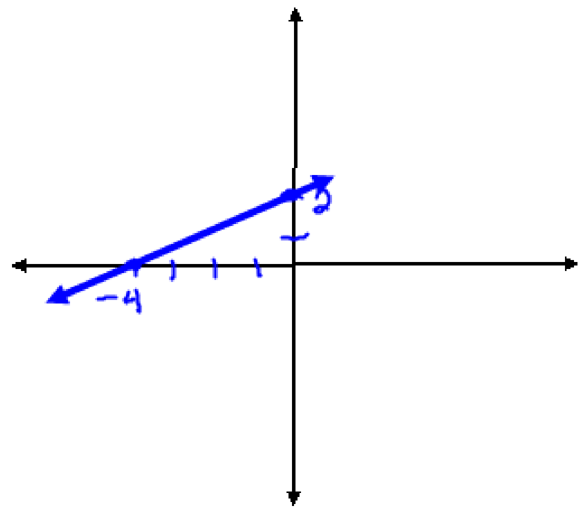
x-intercept:
y-intercept:

b. $y = \frac{1}{2}x + 2$

$y = \frac{x}{2} + 2$

x int
$y=0$
$0 = \frac{x}{2} + \cancel{2}$
$-2 \quad -\cancel{2}$
$(2)(-2) = \frac{x}{\cancel{2}}(\cancel{2})$
$\boxed{-4 = x}$

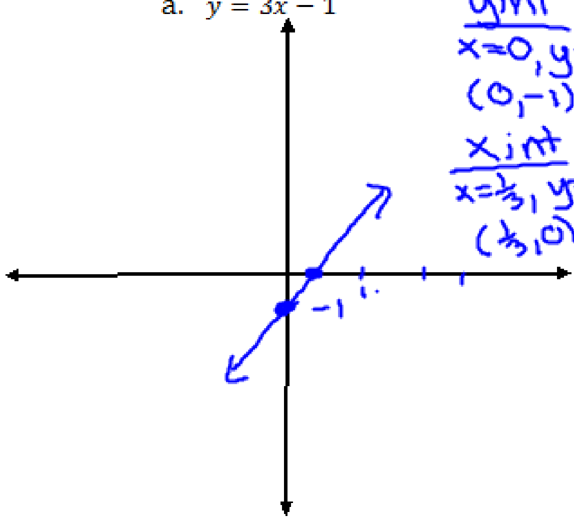
y int
$x=0$
$y = \frac{0}{2} + 2$
$y = 0 + 2$
$\boxed{y = 2}$
$(0, 2)$



x-intercept:
y-intercept:

Example 4: Sketch the following graphs using a graphing calculator. Mark the intercepts on your graph.

a. $y = 3x - 1$



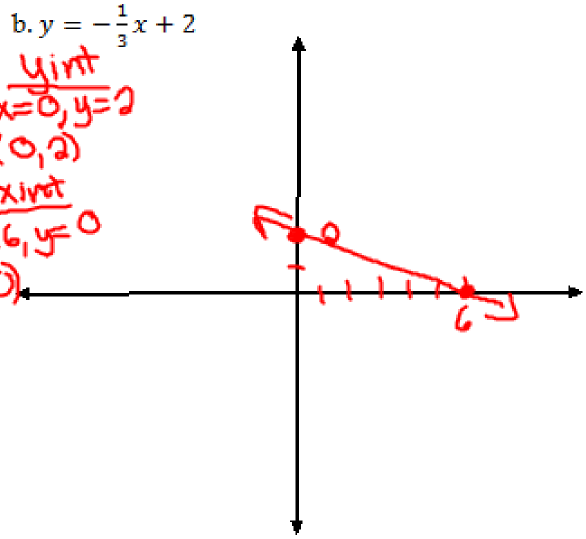
yint
$x=0, y=-1$
$(0, -1)$

xint
$x=\frac{1}{3}, y=0$
$(\frac{1}{3}, 0)$

b. $y = -\frac{1}{3}x + 2$

yint
$x=0, y=2$
$(0, 2)$

xint
$x=6, y=0$
$(6, 0)$



Example 5: Sketching a Graph of a Linear Function in Function Notation

Sketch a graph of the linear function $\boxed{f(x) = -3x + 5.}$

$y = -3x + 5$

x int

$y = 0$

$0 = -3x + \cancel{5}$

$-5 \qquad\qquad -\cancel{5}$

$\frac{-5}{-3} = \frac{-3x}{-3}$

$\frac{5}{3} = x$

$1.6 = x$

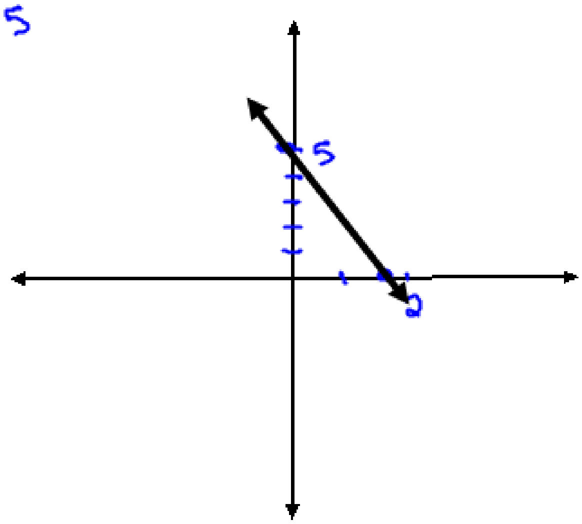
y int

$x = 0$

$y = -3(0) + 5$

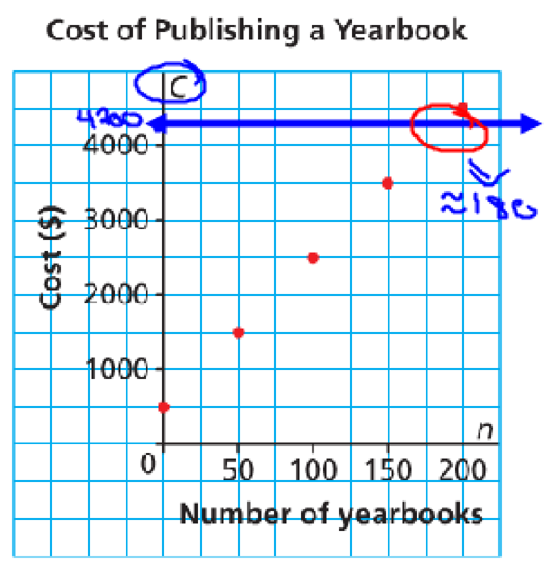
$y = 0 + 5$

$y = 5$



Example 6: Solving a Problem Involving a Linear Function

This graph shows the cost of publishing a school yearbook.



The budget for publishing costs is \$4200. What is the maximum number of books that can be printed?

Method 1: Use the graph to estimate

≈ 180 yearbooks

Method 2: Create a function and solve for when the costs is \$4200.

b = base price \$500

$y = mx + b$

$\boxed{C = 20n + 500}$

$4200 = 20n + \cancel{500}$

$-500 \qquad -\cancel{500}$

$\frac{3700}{20} = \frac{\cancel{20}n}{\cancel{20}}$

$\boxed{185 = n}$

m = rate of change

$= \frac{y_2 - y_1}{x_2 - x_1}$

$(0, 500)$ — x_1, y_1

$(50, 1500)$ — x_2, y_2

$= \frac{1500 - 500}{50 - 0}$

$= \frac{1000}{50}$

$= 20$