Unit 6: Linear Functions

6.1 Slope of a Line (Distance and Midpoint)

The **slope** of a line is the measure of its steepness.

Slope and Direction Investigation

1. Graph the following equations [redacted].
   a. $y = 3x + 2$
   b. $y = -2x - 3$
   c. $y = -0.9x + 5$
   d. $y = 0.5x - 4$

2. Which of the lines rise, or increase, from left to right?

   A , D

3. Which of the lines fall, or decrease, from left to right?

   B , C

4. Which number in the equation of a line determines whether the line rises or falls from left to right?

   $y = mx + b$ m

   m is +'ve → line rises

   m is -'ve → line falls

5. Without graphing, state whether the graphs of the following equations rise or fall from left to right.
   a. $y = -4x - 2$

      fall → -'ve
   b. $y = 5x - 8$

      Rise → +'ve
   c. $y = -0.8x + 7$

      Fall → -'ve
   d. $y = 3x$

      Rise → +'ve

Steepness of Slopes Investigation

1. Graph the following equations ~~u[illegible]~~
   a. $y = 2x + 3$
   b. $y = 6x - 7$
   c. $y = 0.5x + 1$
2. List the above equations in order of Most Steep, to least Steep. (slope or rate of change).

   B, A, C

3. Graph the following equations.
   2 a. $y = -2x + 3$
   1 b. $y = -6x - 7$
   3 c. $y = -0.5x + 1$
4. List the above equations in order of Most Steep, to least Steep. (slope or rate of change).

   B, A, C

5. Use your results from questions 2 and 4 to state which number in the equation of a line determines the steepness of the slope.

   m → The larger m is, the more steep your line.

6. Given the following two equations $\underbrace{y = 3x + 3}_{}$ (m = 3) and $\underbrace{y = 1x - 1}_{}$ (m = 1) write a NEW equation for a line which
   a. is steeper than both lines

      y = 100x − 5
      y = 5x + 2

   b. not as steep as either line but still increasing

      y = 0.5x + 32

   c. steeper than one line but not as steep as the other line.

      y = 2x − 2

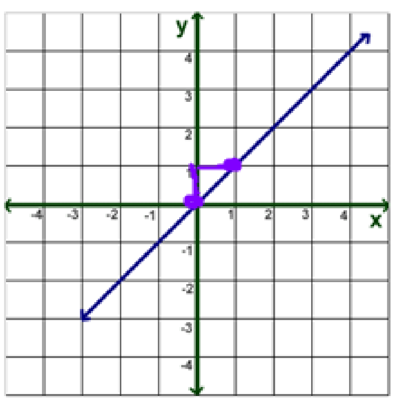
Slope can be represented using several formulas.

$$slope = \frac{rise}{run} = \frac{up/down}{right}$$

$$slope = \frac{\text{change in the dependent variable}}{\text{change in the independent variable}}$$

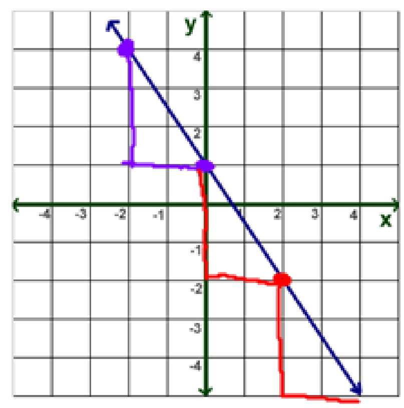
$$* \quad slope = \frac{(y_2 - y_1)}{x_2 - x_1} = \text{rate of change}$$

Example 1: Find the slopes of the following line segments.



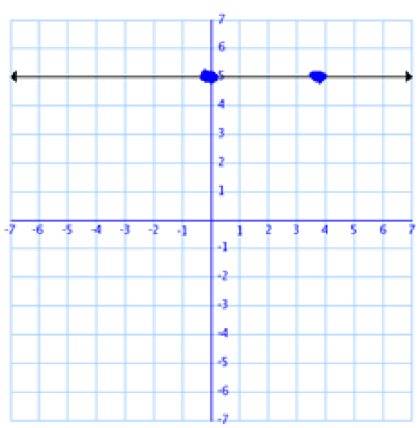
$\frac{Rise}{Run} = \frac{1}{1}$

$m = 1$



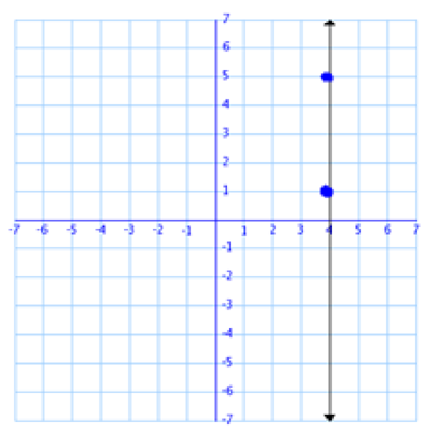
$\frac{Rise}{Run} = \frac{-3}{2}$,

$m = -\frac{3}{2}$



$\frac{Rise}{Run} = \frac{0}{4}$

$m = 0$



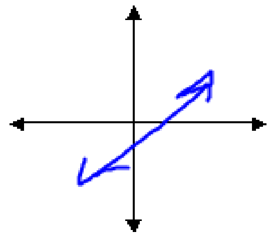
$\frac{Rise}{Run} = \frac{4}{0}$

$m = \frac{4}{0}$

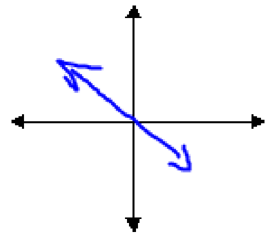
$\therefore m$ is undefined

No slope!

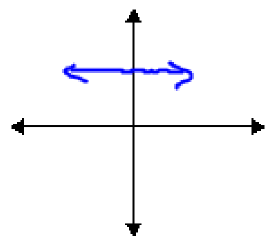
Note: A line which rises up to the right has a positive slope.



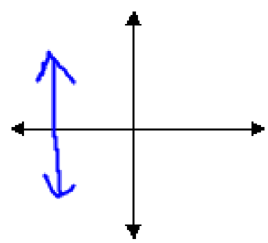
A line which falls to the right has a negative slope.



A horizontal line has a slope of 0.

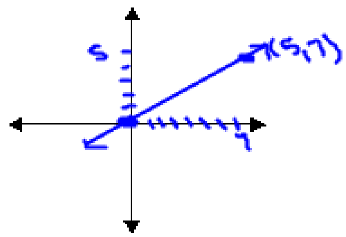


A vertical line has an undefined slope.

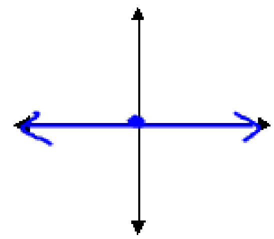


Example 2: Draw a line with the following slopes.

a. $\frac{5}{7}$ $= \frac{\text{rise}}{\text{run}}$

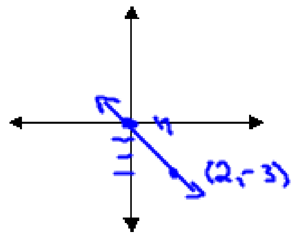


d. 0

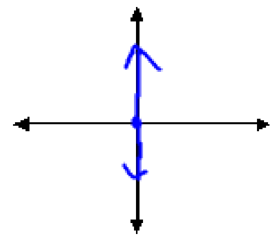


b. $-\frac{3}{2}$ $= \frac{\text{rise}}{\text{run}}$

$= \frac{-3}{2}$

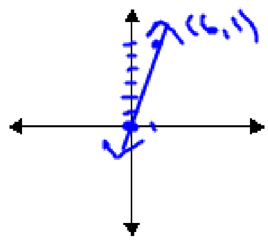


e. undefined



c. 6 $= \frac{\text{rise}}{\text{run}}$

$6 = \frac{6}{1}$



Formula for Slope: $\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$

Example 3: Use the formula to calculate the slope for the line segments joining the following ordered pairs.

$\underset{x_1\,y_1}{\overset{1}{(2,1)}},\underset{x_2\,y_2}{\overset{2}{(5,9)}}$

$m = \frac{9-1}{5-2} = \frac{8}{3}$

$\therefore m = \frac{8}{3}$

$(-3,8),(-5,-2)$

* pg 339 # 5, 6 (slope), 7, 8, 9,
13 i, ii, 17

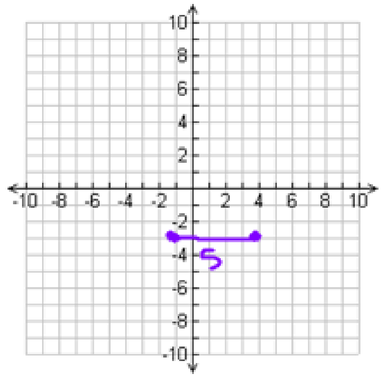
$(0,4),(-2,6)$

Distance

Find the distance between these points which form a horizontal line.

(-1, -3), (4, -3)

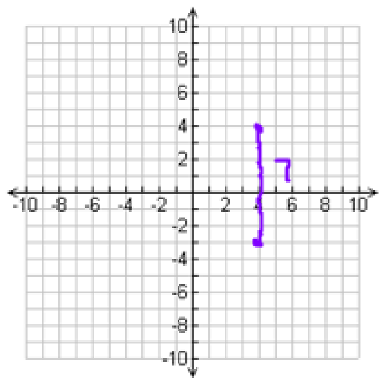
D = 5



Find the distance between these points which form a vertical line.

(4, -3), (4, 4)

D = 7



Use the answer from the last two examples to find the distance between theses points which form an oblique line. (Use the theorem of Pythagoras)

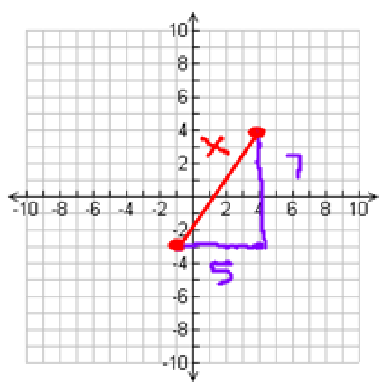
(-1, -3), (4, 4)

$x^2 = 7^2 + 5^2$

$x^2 = 49 + 25$

$x^2 = 74$

$x = \sqrt{74}$



Formula for Distance: $\boxed{d=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}}$

Example 4: Use the formula to calculate the distance for the line segments joining the following ordered pairs.

$\underline{1}$ $\underline{2}$

$(2,1),(5,9)$

$x_1 = 2$ $x_2 = 5$

$y_1 = 1$ $y_2 = 9$

$d=\sqrt{(9-1)^2+(5-2)^2}$

$d=\sqrt{8^2+3^2}$

$d=\sqrt{64+9}$

$d=\sqrt{73}$

$\underline{1}$ $\underline{2}$

$(-3,8),(-5,-2)$

$x_1 = -3$ $x_2 = -5$

$y_1 = 8$ $y_2 = -2$

$d=\sqrt{(-2-8)^2+(-5-{-3})^2}$

$d=\sqrt{(-10)^2+(-2)^2}$

$d=\sqrt{100+4}$

$d=\sqrt{104}$

$\underline{1}$ $\underline{2}$

$(0,4),(-2,6)$

$x_1 = 0$ $x_2 = -2$

$y_1 = 4$ $y_2 = 6$

$d=\sqrt{(6-4)^2+(-2-0)^2}$

$d=\sqrt{2^2+(-2)^2}$

$d=\sqrt{4+4}$

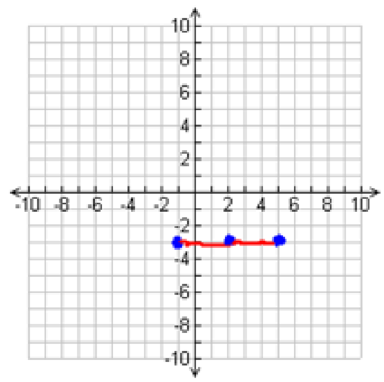
$d=\sqrt{8}$

Midpoint

Find the midpoint between these points which form a horizontal line.

(-1, -3), (5, -3)

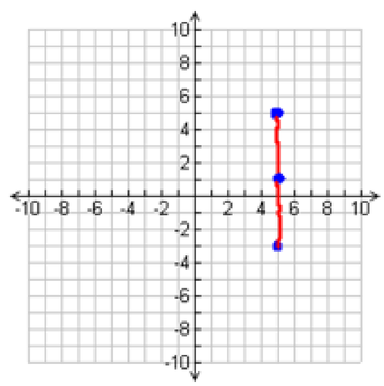
Midpoint = (2, -3)



Find the midpoint between these points which form a vertical line.

(5, -3), (5, 5)

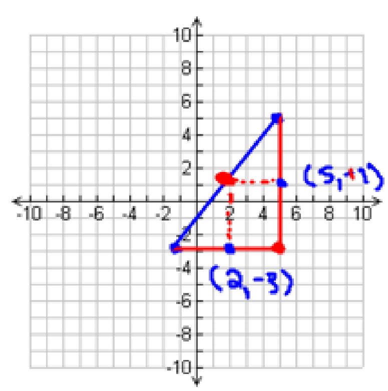
Midpoint = (5, 1)



Use the answer from the last two examples to find the midpoint between theses points which form an oblique line.

(-1, -3), (5, 5)

Midpoint = (2, 1)



Formula for Midpoint: $midpt. = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

Example 5: Use the formula to calculate the midpoint for the following ordered pairs.

$(2,1),(5,9)$

Midpoint = $\left(\frac{2+5}{2}, \frac{1+9}{2} \right)$

$= \left(\frac{7}{2}, \frac{10}{2} \right)$

$= (3.5, 5)$

$(-3,8),(-5,-2)$

Midpoint = $\left(\frac{-3+-5}{2}, \frac{8+-2}{2} \right)$

$= \left(\frac{-8}{2}, \frac{6}{2} \right)$

$= (-4, 3)$

$(0,4),(-2,6)$

Midpoint = $\left(\frac{\quad}{\quad}, \frac{\quad}{\quad} \right)$

$= (\quad , \quad)$

$= (-1, 5)$

6. What general statement can you make about the equations of parallel lines in relation to $y = mx + b$?

7. Are $y = 3x + 7$ and $y = 3x - 8$ parallel to each other?

Parallel lines have the same slope.

$m = 3$ $m = 3$ $\therefore$ They are parallel.

8. Are $y = \frac{2}{3}x - 2$ and $y = \frac{3}{2}x + 1$ parallel to each other?

No.

9. Name 3 lines that are parallel to $y = 2x - 3$.

$y = 2x - 7$

$y = 2x + 100$

10. Name 3 lines that are not parallel to $y = 5x - 2$.

$y = 3x - 2$ $m \neq 5$

Perpendicular Line Investigation

Graph the points and use a ruler to draw the line that passes through them. Use the designated color to draw each line.

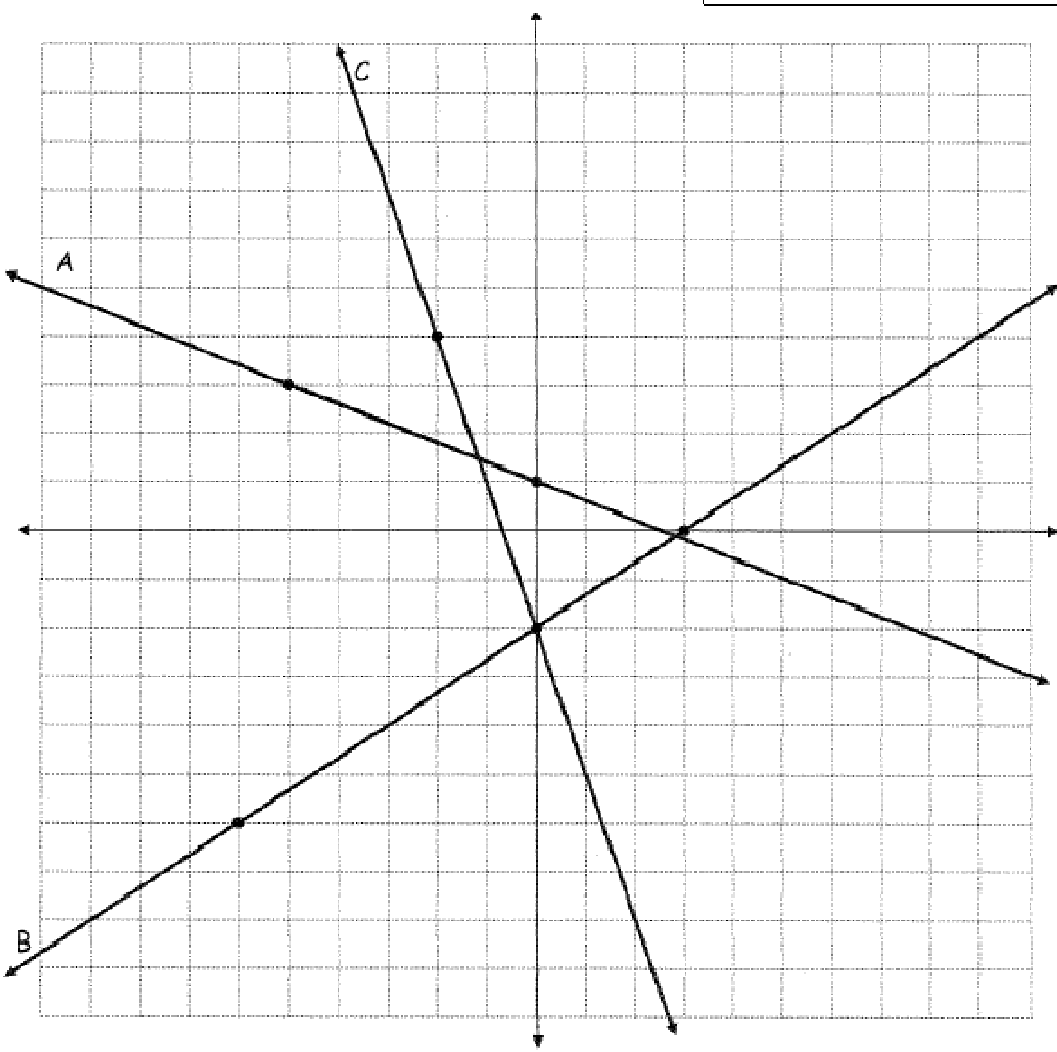
BLUE:	$(0, 2)$	$(2, -1)$
PURPLE:	$(-3, 6)$	$(-6, 5)$
ORANGE:	$(4,0)$	$(6,5)$

Given lines and their points

A: $(0, 1)$ and $(-5, 3)$

B: $(3, 0)$ and $(-6, 6)$

C: $(-2, 4)$ and $(0, -2)$



The equation of Line A is $y = -\frac{2}{5}x + 1$.
The equation of Line B is $y = \frac{2}{3}x - 2$.
The equation of Line C is $y = -3x - 2$.

Use the points given to write the equation of each colored line in slope-intercept form.

BLUE LINE	PURPLE LINE	ORANGE LINE

Use your graph to help answer the following questions.

1. Which colored line is perpendicular to line A? What are the equations of these 2 lines?
2. Which colored line is perpendicular to line B? What are the equations of these 2 lines?
3. Which colored line is perpendicular to line C? What are the equations of these 2 lines?
4. What do you notice about the slopes in each pair of equations?
5. What do you notice about the y-intercepts in each pair of equations?

6. What general statement can you make about the equations of perpendicular lines in relation to $y = mx + b$?

7. Are $y = 3x + 7$ and $y = 3x - 8$ perpendicular to each other?

8. Are $y = \frac{2}{3}x - 2$ and $y = -\frac{3}{2}x + 1$ perpendicular to each other?

9. Name 3 lines that are perpendicular to $y = 2x - 3$.

10. Name 3 lines that are not perpendicular to $y = 5x - 2$.

Example 1: Sketch and find the slopes for the following lines.

$l_1 : (-4, 2), (2, -1)$ → purple

$y = mx + b$
slope yint

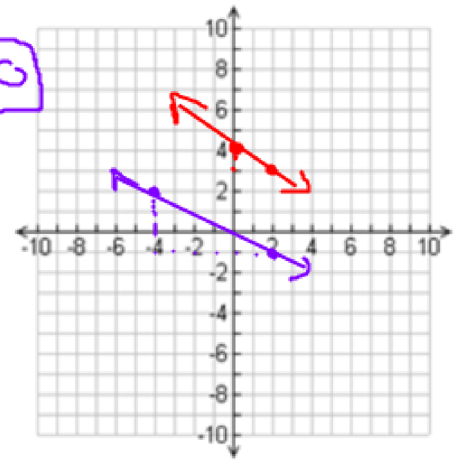
$\boxed{y = \frac{-3}{6}x + 0}$

$l_2 : (0, 4), (2, 3)$

red

Slope $= \frac{-1}{2}$ yint $= 4$

$\boxed{y = \frac{-1}{2}x + 4}$



* **Parallel lines have equal slopes.** $\frac{-1}{2} = \frac{-3}{6}$

Example 2: Sketch and find the slopes for the following lines.

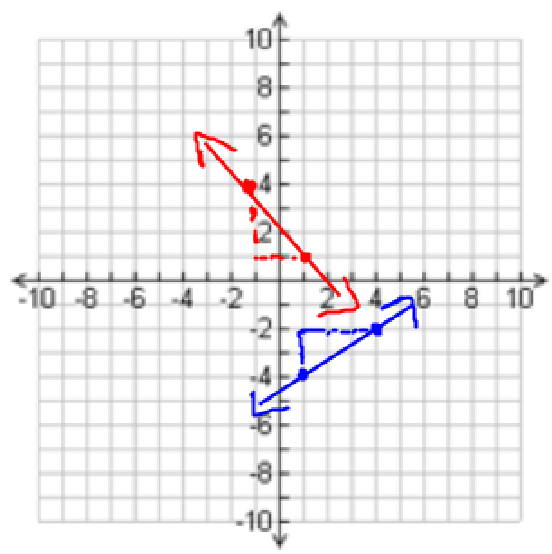
$l_1 : (4, -2), (1, -4)$ → blue

Slope $= \frac{2}{3}$

$l_2 : (-1, 4), (1, 1)$

red

Slope $= \frac{-3}{2}$



* **Perpendicular lines have slopes which are negative reciprocals of each other.**

$\frac{2}{3} \rightarrow -\frac{3}{2}$ flip fraction

Example 3: Find the slope which is parallel and perpendicular to the given slope.

m	Parallel	Perpendicular
$\frac{2}{3}$	$m=\frac{2}{3}$ e.g. $y=mx+b$ $y=\frac{2}{3}x+2$	$m=-\frac{3}{2}$ $y=-\frac{3}{2}x-1$
$\frac{-6}{7}$	$m=\frac{-6}{7}$	$m=-\left(\frac{-7}{6}\right)$ $=\frac{7}{6}$
5	$m=\frac{5}{1}$	$m=\frac{-1}{5}$
0	$m=\frac{0}{1}$ $\longleftrightarrow$	$m=\frac{-1}{0}$ $=$undefined $\updownarrow$

$\perp$

$(3,5),(-4,-8)$

$x_1\, y_1\quad x_2\, y_2$

$m=\frac{y_2-y_1}{x_2-x_1}$

$=\frac{-8-5}{-4-3}$

$=\frac{-13}{-7}=\frac{13}{7}$

$\frac{13}{7}$, $-\frac{7}{13}$

$\boxed{\text{Hwk: pg. 349 \#3-5, 8b,d, 9a, 11a,b}}$

6.4 Slope-Intercept Form of the Equation for a Linear Function

Slope-Intercept Form:

$$\boxed{y = mx + b}$$

where m is the slope and b is the y-intercept.

Example 1: Write the equation of the following lines in slope-intercept form given the slope and y- intercept.

$m = 5, \quad b = -7$

$y = mx + b$

$y = 5x - 7$

$m = -3, \quad b = 5$

$y = -3x + 5$

$m = \frac{9}{2}, \quad b = 0$

$y = \frac{9}{2}x + 0$

$y = \frac{9}{2}x$

$m = 0, \quad b = 5$

$y = 0x + 5$

$y = 0 + 5$

$y = 5$

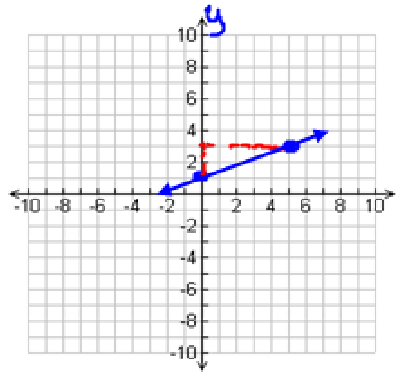
Example 2: Graph the following linear functions.

$y = \frac{2}{5}x + 1$

$m = \frac{2}{5} = \frac{\text{rise}}{\text{run}}$

yint = 1

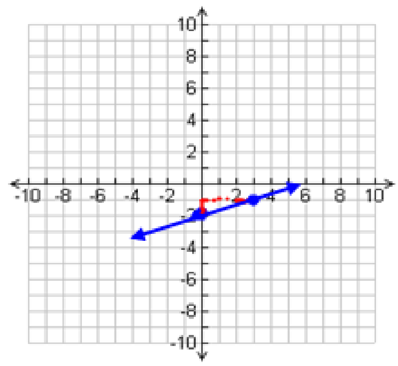
To graph, plot your y-int, then use slope to find more points.



$y = \frac{1}{3}x - 2$

$m = \frac{1}{3} = \frac{\text{rise}}{\text{run}}$

yint = -2

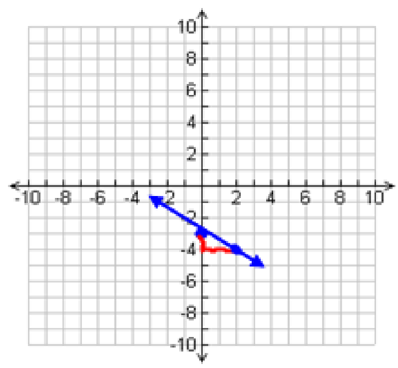


$y = \frac{-1}{2}x - 3$

$m = -\frac{1}{2} = \frac{\text{rise}}{\text{run}}$

→ down 1, across 2

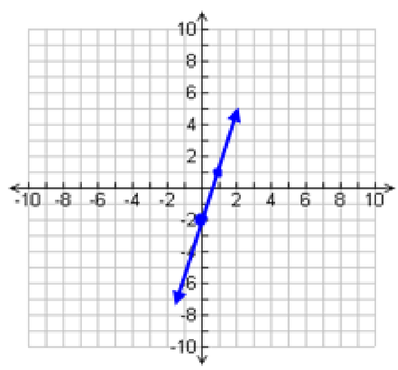
yint = -3



$y = 3x - 2$

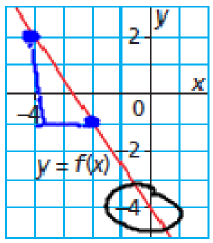
$m = 3 = \frac{3}{1} = \frac{\text{rise}}{\text{run}}$

yint = -2



Example 3: Write the equation in slope-intercept form of the following lines using the given graphs.

a.



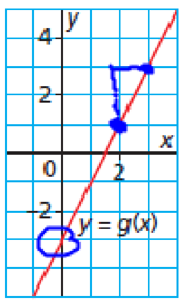
$y = mx + b$

y int $= b = -4$

slope $= m = \frac{-3}{2}$

$\therefore y = \frac{-3}{2}x - 4$

b.



$y = mx + b$

$m = +\frac{2}{1} = 2$

$b = -3$

$\therefore y = 2x - 3$

Example 4: Using an Equation of a Linear Function to Solve a Problem
The student council sponsored a dance. A ticket cost \$5 and the cost for the DJ was \$300.

a. Write an equation for the profit, *P* dollars, on the sale of *t* tickets.

$P = 5t - 300$

b. Suppose 123 people bought tickets. What was the profit?

$123 = t$

$P = 5(123) - 300$

$P = 615 - 300 = \boxed{\$315}$

c. Suppose the profit was \$350, how many people bought tickets?

$P = 350$ find t

$350 = 5t - \cancel{300}$

$+300 \qquad + \cancel{300}$

$\frac{650}{5} = \frac{5t}{5}$

$\therefore t = 130$

d. Could the profit be exactly \$146? Justify your answer.

NO → 146 is not a multiple of 5.

$$\cancel{2x} + y = 22$$
$$\cancel{-2x} \qquad -2x$$

$\Rightarrow$ $y = mx + b$

$\Rightarrow$ Get y by itself.

$$y = 22 - 2x$$

$1 + 2 = 2 + 1$

$$y = -2x + 22$$

6.5 Slope-Point Form of the Equation for a Linear Function

Slope-Point Form:

$y = mx + b$ ↓ slope ↓ yint

$$y - y_1 = \underbrace{m}_{\text{slope}}(x - x_1)$$

where m is the slope and (x_1, y_1) is a point on the line

Example 1: State the slope and a point on the following lines. Graph the lines.

$y - 2 = \frac{1}{3}(x+4)$

$y - y_1 = m(x - x_1)$

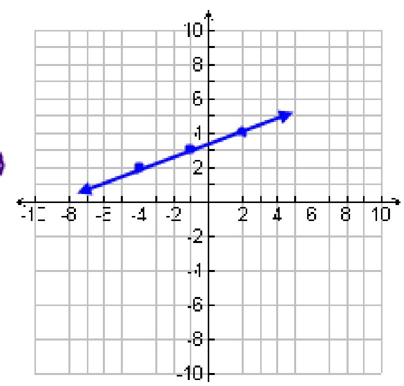
$m = \frac{1}{3}$

$(x+4) = (x - (-4))$

$x_1 = -4$

$y_1 = 2$

$(-4, 2)$



* To graph, start at point, use your slope to find more points *

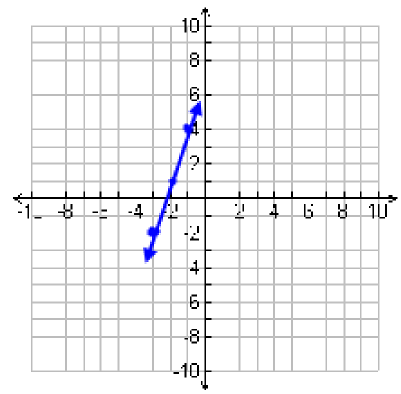
$y + 2 = 3(x+3)$

$m = 3 = \frac{3}{1}$

$x_1 = -3$

$y_1 = -2$

$\therefore (-3, -2)$



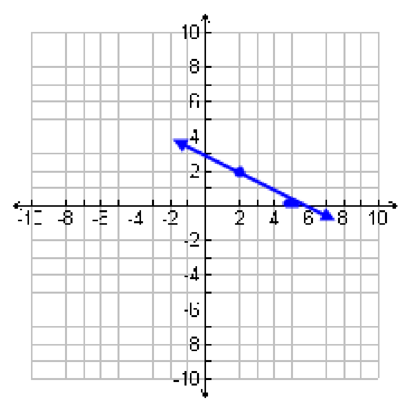
$y - 2 = -\frac{2}{3}(x-2)$

$m = -\frac{2}{3}$

$x_1 = 2$

$y_2 = 2$

$\therefore (2, 2)$



Example 2: Write the equation of each line in slope-point form then rearrange it to slope intercept form. ① $y - y_1 = m(x - x_1)$

$m = -5, \ (3,7)$ ② $y = mx + b$

$x_1 = 3$

$y_1 = 7$

$y - 7 = -5(x-3) + 7$
$+7$

$y = -5(x-3) + 7$

$y = -5x + 15 + 7$ $\therefore y = -5x + 22$

$m = 3, \ (-2,-6)$

$x_1 = -2$

$y_1 = -6$

$y - (-6) = 3(x - (-2))$

$y + 6 = 3(x+2) - 6$
-6

$y = 3x + 6 - 6$ $\therefore y = 3x$

$m = 2, \ (2,-7)$

$x_1 = 2$

$y_1 = -7$

$y - (-7) = 2(x-2)$

$y + 7 = 2x - 4$

$y = 2x - 4 - 7 \longrightarrow \therefore y = 2x - 11$

Example 3: Write the equation of each line in slope-point form.

$\begin{matrix} x_1 & y_1 \\ (2, & 5) \end{matrix}$, $\begin{matrix} x_2 & y_2 \\ (3, & -6) \end{matrix}$ *Find slope $y - y_1 = m(x - x_1)$

*

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$\therefore y - 5 = -11(x - 2)$

$m = \frac{-6-5}{3-2}$

$= \frac{-11}{1}$

$\begin{matrix} x_1 & y_1 \\ (1, & 7) \end{matrix}$, $\begin{matrix} x_2 & y_2 \\ (0, & 3) \end{matrix}$ find slope. Use slope-point form

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$y - y_1 = m(x - x_1)$

$m = \frac{3-7}{0-1}$

$y - 7 = 4(x - 1)$

$= \frac{-4}{-1}$

$= 4$

$(-2, -6)$, $(1, 8)$

Example 4: Write the equation for the line which passes through the point (1, -1) and is:

$y - y_1 = m(x - x_1)$

a. Parallel to: $y = \frac{2}{3}x - 5$

(1, -1)

$\therefore x_1 = 1$

$y_1 = -1$

$m = \frac{2}{3}$

Slope $= \frac{2}{3}$

Parallel lines have the same slope.

$\therefore m = \frac{2}{3}$

$\boxed{\therefore y + 1 = \frac{2}{3}(x - 1)}$

b. Perpendicular to: $y = \frac{2}{3}x - 5$

$x_1 = 1 \quad y_1 = -1$

Slope $= +\frac{2}{3}$

Perpendicular lines have negative/opposite reciprocal slopes.

$\therefore m = -\frac{3}{2}$

$\therefore y + 1 = -\frac{3}{2}(x - 1)$

Example 5: Write the equation for the line which is parallel to $y = \frac{-3}{7}x + 4$ and has the same y-intercept as $y = \frac{6}{5}x - 3$.

6.6 General Form of the Equation for a Linear Function

General Form:

$$* \quad Ax + By + C = 0$$

A, B and C are integers, (x, y) is a point on the line and A is positive.

…-2, -1, 0, 1, 2…

No fractions

~~Standard Form:~~

$$Ax + By = C$$

A, B and C are integers, (x, y) is a point on the line and A is positive.

Example 1: Rewrite each of the following in general form.

$Ax + By + C = 0$ zero

$y = 2x + 4$

$-2x \quad -2x$

$y - 2x = 4$

$-4 \quad -4$

$y - 2x - 4 = 0$

$-2x + y - 4 = 0$

$y = \frac{-2}{5}x - 3$

$+\frac{2}{5}x \quad +\frac{2}{5}x$

$y + \frac{2}{5}x = -3$

$+3 \quad +3$

$5\left(y + \frac{2}{5}x + 3 = 0\right)$

$5y + 2x + 15 = 0$

$2x + 5y + 15 = 0$

$y - 1 = 4(x + 2)$

$y - 1 = 4x + 8$

$-4x \quad -4x$

$y - 1 - 4x = 8$

$-8 \quad -8$

$y - 1 - 4x - 8 = 0$

$y - 4x - 9 = 0$

$-4x + y - 9 = 0$

$\left(y + 3 = \frac{-8}{3}(x - 1)\right)3$

$3y + 9 = -8(x - 1)$

$3y + 9 = -8x + 1$

$+8x \quad +8x$

$8x + 3y + 9 = 1$

$-1 \quad -1$

$8x + 3y + 1 = 0$

Example 2: Determine the x and y intercepts of the following lines.

$3x+2y-12=0$

x int

$y=0$

$3x+2(0)-12=0$

$3x-12=0$
$+12 \quad +12$

$\frac{3x}{3}=\frac{12}{3}$

$\therefore x=4$

y int

$x=0$

$3(0)+2y-12=0$

$2y-12=0$
$+12 \quad +12$

$\frac{2y}{2}=\frac{12}{2}$

$y=6$

$5x-2y+30=0$

x int

$y=0$

$5x+30=0$

$5x=-30 \quad \therefore x=-6$

y int

$x=0$

$-2y+30=0$

$-2y=-30 \quad \therefore y=15$

Example 3: Rewrite the following lines in slope-intercept form.

$y=mx+b$

$3x+2y-12=0$
$+12 \quad +12$

$+3x+2y=12$
$-3x \quad -3x$

$\frac{2y}{2}=\frac{-3x}{2}+\frac{12}{2}$

$y=-\frac{3}{2}x+6$

$5x-2y+30=0$
$-30 \quad -30$

$5x-2y=-30$
$-5x \quad -5x$

$\frac{-2y}{-2}=\frac{-5x}{-2}-\frac{30}{-2}$

$y=\frac{5}{2}x+15$

Example 4: Write the equation of the line parallel to $4x+3y-1=0$ with the same y-intercept as $2x-5y+15=0$. Write the answer in general form.

Parallel to

$4x+3y-1=0$

*same slope

*Change to

$y=mx+b$

$m=$ slope

$4x+3y-1=0$
$\quad\quad\quad +1 \quad +1$

$4x+3y=1$
$-4x \quad\quad -4x$

$\frac{3y}{3}=\frac{-4x}{3}+\frac{1}{3}$

$y=\frac{-4}{3}x+\frac{1}{3}$

$\therefore m=\frac{-4}{3}$

yint of

$2x-5y+15=0$

$x=0$

$\cancel{2(0)}-5y+15=0$

$-5y+15=0$
$\quad\quad -15 \quad -15$

$\frac{-5y}{-5}=\frac{-15}{-5}$

$y=3$

$\therefore$ yint $=3$

$\boxed{\therefore y=mx+b \quad y=\frac{-4}{3}x+3}$