Systems of Linear Equations

7.1 Developing Systems of Linear Equations

A **system of linear equations** (for our purposes) is two equations with two common variables.

$$\begin{cases} 2x + 7y = 4 \\ 3x - 8y = 2 \end{cases}$$

The variables x and y could represent x and y on a coordinate plane or they could represent items in the real world like cars, jackets or desks.

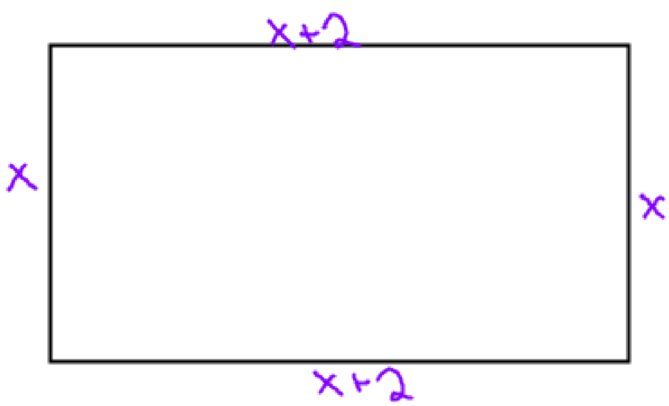
A **solution** to a system is a pair of values that will satisfy both equations.

Example 1: The Nunavut flag has a perimeter of 16 ft. Its length is 2 ft. longer than its width. It has been suggested that the flag is actually 5 ft. long and 3 ft. wide. Use a diagram to create a system of two equations that model this scenario.

$P = 16$

$w = x$

$l = x + 2$



$P = x + x + 2 + x + x + 2$

$P = 4x + 4$

$P = 16$

$x = 3$

$P = 4(3) + 4$

$= 12 + 4$

$= 16$ ✓

Example 2: For a figure skating competition 12 000 tickets were sold. The tickets cost \$35 for an adult and \$20 for a student. The total value of the ticket sales was \$307 500. Set up a table to model this scenario and then set up a system that could be used to solve it. Clearly label all variables.

\$35 → adult
\$20 → student

Total = \$307500

x = adults
y = students

① $x + y = 12000$
② $35x + 20y = 307500$

Example 3: During a clearance sale, all the shirts are on sale at one price and all the sweaters at another price. Two shirts and 4 sweaters cost \$98. One shirt and 3 sweaters cost \$69. Create a system to model this situation.

x = shirts cost
y = sweaters cost

① $2x + 4y = 98$
② $1x + 3y = 69$

7.2 Solving a System of Linear Equations Graphically

Example 1: Graph the following lines using the slope-intercept method.

$y = mx + b$

$x + y = 3$

$-x \quad -x$

$y = -x + 3$

yint = 3

slope = $-\frac{1}{1}$

$2x + y = 4$

$-2x \quad -2x$

$y = -2x + 4$

yint = 4

slope = $\frac{-2}{1}$

$3x - 4y = 12$

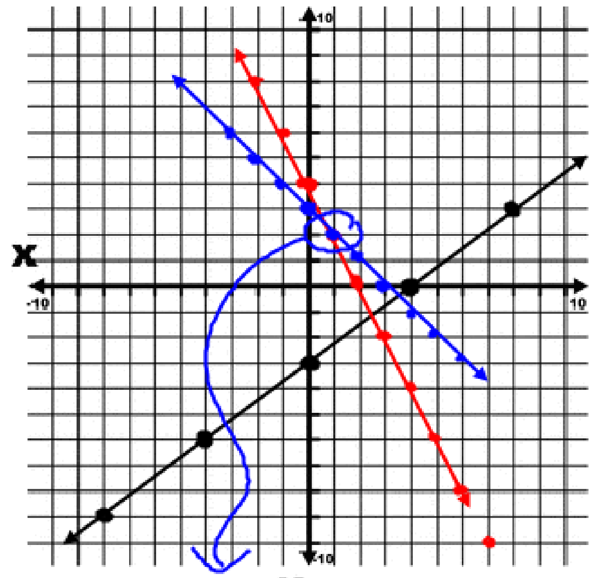
$-3x \quad -3x$

$\frac{-4y}{-4} = \frac{-3x}{-4} + \frac{12}{-4}$

$y = \frac{3}{4}x - 3$

yint = -3

slope = $\frac{3}{4}$



Point of Intersection = (1, 2) = solution

To find the solution of a system means to find an ordered pair (x, y) that satisfies both equations. In other words find the point where both the lines cross.

Example 3: Solve the following system by graphing by hand.

$y = mx + b$

$$\begin{cases} x + y = 5 & (1) \\ 2x - y = 1 & (2) \end{cases}$$

$-x \quad -x$

$-2x \quad -2x$

(1) $y = -x + 5$

yint = 5

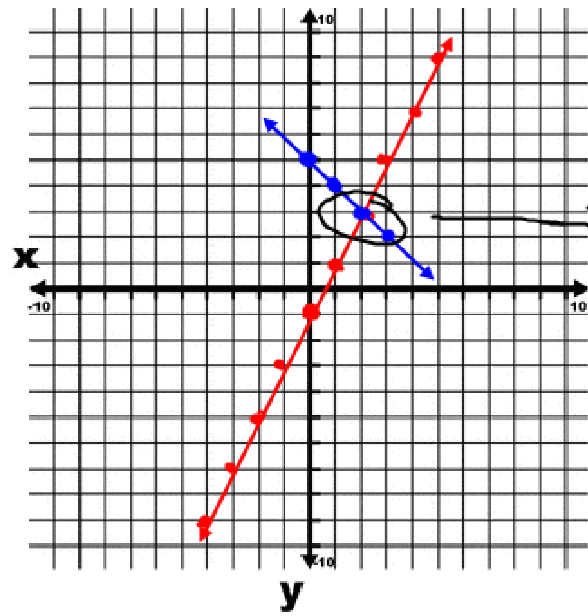
slope = $\frac{-1}{1}$

(2) $\frac{-y}{-1} = \frac{-2x}{-1} + \frac{1}{-1}$

$y = 2x - 1$

yint = -1

slope = $\frac{2}{1}$



Solution (2, 3)

$$\begin{cases} x + y = 4 & (1) \\ 2x + 3y = 9 & (2) \end{cases}$$

$-x \quad -x$

$-2x \quad -2x$

$y = -x + 4$

yint = 4

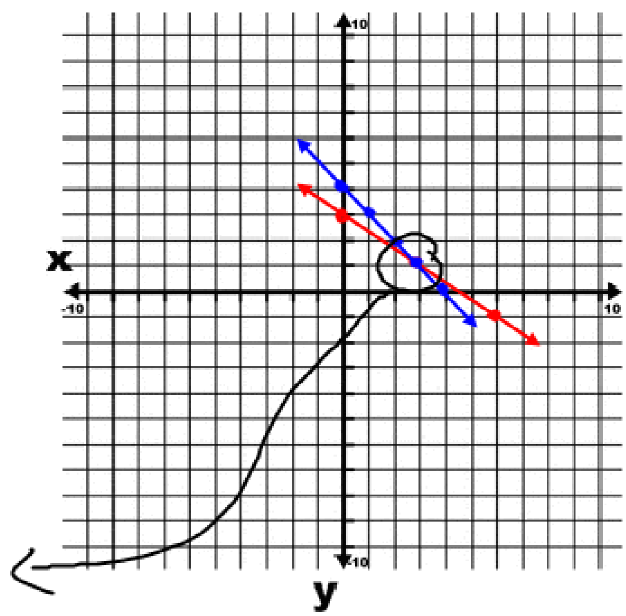
slope = $\frac{-1}{1}$

$\frac{3y}{3} = \frac{-2x}{3} + \frac{9}{3}$

$y = -\frac{2}{3}x + 3$

yint = 3

slope = $\frac{-2}{3}$



Solution: (3, 1)

7.3 Using Graphing Technology to Solve a System of Linear Equations

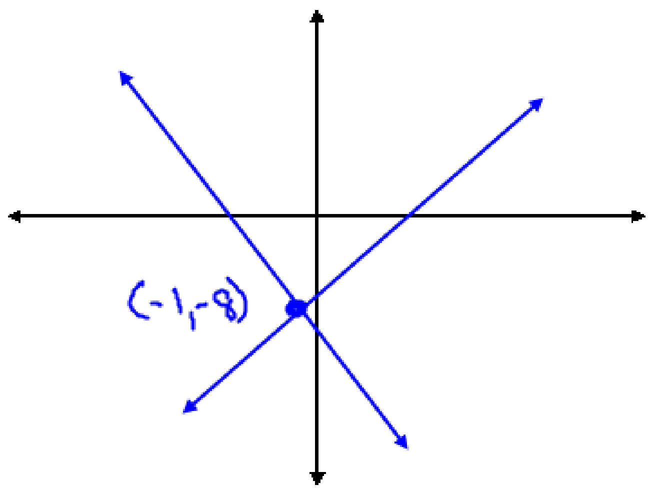
Follow the instructions and key strokes on the handout.

Example 1: Graph the following systems using a graphing calculator.

* Need to be y= form*

$$\begin{cases} y = x - 7 \\ y = -2x - 10 \end{cases}$$

$(-1, -8)$



① $\begin{cases} 2x + 8y = 8 \\ -2x + y = 10 \end{cases}$

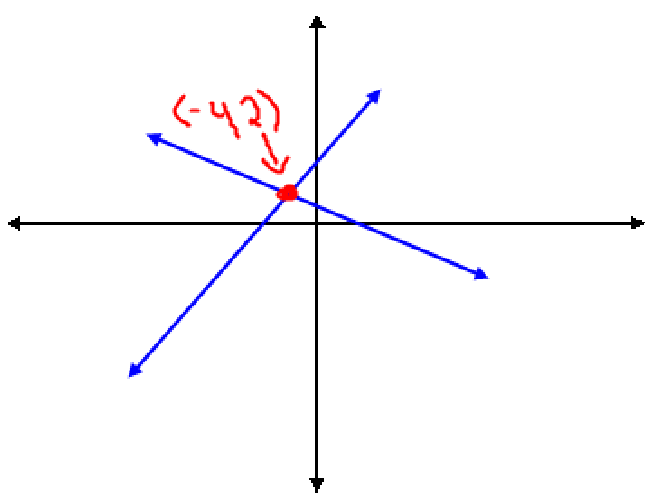
$-2x \quad -2x$

$+2x \quad +2x$

① $\dfrac{8y}{8} = \dfrac{-2x}{8} + \dfrac{8}{8}$

$y = -\frac{2}{8}x + 1$

② $y = 2x + 10$

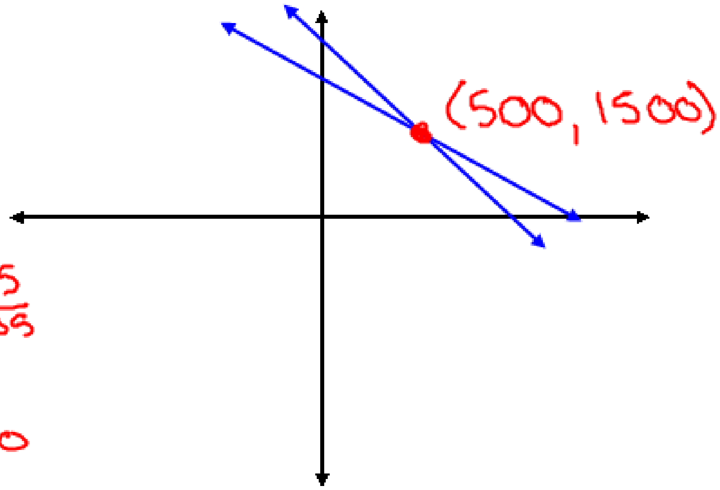


① $\begin{cases} \overset{-x}{x} + y = \overset{-x}{2000} \\ .04x + .05y = 95 \\ -.04x \qquad -0.04x \end{cases}$

① $y = -x + 2000$

② $\frac{0.05y}{0.05} = \frac{-0.04x}{0.05} + \frac{95}{0.05}$

$y = -0.8x + 1900$



Window

xmin → −10
xmax → 2000
ymin → −10
ymax → 2000

7.4 Using a Substitution Strategy to Solve a System of Linear Equations

To solve a system by substitution:

- label the lines in the system 1 and 2
- solve one of the lines for one of the variables, avoiding fractions if possible (let's say x in line 1)
- substitute that equation into the other line (line 2 for us)
- solve for the only variable in that equation (that would be y in line 2 for us)
- substitute the newly found variable into the other equation (y into line 1 for us)
- solve for the remaining variable (x for us)

Example 1: Solve the following systems by the substitution method.

1 $\begin{cases} y = 1 + x \\ y = 4x - 5 \end{cases}$ 2

Substitute $y = 1 + x$ into 2

$1 + x = 4x - 5$
$-4x \quad -4x$
$1 - 3x = -5$
$-1 \quad -1$
$\frac{-3x}{-3} = \frac{-6}{-3}$
$x = 2$

Then solve for x.
Move x's to the left.
Move #s to the right.

Sub $x = 2$ into one equation.
$y = 1 + x$
$y = 1 + 2 = 3$

$\therefore$ Solution $(2, 3)$

1 $\begin{cases} d = 6 \\ d = -3t + 22 \end{cases}$ 2

Sub 1 into 2

$6 = -3t + 22$
$-22 \quad -22$
$\frac{-16}{-3} = \frac{-3t}{-3}$
$\frac{16}{3} = t$

$\therefore$ Solution
$t = \frac{16}{3}$
$d = 6$

① $\begin{cases} x - y = 2 \\ 4x + 2y = 16 \end{cases}$ ②

$+y \quad +y$

Solve for x or y in one equation.

Solve for x in ①

$x = 2 + y$ → Substitute this into ②

$4(2+y) + 2y = 16$

$8 + 4y + 2y = 16$

$8 + 6y = 16$

$-8 \qquad -8$

$6y = 8$

$y = \frac{8}{6}$

Find x by subbing $y = \frac{8}{6}$ into $x = 2 + y$

$\therefore x = \frac{2}{1} + \frac{8}{6}$

$= \frac{12}{6} + \frac{8}{6}$

$= \frac{20}{6}$

$\boxed{\therefore \left(\frac{20}{6}, \frac{8}{6}\right)}$

① $\begin{cases} x - y = 4 \\ 2x + y = -4 \end{cases}$ ②

① $x = 4 + y$

Sub ① into ②

$2(4+y) + y = -4$

$8 + 2y + y = -4$

$-8 \qquad -8$

$2y + y = -12$

$\frac{3y}{3} = \frac{-12}{3}$

$y = -4.$

To find x, sub $y = -4$ into $x = 4 + y$.

$x = 4 + (-4)$

$x = 0$

$\boxed{(0, -4)}$

① $\begin{cases} 2x - y + 3 = 0 \\ 3x + 2y = -1 \end{cases}$ ②

$+y \quad +y$

① $2x + 3 = y$

Sub ① into ②

$3x + 2(2x + 3) = -1$

$3x + 4x + 6 = -1$

$7x + 6 = -1$

$-6 \quad -6$

$\frac{7x}{7} = \frac{-7}{7}$

$x = -1$

To find y, sub $x = -1$ into ①

$y = 2x + 3$

$y = 2(-1) + 3$

$y = -2 + 3$

$y = 1$

$\boxed{\therefore (-1, 1)}$

7.5 Using an Elimination Strategy to Solve a System of Linear Equations

To solve a system by elimination or the add/subtract method:

- label the lines in the system 1 and 2
- rearrange the equations so that the variables and equations are aligned
- multiply one or both equations by a non zero constant so that the coefficients of one of the variables in both lines has the same coefficient
- add or subtract to eliminate that variable (let's say x)
- solve for the remaining variable (y for us)
- substitute that new found value into either equation to find the remaining variable (x for us)

Example 1: Solve the following systems using the elimination or the add/subtract method.

$$\begin{cases} 3x+5y=12 \\ 7x+5y=8 \end{cases}$$

$3x+5y=12$

$-(7x+5y=8)$

$\frac{-4x}{-4}=\frac{4}{-4}$

$x=-1$

* Plug in $x=-1$ to find y.

$3(-1)+5y=12$

$-3+5y=12$

$+3 \qquad +3$

$\frac{5y}{5}=\frac{15}{5}$

$y=3$

$\therefore (-1,3)$

$$2\left(\begin{cases} 3x+y=18) \\ x+2y=11 \end{cases}\right.$$

$6x+2y=36$

$-(x+2y=11)$

$\frac{5x}{5}=\frac{25}{5}$

$x=5$

* Plug $x=5$ into equation 2.

$5+2y=11$

$-5 \qquad -5$

$\frac{2y}{2}=\frac{6}{2}$

$y=3$

$\boxed{(5,3)}$

$$\begin{cases} 6x-5y=-2 \\ 3(2x+3y=18) \end{cases}$$

$6x-5y=-2$

$-(6x+9y=54)$

$\frac{-14y}{-14}=\frac{-56}{-14}$

$y=4$

Find x.

$2x+3(4)=18$

$2x+12=18$

$-12 \quad -12$

$\frac{2x}{2}=\frac{6}{2}$

$x=3$

$\boxed{(3,4)}$