Chapter 6 BLM Answers

BLM 6–1 Chapter 6 Prerequisite Skills

1. a) **i)** II **ii)** IV **iii)** III

b) i) 45° **ii)** $\frac{2\pi}{7}$ **iii)** $\frac{\pi}{4}$

2. a) 310° **b)** $-\frac{\pi}{2}, \frac{7\pi}{2}$

c) 488.4°, –231.6°, –591.6° **d)** $-\frac{7\pi}{4}, \frac{9\pi}{4}$

3. a) $115° + 360°n; n \in \text{I}$

b) $\frac{7\pi}{3} + 2\pi n, n \in \text{I}$

c) 13.4 rad + $2\pi n$ rad; $n \in \text{I}$

4. a) 252° **b)** $\frac{11\pi}{6}$ **c)** 269.3° **d)** –0.88

5. a) negative **b)** negative **c)** positive
d) positive **e)** negative **f)** negative

6. a) $\frac{1}{2}$ **b)** $\frac{2\sqrt{3}}{3}$ **c)** $\frac{-\sqrt{3}}{3}$ **d)** $\frac{3}{4}$

7. a) 0.58 **b)** –1.06
c) 0.41 **d)** 225.68

8. a) $\frac{5\pi}{6}$ **b)** $\frac{8\pi}{3}$ **c)** $\frac{5\pi^2}{8}$ **d)** $\frac{-\pi}{24}$

9. a) $x(x - 1)$ **b)** $(x - 4)(x + 1)$
c) $(x - y)(x + y)$ **d)** $3(x + 1)(x - 2)$
e) $2x(x - 2)(x + 5)$

10. a) $\theta \neq \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$

b) $\theta \neq -\frac{\pi}{2}, -\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$

11. a) $\theta \neq$ odd integral multiples of $\frac{\pi}{2}$

b) $\theta \neq$ odd integral multiples of $\frac{\pi}{2}$, odd integral multiples of $\frac{\pi}{4}$

12. $\theta = \frac{\pi}{6}, \frac{11\pi}{6}$

13. $\theta = \frac{\pi}{6} + 2\pi n; n \in \text{I}$ or $\theta = \frac{11\pi}{6} + 2\pi n; n \in \text{I}$

BLM 6–2 Section 6.1 Extra Practice

1. a) $x \neq \frac{\pi}{2} + \pi n; n \in \text{I}$

b) $x \neq \pi n; n \in \text{I}$ and $x \neq \frac{\pi}{2} + \pi n; n \in \text{I}$

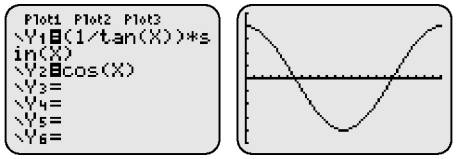
c) $x \neq 2\pi n; n \in \text{I}$ and $x \neq \frac{\pi}{2} + \pi n; n \in \text{I}$

d) $x \neq \frac{\pi}{2} n; n \in \text{I}$

2. $\theta \neq \pi n; n \in \text{I}$

3. a) $\cos x$ **b)** $\cos x$ **c)** $\sin x$ **d)** $\tan x$

4.



5. a) 2 **b)** 1 **c)** $\sec^2 x$ **d)** 1 **e)** $\sin x \cos x$ **f)** 1

6. a) may be an identity
b) not an identity **c)** not an identity

7. a) $\cot x$ **b)** $\sec x$ **c)** $\csc x$

8. Left side $= \sin^4\left(\frac{\pi}{6}\right) - \cos^4\left(\frac{\pi}{6}\right)$

$= \frac{1}{16} - \frac{9}{16}$

$= -\frac{1}{2}$

Right side $= 2\sin^2\left(\frac{\pi}{6}\right) - 1$

$= -\frac{1}{2}$

$=$ Left side

9. Left side $= \sec\left(\frac{\pi}{4}\right) + \sec\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right)$

$= \frac{1}{\cos\left(\frac{\pi}{4}\right)} + \frac{\cos\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right)}$

$= \frac{2}{\sqrt{2}} + 1$

Right side $= 1 + \sec\left(\frac{\pi}{4}\right)$

$= 1 + \frac{1}{\cos\left(\frac{\pi}{4}\right)}$

$= 1 + \frac{2}{\sqrt{2}}$

$=$ Left side

10. a)

LS $= \cos\left(\frac{\pi}{6}\right) \div \left(\sec\left(\frac{\pi}{6}\right) - 1\right) + \cos\left(\frac{\pi}{6}\right) \div \left(\sec\left(\frac{\pi}{6}\right) + 1\right)$

$= \frac{\sqrt{3}}{2} \div \left(\frac{2}{\sqrt{3}} - 1\right) + \frac{\sqrt{3}}{2} \div \left(\frac{2}{\sqrt{3}} + 1\right)$

$= \frac{\sqrt{3}}{2} \div \left(\frac{2 - \sqrt{3}}{\sqrt{3}}\right) + \frac{\sqrt{3}}{2} \div \left(\frac{2 + \sqrt{3}}{\sqrt{3}}\right)$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2 - \sqrt{3}} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2 + \sqrt{3}}$

$= \frac{3}{4 - 2\sqrt{3}} + \frac{3}{4 + 2\sqrt{3}}$

$= \frac{12 + 6\sqrt{3} + 12 - 6\sqrt{3}}{16 - 12}$

$= 6$

RS $= 2\cot^2\left(\frac{\pi}{6}\right)$

$= \frac{2}{\tan^2\left(\frac{\pi}{6}\right)}$

$= 2 \div \frac{1}{3}$

$= 6$

$=$ LS

b) $x \neq 0°, 180°$

11. $\cos^2 x + \sin^2 x = 1$

$\frac{\cos^2 x}{\sin^2 x} + \frac{\sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$

$\cot^2 x + 1 = \csc^2 x$

BLM 6–3 Section 6.2 Extra Practice

1. a) $\sin 63°$ **b)** $\cos 17°$

c) $\cos\left(-\frac{\pi}{6}\right)$ **d)** $\sin\left(\frac{\pi}{12}\right)$

2. a) $\frac{1}{2}$ **b)** $\frac{\sqrt{3}}{2}$ **c)** 1 **d)** $\frac{1}{\sqrt{2}}$

3. a) $\sin\frac{\pi}{3}$ **b)** $\cos\frac{2\pi}{3}$

c) $\cos 30°$ **d)** $\tan\frac{\pi}{3}$

4. a) cos A **b)** –sin A **c)** –sin A **d)** cos A

5. a) cos A **b)** –cos A **c)** cos A **d)** –sin A

6. a) $\cos\theta$ **b)** $\cos(4x)$ **c)** $-\sin\theta$ **d)** $-\sin\theta$

7. a) $-\frac{1}{2}$ **b)** $2 - \sqrt{3}$ **c)** $\frac{\sqrt{3} + 1}{2\sqrt{2}}$ **d)** $-\frac{\sqrt{3}}{2}$

8. a) true **b)** false **c)** true **d)** false

9. a) $\frac{56}{65}$ **b)** $\frac{63}{65}$ **c)** $\frac{-119}{169}$ **d)** $\frac{24}{25}$

10. $-\frac{120}{169}$

BLM 6–4 Section 6.3 Extra Practice

1. a) $\frac{1}{\sin x}$ **b)** $\frac{1}{\sin^2 x}$ **c)** $\cos x$

2. a) $\tan x$ **b)** $\frac{\sin x - 2}{5}$ **c)** $\frac{\cos x + 2}{7}$

d) $\sin x - 1$

3. a) Example:

Left side $= \csc^2 x\left(1 - \cos^2 x\right)$

$= \frac{1}{\sin^2 x}(\sin^2 x)$

$= 1$

$=$ Right side

b) Example:

$$\begin{aligned}\text{Left side} &= (\tan x - 1)^2\\ &= \tan^2 x - 2\tan x + 1\\ &= \frac{\sin^2 x - 2\sin x\cos x + \cos^2 x}{\cos^2 x}\\ \text{Right side} &= \frac{1}{\cos^2 x} - \frac{2\sin x}{\cos x}\\ &= \frac{1 - 2\sin x\cos x}{\cos^2 x}\\ &= \frac{\sin^2 x - 2\sin x\cos x + \cos^2 x}{\cos^2 x}\\ &= \text{Left side}\end{aligned}$$

c) Example:

$$\begin{aligned}\text{Right side} &= \cos x\\ \text{Left side} &= \frac{\sin^2 x + \cos^2 x}{\sec x}\\ &= 1 \div \frac{1}{\cos x}\\ &= \cos x\\ &= \text{Right side}\end{aligned}$$

4. a) Example:

$$\begin{aligned}\text{Right side} &= \tan x\\ \text{Left side} &= \frac{1 + \tan x}{1 + \cot x}\\ &= 1 + \frac{\sin x}{\cos x} \div \left(1 + \frac{\cos x}{\sin x}\right)\\ &= \frac{\cos x + \sin x}{\cos x} \times \frac{\sin x}{\sin x + \cos x}\\ &= \frac{\sin x}{\cos x}\\ &= \tan x\\ &= \text{Right side}\end{aligned}$$

b) Example:

$$\begin{aligned}\text{Right side} &= \cot x\\ \text{Left side} &= \frac{\sec x}{\sin x} - \frac{\sin x}{\cos x}\\ &= \frac{1}{\sin x\cos x} - \frac{\sin^2 x}{\sin x\cos x}\\ &= \frac{\cos^2 x}{\sin x\cos x}\\ &= \cot x\\ &= \text{Right side}\end{aligned}$$

c) Example:

$$\begin{aligned}\text{Right side} &= \csc x\\ &= \frac{1}{\sin x}\\ \text{Left side} &= \frac{\cot x + \tan x}{\sec x}\\ &= \cos x\left(\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}\right)\\ &= \frac{\cos^2 x + \sin^2 x}{\sin x}\\ &= \frac{1}{\sin x}\\ &= \text{Right side}\end{aligned}$$

5. a) Example:

$$\begin{aligned}\text{Left side} &= \frac{\csc x + \cot x}{\tan x + \sin x}\\ &= \frac{1 + \cos x}{\sin x} \div \frac{\sin x + \sin x\cos x}{\cos x}\\ &= \frac{1 + \cos x}{\sin x} \times \frac{\cos x}{\sin x(1 + \cos x)}\\ &= \frac{\cos x}{\sin^2 x}\\ \text{Right side} &= \cot x\csc x\\ &= \frac{\cos x}{\sin^2 x}\\ &= \text{Left side}\end{aligned}$$

b) Example:

$$\begin{aligned}\text{Right side} &= \tan x\\ \text{Left side} &= \frac{\sin x + \tan x}{\cos x + 1}\\ &= \frac{\sin x\cos x + \sin x}{\cos x} \times \frac{1}{\cos x + 1}\\ &= \frac{\sin x(\cos x + 1)}{\cos x} \times \frac{1}{\cos x + 1}\\ &= \tan x\\ &= \text{Right side}\end{aligned}$$

c) Example:

$$\begin{aligned}\text{Right side} &= \cot x\\ \text{Left side} &= \frac{\cos x + 1}{\sin x + \tan x}\\ &= (\cos x + 1) \times \frac{\cos x}{\sin x(\cos x + 1)}\\ &= \cot x\\ &= \text{Right side}\end{aligned}$$

6. a) Example:

$$\text{Right side} = \frac{1+\sin x}{\cos x}$$

$$\begin{aligned}\text{Left side} &= \frac{\cos x}{1-\sin x}\\ &= \frac{\cos x(1+\sin x)}{(1-\sin x)(1+\sin x)}\\ &= \frac{\cos x(1+\sin x)}{1-\sin^2 x}\\ &= \frac{\cos x(1+\sin x)}{\cos^2 x}\\ &= \frac{1+\sin x}{\cos x}\\ &= \text{Right side}\end{aligned}$$

b) Example:

$$\begin{aligned}\text{Left side} &= \frac{1+\cos x}{\sin x}\\ \text{Right side} &= \frac{\sin x}{1-\cos x}\times\frac{1+\cos x}{1+\cos x}\\ &= \frac{\sin x(1+\cos x)}{\sin^2 x}\\ &= \frac{1+\cos x}{\sin x}\\ &= \text{Left side}\end{aligned}$$

c) Example:

$$\begin{aligned}\text{Right side} &= 2\cot^2 x\\ &= \frac{2\cos^2 x}{\sin^2 x}\\ \text{Left side} &= \frac{\cos x}{\sec x-1}+\frac{\cos x}{\sec x+1}\\ &= \cos x\div\frac{1-\cos x}{\cos x}+\cos x\div\frac{1+\cos x}{\cos x}\\ &= \frac{\cos^2 x}{1-\cos x}+\frac{\cos^2 x}{1+\cos x}\\ &= \frac{2\cos^2 x}{1-\cos^2 x}\\ &= \frac{2\cos^2 x}{\sin^2 x}\\ &= \text{Right side}\end{aligned}$$

7. a) Example:

$$\begin{aligned}\text{Right side} &= \cos^2 x-\sin^2 y\\ \text{Left side} &= \cos(x+y)\cos(x-y)\\ &= (\cos x\cos y-\sin x\sin y)(\cos x\cos y+\sin x\sin y)\\ &= \cos^2 x\cos^2 y-\sin^2 x\sin^2 y\\ &= \cos^2 x(1-\sin^2 y)-\sin^2 y(1-\cos^2 x)\\ &= \cos^2 x-\sin^2 y\cos^2 x-\sin^2 y+\sin^2 y\cos^2 x\\ &= \cos^2 x-\sin^2 y\\ &= \text{Right side}\end{aligned}$$

b) Example:

$$\begin{aligned}\text{Right side} &= \cot x\\ &= \frac{\cos x}{\sin x}\\ \text{Left side} &= \frac{1+\cos 2x}{\sin 2x}\\ &= \frac{1+2\cos^2 x-1}{2\sin x\cos x}\\ &= \frac{\cos x}{\sin x}\\ &= \text{Right side}\end{aligned}$$

c) Example:

$$\begin{aligned}\text{Right side} &= (\sin x+\cos x)^2\\ &= \sin^2 x+2\sin x\cos x+\cos^2 x\\ &= 1+2\sin x\cos x\\ \text{Left side} &= 1+\sin 2x\\ &= 1+2\sin x\cos x\\ &= \text{Right side}\end{aligned}$$

d) Example:

$$\begin{aligned}\text{Right side} &= \frac{2}{1+\cos 2x}\\ &= \frac{2}{1+2\cos^2 x-1}\\ &= \frac{1}{\cos^2 x}\\ \text{Left side} &= \sec^2 x\\ &= \frac{1}{\cos^2 x}\\ &= \text{Right side}\end{aligned}$$

8. a) Verify for $x = 30°$:

$$\begin{aligned}\text{Left side} &= \sec^4 30° - \sec^2 30° \\ &= \frac{16}{9} - \frac{4}{3} \\ &= \frac{4}{9} \\ \text{Right side} &= \tan^4 30° + \tan^2 30° \\ &= \frac{1}{9} - \frac{1}{3} \\ &= \frac{4}{9} \\ &= \text{Left side}\end{aligned}$$

Example:

$$\begin{aligned}\text{Left side} &= \sec^4 x - \sec^2 x \\ &= \frac{1}{\cos^2 x}\left(\frac{1 - \cos^2 x}{\cos^2 x}\right) \\ &= \frac{\sin^2 x}{\cos^4 x} \\ \text{Right side} &= \tan^4 x + \tan^2 x \\ &= \tan^2 x(\tan^2 x + 1) \\ &= \frac{\sin^2 x}{\cos^2 x}\left(\frac{1}{\cos^2 x}\right) \\ &= \frac{\sin^2 x}{\cos^4 x} \\ &= \text{Left side}\end{aligned}$$

b) Verify for $x = 30°$:

$$\begin{aligned}\text{Left side} &= \cos 30° + \cos 30° \tan^2 30° \\ &= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{6} \\ &= \frac{2\sqrt{3}}{3} \\ \text{Right side} &= \sec 30° \\ &= \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{2\sqrt{3}}{3} \\ &= \text{Left side}\end{aligned}$$

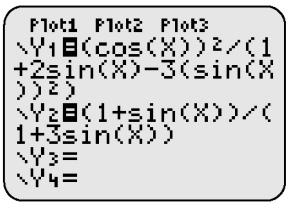
Example:

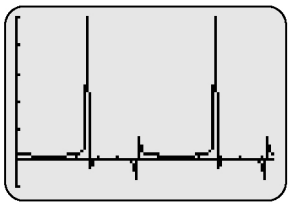
$$\begin{aligned}\text{Left side} &= \cos x + \cos x \tan^2 x \\ &= \frac{\cos^3 x + \cos x \sin^2 x}{\cos^2 x} \\ &= \frac{\cos x(\cos^2 x + \sin^2 x)}{\cos^2 x} \\ &\quad \frac{1}{\cos x} \\ \text{Right side} &= \sec x \\ &= \frac{1}{\cos x} \\ &= \text{Left side}\end{aligned}$$

9. a) Verify for $x = 3.2$:

$$\begin{aligned}\text{Left side} &= \frac{\cos^2 3.2}{1 + 2\sin 3.2 - 3\sin^2 3.2} \cdot \\ &\approx 1.14153 \\ \text{Right side} &= \frac{1 + \sin 3.2}{1 + 3\sin 3.2} \\ &\approx 1.14153\end{aligned}$$

b)





From the graph, the equation appears to be an identity.

10. a) Example:

$$\begin{aligned}\text{Left side} &= \tan\theta \\ \text{Right side} &= \frac{1 - \cos 2\theta}{\sin 2\theta} \\ &= \frac{1 - (1 - 2\sin^2\theta)}{2\sin\theta\cos\theta} \\ &= \frac{\sin\theta}{\cos\theta} \\ &= \tan\theta \\ &= \text{Left side}\end{aligned}$$

b) $\theta \neq \frac{n\pi}{2}; n \in \mathrm{I}$

11. Example:

$$\begin{aligned}\text{Left side} &= 1 + \sin 2x \\ &= 1 + 2\sin x\cos x\end{aligned}$$

$$\begin{aligned}\text{Right side} &= (\sin x + \cos x)^2 \\ &= \sin^2 x + 2\sin x\cos x + \cos^2 x \\ &= 1 + 2\sin x\cos x \\ &= \text{Left side}\end{aligned}$$

12. Example:

$$\begin{aligned}\text{Left side} &= \cos 3x + 1 \\ &= \cos(2x + x) + 1 \\ &= \cos 2x\cos x - \sin 2x\sin x + 1 \\ &= \cos x(2\cos^2 x - 1) - 2\sin x(\sin x\cos x) + 1 \\ &= 2\cos^3 x - \cos x - 2\sin^2 x\cos x + 1 \\ &= 2\cos^3 x - \cos x - 2\cos x(1 - \cos^2 x) + 1 \\ &= 4\cos^3 x - 3\cos x + 1\end{aligned}$$

$$\begin{aligned}\text{Right side} &= 4\cos^3 x - 3\cos x + 1 \\ &= \text{Left side}\end{aligned}$$

BLM 6–5 Section 6.4 Extra Practice

1. a) $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

b) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$, and $\frac{7\pi}{4}$

c) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$, and $\frac{7\pi}{4}$ **d)** π

2. a) 0°, 72°, 144°, 216°, 288° **b)** 270°
c) 90°, 270°

3. a) $\frac{2\pi}{3}, \frac{4\pi}{3}$

b) no solution

c) $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$

4. –135°, –45°, 45°, 135°

5. $\frac{3\pi}{4}, \frac{7\pi}{4}$

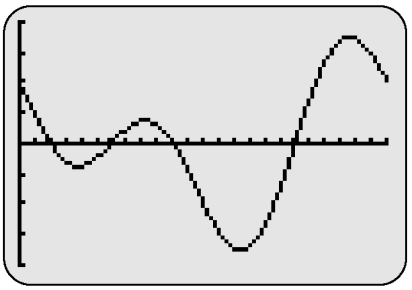
6. The error was in dividing 1 by 2.

$$\begin{aligned}\sin 2x &= 1 \\ 2x &= 90° \\ x &= 45°\end{aligned}$$

7. a) The student used 2π rather than π; because the equation is $\sin 2x = 1$, the period of the function is π.

b) $\frac{\pi}{4} + \pi n, \frac{5\pi}{4} + \pi n; n \in \text{I}$

8. a) Graph the function $Y_1 = \cos x - 2\sin x\cos x$ using Xmin = 0 and Xmax = 2π. The x-intercepts are the solutions.



b) $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

9. $\frac{\pi}{2} + 2\pi n, \frac{\pi}{4} + \pi n; n \in \text{I}$

10. 60°, 120°, 240°, 300°
General Solution: $60° + 180°n$, $120° + 180°n$; $n \in \text{I}$

BLM 6–7 Chapter 6 Test

1. A **2.** C
3. A **4.** B **5.** D

6. $\frac{-\sqrt{6} - \sqrt{2}}{4}$

7. 0.23

8. 150°

9. $\frac{5}{13}$

10. $\sin\left(\frac{5y}{2}\right)$

11. a)

$$\begin{aligned}\text{Left side} &= \sin\left(x + \frac{\pi}{2}\right) \\ &= \sin\left(\frac{\pi}{2} + \frac{\pi}{2}\right) \\ &= \sin(\pi) \\ &= 0\end{aligned}$$

$$\begin{aligned}\text{Right side} &= \csc x - 1 \\ &= \csc\frac{\pi}{2} - 1 \\ &= 1 - 1 \\ &= 0\end{aligned}$$

b) No; it is not true for all permissible values of x.

12. a)

$$\begin{aligned}\text{Left side} &= \sin^2 x + \cos^4 x \\ &= \sin^2 30° + \cos^4 30° \\ &= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^4 \\ &= \frac{13}{16}\end{aligned}$$

$$\begin{aligned}\text{Right side} &= \cos^2 x + \sin^4 x \\ &= \cos^2 30° + \sin^4 30° \\ &= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^4 \\ &= \frac{13}{16}\end{aligned}$$

b) Example:

$$\begin{aligned}\text{Left side} &= \sin^2 x + \cos^4 x \\ &= \sin^2 x + (1 - \sin^2 x)^2 \\ &= \sin^2 x + 1 - 2\sin^2 x + \sin^4 x \\ &= 1 - \sin^2 x + \sin^4 x \\ &= \cos^2 x + \sin^4 x \\ &= \text{Right side}\end{aligned}$$

$$\text{Right side} = \cos^2 x + \sin^4 x$$

13. a) $x \neq 0°, 90°, 180°, 270°, 360°$

b) Example:

$$\begin{aligned}\text{Left side} &= \frac{\tan x + \sec x}{\cot x} \\ &= \left(\frac{\sin x}{\cos x} + \frac{1}{\cos x}\right) \div \cot x \\ &= \left(\frac{\sin x + 1}{\cos x}\right) \div \left(\frac{\cos x}{\sin x}\right) \\ &= \left(\frac{\sin x + 1}{\cos x}\right) \times \left(\frac{\sin x}{\cos x}\right) \\ &= \frac{(\sin x + 1)\sin x}{1 - \sin^2 x} \\ &= \frac{(\sin x + 1)\sin x}{(1 - \sin x)(1 + \sin x)} \\ &= \frac{\sin x}{(1 - \sin x)} \\ &= \text{Right side}\end{aligned}$$

$$\text{Right side} = \frac{\sin x}{1 - \sin x}$$

14. $\pm\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

15. $\frac{\pi}{3} + \pi n, \frac{2\pi}{3} + \pi n; n \in \mathrm{I}$

BLM U2–2 Unit 2 Test

1. A

2. D

3. A

4. A

5. B

6. B

7. D

8. 2.18 m

9. $\frac{\sqrt{3} - 1}{2}$

10. $\frac{\pi}{3} + \pi n, \frac{2\pi}{3} + \pi n, n \in \mathrm{I}$

11. $y = 6\cos\frac{\pi}{5}(x - 3) + 4$

12. 2

13. $\mathrm{B} \neq 180°n, n \in \mathrm{I}$

14. $\tan 60°; \sqrt{3}$

15. a) Period: 4π; phase shift: 4 left

b) The graph of $f(\theta)$ is translated left $\frac{9\pi}{2}$

c) -1.9

16. a)

$$\begin{aligned}\text{Left side} &= \frac{1 - \tan^2 30°}{1 + \tan^2 30°} \\ &= \frac{1 - \left(\frac{1}{\sqrt{3}}\right)^2}{1 + \left(\frac{1}{\sqrt{3}}\right)^3} \\ &= \frac{1}{2}\end{aligned}$$

$$\begin{aligned}\text{Right side} &= \cos(2 \times 30°) \\ &= \cos 60° \\ &= \frac{1}{2}\end{aligned}$$

$$\text{Right side} = \text{Left side}$$

b) Left side $= \dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}$

$= \left(1 - \dfrac{\sin^2\theta}{\cos^2\theta}\right) \div \sec^2\theta$

$= \left(\dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}\right) \times \cos^2\theta$

$= \cos^2\theta - \sin^2\theta$

$= \cos 2\theta$

Left side $=$ Right side

c) $x \neq \dfrac{\pi}{2} + \pi n, n \in \mathrm{I}$

17. a) Amplitude: 2; range: $1 \leq y \leq 5$

b) $y = 2\sin 3\left(x + \dfrac{\pi}{6}\right) + 3$; $y = 2\cos 3x + 3$

18. a) $b = 3$

b) original graph is horizontally stretched by a factor of 6

19. a) Yes;
$2\sin^2 x - \sin x - 1 = (2\sin x + 1)(\sin x - 1)$

b) 90°, 210°, 330°