## Lesson 1.3 Exercises, pages 35–42

**A**

**3.** Which sequences could be geometric? If a sequence is geometric, state its common ratio.

**a)** 1, 2, 4, 8, 16, . . .

The sequence is geometric.
$r$ is: $\frac{2}{1} = 2$

**b)** 4, 9, 16, 25, 36, . . .

The sequence is not geometric.

**c)** $-3$, 2, 7, 12, 17, . . .

The sequence is not geometric.

**d)** 6, 0.6, 0.06, 0.006, . . .

The sequence is geometric.
$r$ is: $\frac{0.6}{6} = 0.1$

**e)** 10, 100, 1000, 10 000

The sequence is geometric.
$r$ is: $\frac{100}{10} = 10$

**f)** 2, 4, 6, 8, 10, . . .

The sequence is not geometric.

4. State the common ratio, then write the next 3 terms of each geometric sequence.

a) $-1, -3, -9, \ldots$

$r$ is $\frac{-3}{-1} = 3$

The next 3 terms are:

$-27, -81, -243$

b) $48, 24, 12, \ldots$

$r$ is $\frac{24}{48} = 0.5$

The next 3 terms are:

$6, 3, 1.5$

c) $4, -2, 1, \ldots$

$r$ is $\frac{-2}{4} = -0.5$

The next 3 terms are:

$-0.5, 0.25, -0.125$

d) $\frac{1}{2}, \frac{1}{6}, \frac{1}{18}, \ldots$

$r$ is $\frac{1}{6} \div \frac{1}{2} = \frac{1}{3}$

The next 3 terms are:

$\frac{1}{54}, \frac{1}{162}, \frac{1}{486}$

5. For each geometric sequence, determine the indicated value.

a) $3, 6, 12, \ldots$; determine $t_7$

$r$ is: $\frac{6}{3} = 2$

Use: $t_n = t_1 r^{n-1}$

Substitute: $n = 7$, $t_1 = 3$, $r = 2$

$t_7 = 3(2)^{7-1}$

$t_7 = 192$

b) $18, 9, 4.5, \ldots$; determine $t_6$

$r$ is: $\frac{9}{18} = 0.5$

Use: $t_n = t_1 r^{n-1}$

Substitute: $n = 6$, $t_1 = 18$, $r = 0.5$

$t_6 = 18(0.5)^{6-1}$

$t_6 = 0.5625$

c) $23, -46, 92, \ldots$; determine $t_{10}$

$r$ is: $\frac{-46}{23} = -2$

Use: $t_n = t_1 r^{n-1}$

Substitute:

$n = 10$, $t_1 = 23$, $r = -2$

$t_{10} = 23(-2)^{10-1}$

$t_{10} = -11\ 776$

d) $2, \frac{1}{2}, \frac{1}{8}, \ldots$; determine $t_5$

$r$ is: $\frac{\frac{1}{2}}{2} = \frac{1}{4}$

Use: $t_n = t_1 r^{n-1}$

Substitute:

$n = 5$, $t_1 = 2$, $r = \frac{1}{4}$

$t_5 = 2\left(\frac{1}{4}\right)^{5-1}$

$t_5 = \frac{1}{128}$

**B**

6. Write the first 4 terms of each geometric sequence, given the 1st term and the common ratio. Identify the sequence as decreasing, increasing, or neither. Justify your answers.

a) $t_1 = -3$; $r = 4$

$t_1 = -3$

$t_2$ is $(-3)(4) = -12$

$t_3$ is $(-12)(4) = -48$

$t_4$ is $(-48)(4) = -192$

The sequence is decreasing because the terms are decreasing.

b) $t_1 = 5$; $r = 2$

$t_1 = 5$

$t_2$ is $(5)(2) = 10$

$t_3$ is $(10)(2) = 20$

$t_4$ is $(20)(2) = 40$

The sequence is increasing because the terms are increasing.

**c)** $t_1 = -0.5; r = -3$

$t_1 = -0.5$
$t_2$ is $(-0.5)(-3) = 1.5$
$t_3$ is $(1.5)(-3) = -4.5$
$t_4$ is $(-4.5)(-3) = 13.5$
The sequence neither increases nor decreases because the terms alternate signs.

**d)** $t_1 = \frac{1}{2}; r = \frac{2}{3}$

$t_1 = \frac{1}{2}$
$t_2$ is $\left(\frac{1}{2}\right)\left(\frac{2}{3}\right) = \frac{1}{3}$
$t_3$ is $\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = \frac{2}{9}$
$t_4$ is $\left(\frac{2}{9}\right)\left(\frac{2}{3}\right) = \frac{4}{27}$
The sequence is decreasing because the terms are decreasing.

**7.** Write the first 5 terms of a geometric sequence where:

**a)** the 6th term is 64

Sample response: $t_6 = 64$
Divide by a common ratio that is a factor of 64, such as $-2$.

$t_6 = 64$ $\quad t_5 = \frac{64}{-2}$
$t_5 = -32$
$t_4 = \frac{-32}{-2}$ $\quad t_3 = \frac{16}{-2}$
$t_4 = 16$ $\quad t_3 = -8$
$t_2 = \frac{-8}{-2}$ $\quad t_1 = \frac{4}{-2}$
$t_2 = 4$ $\quad t_1 = -2$
A possible geometric sequence is:
$-2, 4, -8, 16, -32, \ldots$

**b)** the 1st term is $\frac{3}{4}$

Sample response:
Choose a value for $r$, such as $r = 4$.

$t_1 = \frac{3}{4}$ $\quad t_2 = \left(\frac{3}{4}\right)(4)$
$t_2 = 3$
$t_3 = (3)(4)$ $\quad t_4 = (12)(4)$
$t_3 = 12$ $\quad t_4 = 48$
$t_5 = (48)(4)$
$t_5 = 192$
A possible geometric sequence is:
$\frac{3}{4}, 3, 12, 48, 192, \ldots$

**c)** every term is negative

Sample response: Choose a negative 1st term and a positive common ratio, such as $t_1 = -5$ and $r = 2$.
$t_1 = -5$ $\quad t_2 = (-5)(2)$
$t_2 = -10$
$t_3 = (-10)(2)$ $\quad t_4 = (-20)(2)$
$t_3 = -20$ $\quad t_4 = -40$
$t_5 = (-40)(2)$
$t_5 = -80$
A possible geometric sequence is:
$-5, -10, -20, -40, -80, \ldots$

**d)** every term is an even number

Sample response: Choose an even 1st term and an odd or even common ratio, such as $t_1 = 4$ and $r = 3$
$t_1 = 4$ $\quad t_2 = (4)(3)$
$t_2 = 12$
$t_3 = (12)(3)$ $\quad t_4 = (36)(3)$
$t_3 = 36$ $\quad t_4 = 108$
$t_5 = (108)(3)$
$t_5 = 324$
A possible geometric sequence is:
4, 12, 36, 108, 324, . . .

**8.** Use the given data about each finite geometric sequence to determine the indicated values.

**a)** Given $t_1 = -1$ and $r = -2$

**i)** Determine $t_9$.

Use: $t_n = t_1r^{n-1}$ Substitute: $n = 9$, $t_1 = -1$, $r = -2$

$t_9 = (-1)(-2)^{9-1}$

$t_9 = (-1)(-2)^8$

$t_9 = -256$

**ii)** The last term is $-4096$. How many terms are in the sequence?

Use $t_n = t_1r^{n-1}$ to determine $n$.

Substitute: $t_n = -4096$, $t_1 = -1$, $r = -2$

$-4096 = (-1)(-2)^{n-1}$ Divide by $-1$.

$4096 = (-2)^{n-1}$

$(-2)^{12} = (-2)^{n-1}$ Equate exponents.

$12 = n - 1$

$n = 13$

There are 13 terms in the sequence.

**b)** Given $t_1 = 0.002$ and $t_4 = 2$

**i)** Determine $t_7$.

Use $t_n = t_1r^{n-1}$ to determine the common ratio, $r$.

Substitute: $n = 4$, $t_4 = 2$, $t_1 = 0.002$

$2 = 0.002r^{4-1}$

$2 = 0.002r^3$ Divide each side by 0.002.

$1000 = r^3$

$\sqrt[3]{1000} = r$

$r = 10$

To determine $t_7$, use: $t_n = t_1r^{n-1}$

Substitute: $n = 7$, $t_1 = 0.002$, and $r = 10$

$t_7 = 0.002(10)^{7-1}$

$t_7 = 0.002(10)^6$

$t_7 = 2000$

**ii)** Determine which term has the value 20 000.

Use $t_n = t_1r^{n-1}$ to determine $n$.

Substitute: $t_n = 20\,000$, $t_1 = 0.002$, $r = 10$

$20\,000 = 0.002(10)^{n-1}$

$10\,000\,000 = 10^{n-1}$

$10^7 = 10^{n-1}$

$7 = n - 1$

$n = 8$

20 000 is the 8th term.

**9.** **a)** Insert three numbers between 8 and 128, so the five numbers form an arithmetic sequence. Explain what you did.

**The sequence has the form: 8, $8 + d$, $8 + 2d$, $8 + 3d$, 128**
**Write $128 = 8 + 4d$, then solve for $d$ to get $d = 30$.**
**The arithmetic sequence is: 8, 38, 68, 98, 128**

**b)** Insert three numbers between 8 and 128, so the five numbers form a geometric sequence. Explain what you did.

**The sequence has the form: 8, $8r$, $8r^2$, $8r^3$, 128**
**Write $128 = 8r^4$, then solve for $r$ to get $r^4 = 16$, so $r = 2$ or $-2$.**
**The geometric sequence is: 8, 16, 32, 64, 128;**
**or 8, −16, 32, −64, 128**

**c)** What was similar about your strategies in parts a and b? What was different?

**For each sequence, I wrote an equation for the 5th term, then solved the equation to determine the common difference and common ratio. For the arithmetic sequence, I added the common difference to get the next terms. For the geometric sequences, there were two possible common ratios, and I multiplied by each common ratio to get the next terms.**

**10.** Suppose a person is given 1¢ on the first day of April; 3¢ on the second day; 9¢ on the third day, and so on. This pattern continues throughout April.

**a)** About how much money will the person be given on the last day of April?

**There are 30 days in April.**
**The daily amounts, in cents, form this geometric sequence:**
**$1, 1(3), 1(3)^2, \ldots, 1(3)^{29}$**
**The amount on the last day, in cents, is $1(3)^{29} \doteq 6.863 \times 10^{13}$**
**Divide by 100 to convert the amount to dollars:**
**approximately $\$6.863 \times 10^{11}$**

**b)** Why might it be difficult to determine the exact amount using a calculator?

**A calculator screen shows only 10 digits, and the number of digits in the amount of money in dollars is greater than 10.**

**11.** In a geometric sequence, $t_3 = 9$ and $t_6 = 1.125$; determine $t_7$ and $t_9$.

Use $t_n = t_1r^{n-1}$ twice to get two equations.
For $t_3$, substitute: $n = 3$, $t_n = 9$
$9 = t_1r^{3-1}$
$9 = t_1r^2$ ①
For $t_6$, substitute: $n = 6$, $t_n = 1.125$
$1.125 = t_1r^{6-1}$
$1.125 = t_1r^5$ ②
Write equation ② as: $1.125 = t_1r^2(r^3)$
From equation ①, substitute $t_1r^2 = 9$
$1.125 = 9r^3$ Divide by 9.
$0.125 = r^3$
$r = \sqrt[3]{0.125}$
$r = 0.5$
So, $t_7 = t_6r$
$= 1.125(0.5)$
$= 0.5625$
and, $t_9 = t_7r^2$
$= 0.5625(0.5)^2$
$= 0.140\,625$

**12.** An arithmetic sequence and a geometric sequence have the same first term. The common difference and common ratio are equal and greater than 1. Which sequence increases more rapidly as more terms are included? Use examples to explain.

This arithmetic sequence has $t_1 = 3$ and $d = 4$:
3, 7, 11, 15, 19, 23, . . .
This geometric sequence has $t_1 = 3$ and $r = 4$:
3, 12, 48, 192, 768, 3072, . . .
The geometric sequence increases more rapidly because we are multiplying instead of adding to get the next term.

**13.** A ream of paper is about 2 in. thick. Imagine a ream of paper that is repeatedly cut in half and the two halves stacked one on top of the other. How many cuts have to be made before the stack of paper is taller than 318 ft., the height of Le Chateau York in Winnipeg, Manitoba?

Let the number of cuts be $n$.
The heights of the stacks of paper form, in inches, a geometric sequence with 1st term 2 and common ratio 2:
$2, \quad 2(2), \quad 2(2)^2, \quad 2(2)^3, \quad \ldots \quad 2(2)^n$
Write 318 ft. in inches: 318(12 in.) = 3816 in.
Write an equation:
$2(2)^n = 3816$ Solve for $n$.
$2^n = 1908$
Use guess and test: $2^{10} = 1024$; $2^{11} = 2048$
10 cuts will not be enough.
11 cuts will produce a stack that is: $2(2)^{11}$ in. = 4096 in. high
11 cuts have to be made.

**14.** Between the Canadian censuses in 2001 and 2006, the number of people who could converse in Cree had increased by 7%. In 2006, 87 285 people could converse in Cree. Assume the 5-year increase continues to be 7%. Estimate to the nearest hundred how many people will be able to converse in Cree in 2031.

**To model a growth rate of 7%, multiply by 1.07.**
**The number of people every 5 years form a geometric sequence with first term 87 285 and common ratio 1.07.**
**Every 5 years is: 2006, 2011, 2016, 2021, 2026, 2031, . . .**
**So, the number of people in 2031 is:**
**$87\,285(1.07)^5 = 122\,421.7278\ldots$**
**The number of people who will be able to converse in Cree in 2031 will be approximately 122 400.**

**C**

**15.** A farmer in Saskatchewan wants to estimate the value of a new combine after several years of use. A new combine worth \$370 000 depreciates in value by about 10% each year.

**a)** Estimate the value of the combine at the end of each of the first 5 years. Write the values as a sequence.

**When the value decreases by 10%, the new value is 90% of the original value.**
**To determine a depreciation value of 10%, multiply by 0.9.**
**The values, in dollars, at the end of each of the first 5 years are:**
**$370\,000(0.9)$, $370\,000(0.9)^2$, $370\,000(0.9)^3$, $370\,000(0.9)^4$, $370\,000(0.9)^5$**
**The values, to the nearest dollar, are: \$333 000, \$299 700, \$269 730, \$242 757, \$218 481**

**b)** What type of sequence did you write in part a? Explain your reasoning.

**The sequence is geometric because I multiplied by a constant to get each value from the preceding value.**

**c)** Predict the value of the combine at the end of 10 years.

**At the end of 10 years, to the nearest dollar, the value is:**
**$\$370\,000(0.9)^{10} = \$129\,011$**

**16. a)** Show that squaring each term in a geometric sequence produces the same type of sequence. What is the common ratio?

**Consider the sequence: $t_1, t_1r, t_1r^2, t_1r^3, t_1r^4, \ldots, t_1r^{n-1}$**
**Square each term. The new sequence is:**
**$t_1^2, t_1^2r^2, t_1^2r^4, t_1^2r^6, t_1^2r^8, \ldots, t_1^2r^{2n-2}$**
**This is a geometric sequence with 1st term $t_1^2$ and common ratio $r^2$.**

**b)** Show that raising each term in a geometric sequence to the *m*th power of each term produces the same type of sequence. What is the common ratio?

**Consider the sequence: $t_1, t_1r, t_1r^2, t_1r^3, \ldots, t_1r^{n-1}$**
**Raise each term to the *m*th power.**
**The new sequence is: $t_1^m, t_1^m r^m, t_1^m r^{2m}, t_1^m r^{3m}, \ldots, t_1^m r^{mn-m}$**
**This is a geometric sequence with 1st term $t_1^m$ and common ratio $r^m$.**

 