Checkpoint 2: Assess Your Understanding, pages 55–57

1.3

1. **Multiple Choice** For this geometric sequence: $-5000, 500, -50, \ldots$; which number below is the value of t_9?

   A. 0.0005   **B.** -0.0005 **C.** 0.000 05   **D.** $-0.000\ 05$

2. This sequence is geometric: $2, -6, 18, -54, \ldots$

   a) Write a rule to determine the nth term.

   Use: $t_n = t_1 r^{n-1}$ Substitute: $t_1 = 2, r = -3$
   $t_n = 2(-3)^{n-1}$

   b) Use your rule to determine the 10th term.

   Use: $t_n = 2(-3)^{n-1}$ Substitute: $n = 10$
   $t_{10} = 2(-3)^9$
   $t_{10} = -39\ 366$
   The 10th term is $-39\ 366$.

3. Use the given data about each geometric sequence to determine the indicated value.

a) $t_4 = -5$ and $t_7 = 135$; determine t_1

Use $t_n = t_1 r^{n-1}$ to determine r.
First substitute: $n = 4$, $t_n = -5$
$-5 = t_1 r^{4-1}$
$-5 = t_1 r^3$ ①
Then substitute: $n = 7$, $t_n = 135$
$135 = t_1 r^{7-1}$
$135 = t_1 r^6$ ②
Write equation ② as: $135 = t_1 r^3(r^3)$
From equation ①, substitute $t_1 r^3 = -5$
$135 = -5r^3$ Divide by -5.
$-27 = r^3$
$r = \sqrt[3]{-27}$
$r = -3$
Substitute $r = -3$ in equation ①.
$-5 = t_1(-3)^3$
$t_1 = \dfrac{5}{27}$

b) $t_1 = -1$ and $t_4 = -19\ 683$; determine r

Use: $t_n = t_1 r^{n-1}$
Substitute: $n = 4$, $t_4 = -19\ 683$, $t_1 = -1$
$-19\ 683 = -1r^{4-1}$
$-19\ 683 = -1r^3$
$19\ 683 = r^3$
$\sqrt[3]{19\ 683} = r$
$r = 27$

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4. Multiple Choice The sum of the first 5 terms of a geometric series is $\dfrac{121}{81}$. The common ratio is $\dfrac{1}{3}$. What is the 2nd term?

A. $\dfrac{1}{3}$ **B.** 1   **C.** $\dfrac{121}{3}$ **D.** $\dfrac{1}{9}$

5. Use the given data about each geometric series to determine the indicated value.

a) $t_1 = -4, r = 3$; determine S_5

Use: $S_n = \dfrac{t_1(1 - r^n)}{1 - r}, r \neq 1$

Substitute: $n = 5, t_1 = -4, r = 3$

$S_5 = \dfrac{-4(1 - 3^5)}{1 - 3}$

$S_5 = -484$

b) $3125 + 625 + 125 + \ldots + \dfrac{1}{25}$; determine n

r is: $\dfrac{625}{3125} = \dfrac{1}{5}$

Use: $t_n = t_1 r^{n-1}$

Substitute: $t_n = \dfrac{1}{25}, t_1 = 3125, r = \dfrac{1}{5}$

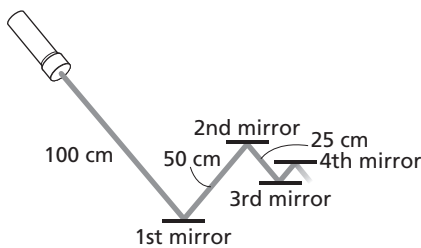$\dfrac{1}{25} = 3125\left(\dfrac{1}{5}\right)^{n-1}$

$\dfrac{1}{78\,125} = \left(\dfrac{1}{5}\right)^{n-1}$

$\left(\dfrac{1}{5}\right)^7 = \left(\dfrac{1}{5}\right)^{n-1}$

$7 = n - 1$

$n = 8$

6. The diagram shows a path of light reflected by mirrors. After the first path, the length of each path is one-half the preceding length.

100 cm · 50 cm · 2nd mirror · 25 cm · 4th mirror · 3rd mirror · 1st mirror

a) What is the length of the path from the 4th mirror to the 5th mirror?

The lengths of the paths, in centimetres, form this geometric sequence: 100, 50, 25, 12.5, 6.25, . . .
The length of the path from the 4th mirror to the 5th mirror is the 5th term: 6.25 cm

b) To the nearest hundredth of a centimetre, what is the total length of the path from the 1st mirror to the 10th mirror?

The total length of the path is the sum of the first 10 terms of this geometric series: $100 + 50 + 25 + 12.5 + 6.25 + \ldots$

Use: $S_n = \dfrac{t_1(1 - r^n)}{1 - r}, r \neq 1$ Substitute: $n = 10, t_1 = 100, r = 0.5$

$S_{10} = \dfrac{100(1 - 0.5^{10})}{1 - 0.5}$

$S_{10} = 199.8046\ldots$
The path is approximately 199.80 cm long.