PRACTICE TEST, pages 80–82

1. Multiple Choice What is the sum of the first 30 terms of this arithmetic series? $-5 - 2 + 1 + 4 + \ldots$

A. 1152 **B.** 1155 **C.** 1158 **D.** 1161

2. Multiple Choice What is the sum of the first 10 terms of this geometric series? $-12\,800 + 6400 - 3200 + 1600 - \ldots$

A. 8525 **B.** −8525 **C.** −8537.5 **D.** 8537.5

3. a) Which sequence below appears to be arithmetic? Justify your answer.

i) 4, −10, 16, −22, 28, . . . **ii)** 4, −10, −24, −38, −52, . . .

In part i, the differences of consecutive terms are:
−14, 26, −38, 50. Since these differences are not equal, the sequence is not arithmetic.
In part ii, the differences of consecutive terms are:
−14, −14, −14, −14. Since these differences are equal, the sequence appears to be arithmetic.

b) Assume that the sequence you identified in part a is arithmetic. Determine:

i) a rule for t_n **ii)** t_{17}

The arithmetic sequence is:
4, −10, −24, −38, −52, . . .
Use: $t_n = t_1 + d(n - 1)$
Substitute: $t_1 = 4, d = -14$
$t_n = 4 - 14(n - 1)$

Use: $t_n = 4 - 14(n - 1)$
Substitute: $n = 17$
$t_{17} = 4 - 14(17 - 1)$
$t_{17} = -220$

iii) the term that has value −332

Use: $t_n = 4 - 14(n - 1)$ **Substitute:** $t_n = -332$
$-332 = 4 - 14(n - 1)$
$24 = n - 1$
$n = 25$
The 25th term has value −332.

4. For a geometric sequence, $t_4 = -1000$ and $t_7 = 1$; determine:

a) t_1

Use: $t_7 = t_4r^3$
Substitute:
$t_7 = 1$, $t_4 = -1000$
$1 = -1000r^3$
$r^3 = \frac{1}{-1000}$
$r = -\frac{1}{10}$, or -0.1
Use: $t_n = t_1r^{n-1}$
Substitute:
$n = 7$, $t_7 = 1$, $r = -0.1$
$1 = t_1(-0.1)^{7-1}$
$t_1 = \frac{1}{(-0.1)^6}$
$t_1 = 1\,000\,000$

b) the term with value 0.0001

Use: $t_n = t_1r^{n-1}$
Substitute:
$t_n = 0.0001$, $t_1 = 1\,000\,000$,
$r = -0.1$
$0.0001 = 1\,000\,000(-0.1)^{n-1}$
$0.000\,000\,000\,1 = (-0.1)^{n-1}$
$(-0.1)^{10} = (-0.1)^{n-1}$
$n = 11$
The 11th term has value 0.0001.

5. a) For the infinite geometric series below, identify which series converges and which series diverges. Justify your answer.

i) $100 - 150 + 225 - 337.5 + \ldots$

The common ratio, r, is: $\frac{-150}{100} = -1.5$
Since r is less than -1, the series diverges.

ii) $10 + 5 + 2.5 + 1.25 + \ldots$

r is: $\frac{5}{10} = \frac{1}{2}$
Since r is between -1 and 1, the series converges.

b) For which series in part a can you determine its sum? Explain why, then determine this sum.

I can determine the sum of an infinite geometric series that converges; that is, the series in part a ii).

Use: $S_\infty = \frac{t_1}{1 - r}$ Substitute: $t_1 = 10$, $r = \frac{1}{2}$, or 0.5
$S_\infty = \frac{10}{1 - 0.5}$, or 20
The sum of the series is 20.

6. This sequence represents the approximate lengths in centimetres of a spring that is stretched by loading it with from one to four 5-kg masses: 50, 54, 58, 62, . . .
Suppose the pattern in the sequence continues. What will the length of the spring be when it is loaded with ten 5-kg masses? Explain how you found out.

Since the differences between consecutive terms are equal, then the series appears to be arithmetic. The length of the spring, in centimetres, will be the 10th term of the arithmetic sequence.

Use: $t_n = t_1 + d(n - 1)$ **Substitute:** $n = 10$, $t_1 = 50$, $d = 4$

$t_n = 50 + 4(10 - 1)$

$t_n = 86$

The spring will be 86 cm long.

7. As part of his exercise routine, Earl uses a program designed to help him eventually do 100 consecutive push-ups. He started with 17 push-ups in week 1 and planned to increase the number of push-ups by 2 each week.

a) In which week does Earl expect to reach his goal?

The number of push-ups each week form an arithmetic sequence with $t_1 = 17$ **and** $d = 2$**. Determine** n **for** $t_n = 100$**.**

Use: $t_n = t_1 + d(n - 1)$ **Substitute:** $t_n = 100$, $t_1 = 17$, $d = 2$

$100 = 17 + 2(n - 1)$

$83 = 2n - 2$

$2n = 85$

$n = 42.5$

Earl should reach his goal in the 43rd week.

b) What is the total number of push-ups he will have done when he reaches his goal? Explain how you know.

The total number of push-ups is the sum of the first 43 terms of the arithmetic sequence.

Use: $S_n = \dfrac{n[2t_1 + d(n - 1)]}{2}$ **Substitute:** $n = 43$, $t_1 = 17$, $d = 2$

$$S_{43} = \frac{43[2(17) + 2(43 - 1)]}{2}$$

$S_{43} = 2537$

Earl will have done 2537 push-ups when he reaches his goal.