

# REVIEW, pages 76–79

## 1.1

1. During the 2003 fire season, the Okanagan Mountain Park fire was the most significant wildfire event in B.C. history. By September 7, the area burned had reached about 24 900 ha and the fire was spreading at a rate of about 150 ha/h.

- a) Suppose the fire continued to spread at the same rate. Create terms of a sequence to represent the area burned for each of the next 6 h. Why is the sequence arithmetic?

Each hour, the area increases by 150 ha. So, for each of the next 6 h, the area burned in hectares is:

25 050, 25 200, 25 350, 25 500, 25 650, 25 800, . . .

This sequence is arithmetic because the difference between consecutive terms is constant.

- b) Write a rule for the general term of the sequence in part a. Use the rule to predict the area burned after 24 h. What assumptions did you make?

Use:  $t_n = t_1 + d(n - 1)$       Substitute:  $t_1 = 25\,050$ ,  $d = 150$

The general term is:  $t_n = 25\,050 + 150(n - 1)$

Substitute:  $n = 24$

$t_{24} = 25\,050 + 150(24 - 1)$

$t_{24} = 28\,500$

After 24 h, the area burned was 28 500 ha; I assumed that the fire continued to spread at the same rate.

## 1.2

2. Use the given data about each arithmetic series to determine the indicated value.

- a)  $5 + 3\frac{1}{2} + 2 + \frac{1}{2} - 1 - \dots$ ;      b)  $S_{12} = 78$  and  $t_1 = -21$ ;  
determine  $S_{21}$       determine  $t_{12}$

$$\text{Use: } S_n = \frac{n[2t_1 + d(n - 1)]}{2}$$

Substitute:

$$n = 21, t_1 = 5, d = -1.5$$

$$S_{21} = \frac{21[2(5) - 1.5(21 - 1)]}{2}$$

$$S_{21} = -210$$

$$\text{Use: } S_n = \frac{n(t_1 + t_n)}{2}$$

Substitute:

$$n = 12, S_{12} = 78, t_1 = -21$$

$$78 = \frac{12(-21 + t_{12})}{2}$$

$$78 = 6(-21 + t_{12})$$

$$13 = -21 + t_{12}$$

$$t_{12} = 34$$

**1.3**

3. Explain the meaning of this newspaper headline.

**I-Pod Sales Grew Geometrically from 2001 to 2006**

In 2002, the number of sales was equal to a constant multiplied by the number of sales in 2001. In 2003, the number of sales was equal to the same constant multiplied by the number of sales in 2002. This pattern continued up to 2006.

4. A soapstone carving was appraised at \$2500. The value of the carving is estimated to increase by 12% each year. What will be the approximate value of the carving after 15 years?

The values of the carving, in dollars, form a geometric sequence with  $t_1 = 2500$  and  $r = 1.12$ . The value, in dollars, after 15 years is  $t_{16}$ .

Use  $t_n = t_1 r^{n-1}$       Substitute:  $n = 16, t_1 = 2500, r = 1.12$

$$t_{16} = 2500(1.12)^{15}$$

$$t_{16} = 13\,683.9144 \dots$$

After 15 years, the carving will be worth approximately \$13 684.

**1.4**

5. Determine the sum of the geometric series below. Give the answer to 3 decimal places.

$$-700 + 350 - 175 + \dots + 5.468\,75$$

Use:  $t_n = t_1 r^{n-1}$  to determine  $n$ .

Substitute:  $t_n = 5.468\,75, t_1 = -700, r$  is:  $\frac{350}{-700} = -0.5$

$$5.468\,75 = -700(-0.5)^{n-1}$$

$$-0.007\,812\,5 = (-0.5)^{n-1}$$

$$(-0.5)^7 = (-0.5)^{n-1}$$

$$n = 8$$

Then, use:  $S_n = \frac{t_1(1-r^n)}{1-r}, r \neq 1$

Substitute:  $n = 8, t_1 = -700, r = -0.5$

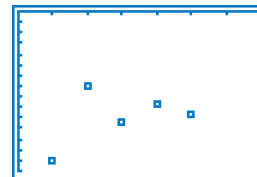
$$S_8 = \frac{-700(1 - (-0.5)^8)}{1 - (-0.5)}$$

$$S_8 \doteq -464.844$$

**1.5**

6. Use a graphing calculator or graphing software.

Use the series from question 5. Graph the first 5 partial sums. Explain how the graph shows whether the series converges or diverges.



**Sample response:** The series has  $t_1 = -700$  and  $r = -0.5$ ; its partial sums are:  $-700, -350, -525, -437.5, -481.25, \dots$

The series converges because the points appear to approach a constant value of approximately  $-450$ .

**1.6**

7. Explain how you can use the common ratio of a geometric series to identify whether the series is convergent or divergent.

A geometric series with a common ratio less than 1 and greater than  $-1$  converges. A geometric series with a common ratio less than or equal to  $-1$  or greater than or equal to 1 diverges.

8. Identify each infinite geometric series that converges. Determine the sum of any series that converges.

a)  $2 - 3 + 4.5 - 6.75 + \dots$       b)  $\frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \frac{8}{81} + \dots$

$r$  is  $\frac{-3}{2} = -1.5$ ,  
so the series diverges.

$r$  is  $\frac{\frac{2}{9}}{\frac{1}{3}} = \frac{2}{3}$ ,  
so the series converges.

Use:  $S_\infty = \frac{t_1}{1-r}$

Substitute:  $t_1 = \frac{1}{3}, r = \frac{2}{3}$

$$S_\infty = \frac{\frac{1}{3}}{1 - \frac{2}{3}}, \text{ or } 1$$

9. A small steel ball bearing is moving vertically between two electromagnets whose relative strength varies each second. The ball bearing moves 10 cm up in the 1st second, then 5 cm down in the 2nd second, then 2.5 cm up in 3rd second, and so on. This pattern continues.

a) Assume the distance the ball bearing moves up is positive; the distance it moves down is negative.

i) Write a series to represent the distance travelled in 5 s.

**Each second, the distance is halved.**

**In the first 5 s, the distance in centimetres is:**

$$10 - 5 + 2.5 - 1.25 + 0.625$$

ii) Calculate the sum of the series. What does this sum represent?

Use:  $S_n = \frac{t_1(1 - r^n)}{1 - r}$ ,  $r \neq 1$     Substitute:  $n = 5$ ,  $t_1 = 10$ ,  $r = -0.5$

$$S_5 = \frac{10(1 - (-0.5)^5)}{1 - (-0.5)}$$

$$S_5 = 6.875$$

**After 5 s, the ball bearing is 6.875 cm above the magnet from which it started.**

b) Suppose this process continues indefinitely. What is the sum of the series?

**The series is infinite and converges.**

Use:  $S_\infty = \frac{t_1}{1 - r}$     Substitute:  $t_1 = 10$ ,  $r = -0.5$

$$S_\infty = \frac{10}{1 - (-0.5)}$$

$$S_\infty = 6.\bar{6}$$

**The sum of the series is  $6.\bar{6}$  cm.**