Lesson 2.3 Exercises, pages 114–119

A

3. **a)** Simplify each radical, if possible.

$\sqrt{27}$ $\quad 3\sqrt{2}$ $\quad \sqrt{8}$ $\quad \sqrt{32}$

$\sqrt{27} = \sqrt{9 \cdot 3} = 3\sqrt{3}$ $\quad 3\sqrt{2}$ $\quad \sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$ $\quad \sqrt{32} = \sqrt{16 \cdot 2} = 4\sqrt{2}$

$\sqrt{6}$ $\quad 2\sqrt{3}$ $\quad \sqrt{48}$ $\quad \sqrt{18}$

$\sqrt{6}$ $\quad 2\sqrt{3}$ $\quad \sqrt{48} = \sqrt{16 \cdot 3} = 4\sqrt{3}$ $\quad \sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2}$

b) Group the radicals in part a into sets of like radicals.

All radicals have index 2.

Radicals with radicand 2: $3\sqrt{2}$, $2\sqrt{2}$, $4\sqrt{2}$, $3\sqrt{2}$; that is, $3\sqrt{2}$, $\sqrt{8}$, $\sqrt{32}$, $\sqrt{18}$

Radicals with radicand 3: $3\sqrt{3}$, $2\sqrt{3}$, $4\sqrt{3}$; that is, $\sqrt{27}$, $2\sqrt{3}$, $\sqrt{48}$

4. Simplify by adding or subtracting like terms.

a) $3\sqrt{2} + 2\sqrt{2} - 5\sqrt{2}$

$= (3 + 2 - 5)\sqrt{2}$
$= 0$

b) $\sqrt{108} - 2\sqrt{3} - \sqrt{75}$

$= \sqrt{36 \cdot 3} - 2\sqrt{3} - \sqrt{25 \cdot 3}$
$= 6\sqrt{3} - 2\sqrt{3} - 5\sqrt{3}$
$= -\sqrt{3}$

c) $5\sqrt{7} + 2\sqrt{5} - \sqrt{28} + \sqrt{45}$

$= 5\sqrt{7} + 2\sqrt{5} - \sqrt{4 \cdot 7} + \sqrt{9 \cdot 5}$
$= 5\sqrt{7} + 2\sqrt{5} - 2\sqrt{7} + 3\sqrt{5}$
$= 3\sqrt{7} + 5\sqrt{5}$

d) $\sqrt[3]{16} + \sqrt[3]{375} - 3\sqrt[3]{2}$

$= \sqrt[3]{8 \cdot 2} + \sqrt[3]{125 \cdot 3} - 3\sqrt[3]{2}$
$= 2\sqrt[3]{2} + 5\sqrt[3]{3} - 3\sqrt[3]{2}$
$= -\sqrt[3]{2} + 5\sqrt[3]{3}$

5. Simplify.

a) $6\sqrt{x} - 4\sqrt{x} + 2\sqrt{x} + \sqrt{x}, x \geq 0$

$= (6 - 4 + 2 + 1)\sqrt{x}$
$= 5\sqrt{x}$

b) $\sqrt{4a} + \sqrt{16a} - \sqrt{9a}, a \geq 0$

$= \sqrt{4 \cdot a} + \sqrt{16 \cdot a} - \sqrt{9 \cdot a}$
$= 2\sqrt{a} + 4\sqrt{a} - 3\sqrt{a}$
$= 3\sqrt{a}$

c) $\sqrt[3]{27x^2} - \sqrt[3]{8x^2} + \sqrt[3]{64x^2}, x \in \mathbb{R}$

$= \sqrt[3]{27 \cdot x^2} - \sqrt[3]{8 \cdot x^2} + \sqrt[3]{64 \cdot x^2}$
$= 3\sqrt[3]{x^2} - 2\sqrt[3]{x^2} + 4\sqrt[3]{x^2}$
$= 5\sqrt[3]{x^2}$

B

6. Explain why it is necessary to write $\sqrt[4]{x^4}$ as $|x|$.

$\sqrt[4]{x^4}$ is defined for $x \in \mathbb{R}$. The radical sign indicates the principal root, so the value of $\sqrt[4]{x^4}$ cannot be negative. Although x^4 is always positive or zero, x can be negative. So, it is necessary to write $\sqrt[4]{x^4}$ as $|x|$.

7. Identify the values of the variables for which each radical is defined, then simplify.

a) $7\sqrt{-x} + 15\sqrt{-x} - 13\sqrt{-x}$

The radicand cannot be negative, so $-x \geq 0$; that is, $x \leq 0$.
$7\sqrt{-x} + 15\sqrt{-x} - 13\sqrt{-x} = (7 + 15 - 13)\sqrt{-x}$
$= 9\sqrt{-x}$

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b) $\sqrt{28m^4n} + m^2\sqrt{63n}$

The radicand cannot be negative.
Since $m^4 \geq 0$, $m \in \mathbb{R}$.
n cannot be negative, so $n \geq 0$.

$$\begin{aligned}\sqrt{28m^4n} + m^2\sqrt{63n} &= \sqrt{4 \cdot 7 \cdot m^4 \cdot n} + m^2\sqrt{9 \cdot 7 \cdot n}\\ &= 2m^2\sqrt{7n} + 3m^2\sqrt{7n}\\ &= 5m^2\sqrt{7n}\end{aligned}$$

c) $4\sqrt[3]{2p^4q} - 6p\sqrt[3]{2pq}$

The cube root of a number is defined for all real numbers. So, each radical is defined for $p, q \in \mathbb{R}$.

$$\begin{aligned}4\sqrt[3]{2p^4q} - 6p\sqrt[3]{2pq} &= 4\sqrt[3]{2 \cdot p^3 \cdot pq} - 6p\sqrt[3]{2pq}\\ &= 4p\sqrt[3]{2pq} - 6p\sqrt[3]{2pq}\\ &= -2p\sqrt[3]{2pq}\end{aligned}$$

8. Simplify.

a) $\sqrt{5b} + 4\sqrt{5b} - 3\sqrt[3]{5b} - 2\sqrt{5b}, b \geq 0$

$$\begin{aligned}&= \sqrt{5b} + 4\sqrt{5b} - 2\sqrt{5b} - 3\sqrt[3]{5b}\\ &= 3\sqrt{5b} - 3\sqrt[3]{5b}\end{aligned}$$

b) $3\sqrt{x^3} + 5\sqrt{2x} - \sqrt{4x^3}, x \geq 0$

$$\begin{aligned}&= 3\sqrt{x^2 \cdot x} + 5\sqrt{2x} - \sqrt{4 \cdot x^2 \cdot x}\\ &= 3x\sqrt{x} + 5\sqrt{2x} - 2x\sqrt{x}\\ &= x\sqrt{x} + 5\sqrt{2x}\end{aligned}$$

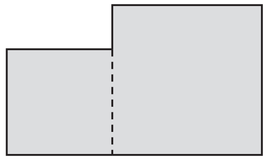
c) $5e\sqrt{24e^3} - 7\sqrt{54e^5} + e^2\sqrt{6e} + 6e, e \geq 0$

$$\begin{aligned}&= 5e\sqrt{4 \cdot 6 \cdot e^2 \cdot e} - 7\sqrt{9 \cdot 6 \cdot e^4 \cdot e} + e^2\sqrt{6e} + 6e\\ &= 5e(2e)\sqrt{6e} - 7(3e^2)\sqrt{6e} + e^2\sqrt{6e} + 6e\\ &= 10e^2\sqrt{6e} - 21e^2\sqrt{6e} + e^2\sqrt{6e} + 6e\\ &= -10e^2\sqrt{6e} + 6e\end{aligned}$$

d) $\sqrt[3]{16v^5} + \sqrt[3]{3w^4} + 2w\sqrt[3]{24w} - 5v\sqrt[3]{54v^2}, v, w \in \mathbb{R}$

$$\begin{aligned}&= \sqrt[3]{8 \cdot 2 \cdot v^3 \cdot v^2} + \sqrt[3]{3 \cdot w^3 \cdot w} + 2w\sqrt[3]{8 \cdot 3 \cdot w} - 5v\sqrt[3]{27 \cdot 2 \cdot v^2}\\ &= 2v\sqrt[3]{2v^2} + w\sqrt[3]{3w} + 2w(2)\sqrt[3]{3w} - 5v(3)\sqrt[3]{2v^2}\\ &= 2v\sqrt[3]{2v^2} + w\sqrt[3]{3w} + 4w\sqrt[3]{3w} - 15v\sqrt[3]{2v^2}\\ &= -13v\sqrt[3]{2v^2} + 5w\sqrt[3]{3w}\end{aligned}$$

9. A square with area 24 square units is placed beside a square with area 50 square units. In simplest form, write a radical expression for the perimeter of the shape formed.



The side length of a square is the square root of its area.

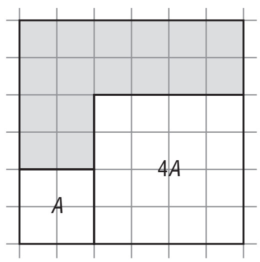
Small square:
Its area is 24, so its side length is $\sqrt{24}$.

Large square:
Its area is 50, so its side length is $\sqrt{50}$.

The perimeter of the shape formed consists of 3 sides of each square and the length that is the difference in their side lengths.

$$\begin{aligned}\text{Perimeter of shape formed} &= 3\sqrt{24} + 3\sqrt{50} + (\sqrt{50} - \sqrt{24})\\ &= 3\sqrt{24} + 3\sqrt{50} + \sqrt{50} - \sqrt{24}\\ &= 2\sqrt{24} + 4\sqrt{50}\\ &= 2\sqrt{4 \cdot 6} + 4\sqrt{25 \cdot 2}\\ &= 4\sqrt{6} + 20\sqrt{2}\end{aligned}$$

10. Two squares are enclosed in a large square as shown. The area of the smallest square is A square units. The area of the middle square is $4A$ square units. Determine the area and perimeter of the shaded region in terms of A.



The side length of a square is the square root of its area.
So, the side length of the small square is: $\sqrt{A}$ units
The side length of the middle square is: $\sqrt{4A}$, or $2\sqrt{A}$ units
The side length of the large square is the sum of the side lengths of the other 2 squares: $\sqrt{A} + 2\sqrt{A}$, or $3\sqrt{A}$
Area of shaded region
= area of large square − area of small square − area of middle square
$= (3\sqrt{A})^2 - A - 4A$
$= 9A - 5A$
$= 4A$

From the diagram, the length of 2 grid squares is $\sqrt{A}$.
The perimeter of the shaded region is the length of 20 grid squares.

$$\begin{aligned}\text{So, perimeter} &= 20 \text{ grid squares}\\ &= 10 \cdot (2 \text{ grid squares})\\ &= 10\sqrt{A}\end{aligned}$$

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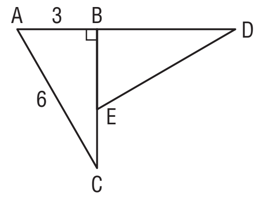
C

11. In right $\triangle ABC$, AB has length 3 units and AC has length 6 units. A congruent triangle is placed adjacent to $\triangle ABC$ as shown. Determine the perimeter of the shape formed.



Use the Pythagorean Theorem in ΔABC to determine the length of BC.

$$(AC)^2 = (BC)^2 + (AB)^2$$
$$6^2 = (BC)^2 + 3^2$$
$$27 = (BC)^2$$
$$\sqrt{27} = BC$$
$$3\sqrt{3} = BC$$

The perimeter of each triangle is: $6 + 3 + 3\sqrt{3} = 9 + 3\sqrt{3}$

BE = AB = 3

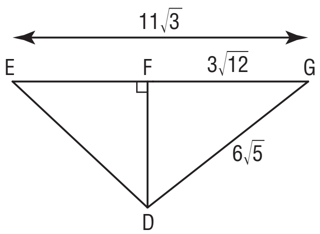
So, the perimeter of the shape formed is:

2 times the perimeter of ΔABC -2 times BE

$$= 2(9 + 3\sqrt{3}) - 2(3)$$
$$= 18 + 6\sqrt{3} - 6$$
$$= 12 + 6\sqrt{3}$$

So, the perimeter of the shape formed is $(12 + 6\sqrt{3})$ units.

12. Determine whether $\triangle EDG$ is a right triangle. How did you find out?



Use the Pythagorean Theorem in right ΔDFG to determine the length of DF.

$$(DG)^2 = (FG)^2 + (DF)^2$$
$$(6\sqrt{5})^2 = (3\sqrt{12})^2 + (DF)^2$$
$$180 = 108 + (DF)^2$$
$$(DF)^2 = 72$$
$$DF = \sqrt{72}$$
$$DF = 6\sqrt{2}$$

Use the Pythagorean Theorem in right ΔDEF to determine the length of ED.

$$(ED)^2 = (EF)^2 + (DF)^2$$
$$(ED)^2 = (11\sqrt{3} - 3\sqrt{12})^2 + (6\sqrt{2})^2$$
$$(ED)^2 = 75 + 72$$
$$(ED)^2 = 147$$
$$ED = \sqrt{147}$$
$$ED = 7\sqrt{3}$$

To determine whether ΔEDG is a right triangle, use the Pythagorean Theorem to check whether $(EG)^2 = (ED)^2 + (DG)^2$

L.S. $= (EG)^2$
$= (11\sqrt{3})^2$
$= 363$

R.S. $= (ED)^2 + (DG)^2$
$= (7\sqrt{3})^2 + (6\sqrt{5})^2$
$= 147 + 180$
$= 327$

Since L.S. $\neq$ R.S., ΔEDG is not a right triangle.

13. Determine if there are any values of x and y such that $\sqrt{x + y}$ and $\sqrt{x} + \sqrt{y}$ are equal. Explain your reasoning.

$x, y \geq 0$

$$\sqrt{x + y} = \sqrt{x} + \sqrt{y}$$
$$(\sqrt{x + y})^2 = (\sqrt{x} + \sqrt{y})^2$$
$$x + y = x + 2\sqrt{xy} + y$$
$$2\sqrt{xy} = 0$$

For $xy = 0$, $x = 0$, or $y = 0$, or both $x = 0$ and $y = 0$

So, there are values of x and y such that $\sqrt{x + y} = \sqrt{x} + \sqrt{y}$.

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