Checkpoint 2: Assess Your Understanding, pages 136–138

2.3

1. Group the radicals into sets of like radicals. Simplify first if necessary.

$\sqrt{50}, \sqrt{27}, \sqrt{48}, 3\sqrt{72}, \sqrt{98}, 2\sqrt{3}$

$$\sqrt{50} = \sqrt{25 \cdot 2} = 5\sqrt{2} \qquad \sqrt{27} = \sqrt{9 \cdot 3} = 3\sqrt{3} \qquad \sqrt{48} = \sqrt{16 \cdot 3} = 4\sqrt{3}$$

$$3\sqrt{72} = 3\sqrt{36 \cdot 2} = 18\sqrt{2} \qquad \sqrt{98} = \sqrt{49 \cdot 2} = 7\sqrt{2} \qquad 2\sqrt{3}$$

All radicals have index 2.

Radicals with radicand 2: $5\sqrt{2}, 18\sqrt{2}, 7\sqrt{2}$; that is, $\sqrt{50}, 3\sqrt{72}, \sqrt{98}$

Radicals with radicand 3: $3\sqrt{3}, 4\sqrt{3}, 2\sqrt{3}$; that is, $\sqrt{27}, \sqrt{48}, 2\sqrt{3}$

2. **Multiple Choice** Which list contains only like radicals? Assume all variables are non-negative.

A. $\sqrt{25x}, \sqrt{25x^2}, \sqrt[3]{25x}$

B. $3\sqrt{x}, 3\sqrt{y}, 3\sqrt{a}$

C. $\sqrt[3]{x^4}, 5\sqrt{x^4}, \sqrt[3]{5x^4}$

[✓ **D.**] $\sqrt[3]{x^5}, 2x\sqrt[3]{x^2}, -3\sqrt[3]{8x^5}$

3. Identify the values of the variables for which each radical is defined, then simplify.

a) $\sqrt{25x} + \sqrt{36x} - \sqrt{4x}$

The radicands cannot be negative, so $x \geq 0$.

$$\begin{aligned}\sqrt{25x} + \sqrt{36x} - \sqrt{4x} &= 5\sqrt{x} + 6\sqrt{x} - 2\sqrt{x}\\ &= 9\sqrt{x}\end{aligned}$$

b) $\sqrt{8a} - 3\sqrt{b} + 5\sqrt{2a} + \sqrt{4b}$

The radicands cannot be negative, so $a \geq 0$ and $b \geq 0$.

$$\begin{aligned}\sqrt{8a} - 3\sqrt{b} + 5\sqrt{2a} + \sqrt{4b} &= 2\sqrt{2a} - 3\sqrt{b} + 5\sqrt{2a} + 2\sqrt{b}\\ &= 7\sqrt{2a} - \sqrt{b}\end{aligned}$$

c) $\sqrt{75c^4d} + c^2\sqrt{12d} + \sqrt{48cd^4} - 3d^2\sqrt{27c}$

$\sqrt{12d} \geq 0$, so $d \geq 0$ and $\sqrt{27c} \geq 0$, so $c \geq 0$

$$\begin{aligned}&\sqrt{75c^4d} + c^2\sqrt{12d} + \sqrt{48cd^4} - 3d^2\sqrt{27c}\\ &= \sqrt{25 \cdot 3 \cdot c^4 \cdot d} + c^2\sqrt{4 \cdot 3d} + \sqrt{16 \cdot 3 \cdot c \cdot d^4} - 3d^2\sqrt{9 \cdot 3 \cdot c}\\ &= 5c^2\sqrt{3d} + 2c^2\sqrt{3d} + 4d^2\sqrt{3c} - 9d^2\sqrt{3c}\\ &= 7c^2\sqrt{3d} - 5d^2\sqrt{3c}\end{aligned}$$

d) $\sqrt[3]{8a} + \sqrt[3]{16a^4} - \sqrt[3]{-128a}$

The cube root of a number is defined for all real numbers. So, each radical is defined for $a \in \mathbb{R}$.

$$\begin{aligned}\sqrt[3]{8a} + \sqrt[3]{16a^4} - \sqrt[3]{-128a} &= \sqrt[3]{8 \cdot a} + \sqrt[3]{8 \cdot 2 \cdot a^3 \cdot a} - \sqrt[3]{-64 \cdot 2 \cdot a}\\ &= 2\sqrt[3]{a} + 2a\sqrt[3]{2a} - (-4)\sqrt[3]{2a}\\ &= 2\sqrt[3]{a} + 2a\sqrt[3]{2a} + 4\sqrt[3]{2a}\end{aligned}$$

2.4

4. Expand and simplify.

a) $(\sqrt{7} + \sqrt{2})^2$

$$\begin{aligned}&= (\sqrt{7} + \sqrt{2})(\sqrt{7} + \sqrt{2})\\ &= \sqrt{7}(\sqrt{7} + \sqrt{2})\\ &\quad + \sqrt{2}(\sqrt{7} + \sqrt{2})\\ &= 7 + \sqrt{14} + \sqrt{14} + 2\\ &= 9 + 2\sqrt{14}\end{aligned}$$

b) $(\sqrt{7} - \sqrt{2})^2$

$$\begin{aligned}&= (\sqrt{7} - \sqrt{2})(\sqrt{7} - \sqrt{2})\\ &= \sqrt{7}(\sqrt{7} - \sqrt{2}) - \sqrt{2}(\sqrt{7} - \sqrt{2})\\ &= 7 - \sqrt{14} - \sqrt{14} + 2\\ &= 9 - 2\sqrt{14}\end{aligned}$$

c) $(\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2})$

$$\begin{aligned}&= \sqrt{7}(\sqrt{7} - \sqrt{2})\\ &\quad + \sqrt{2}(\sqrt{7} - \sqrt{2})\\ &= 7 - \sqrt{14} + \sqrt{14} - 2\\ &= 5\end{aligned}$$

d) $(2\sqrt{7} + 3\sqrt{2})(\sqrt{7} - 2\sqrt{2})$

$$\begin{aligned}&= 2\sqrt{7}(\sqrt{7} - 2\sqrt{2}) + 3\sqrt{2}(\sqrt{7} - 2\sqrt{2})\\ &= 14 - 4\sqrt{14} + 3\sqrt{14} - 12\\ &= 2 - \sqrt{14}\end{aligned}$$

5. Multiple Choice Which expression represents

$(3\sqrt{x} - 2\sqrt{y})(3\sqrt{x} + 2\sqrt{y}) - (2\sqrt{x} - 3\sqrt{y})^2, x \geq 0, y \geq 0,$

in simplest form?

A. $5x + 5y + 12xy$

B. $5x - 5y + 12\sqrt{xy}$

C. $5x - 13y + 12xy$

[✓ **D.**] $5x - 13y + 12\sqrt{xy}$

6. Rationalize the denominator.

a) $\dfrac{8\sqrt{3}+1}{\sqrt{3}}$

$$= \frac{(8\sqrt{3}+1)}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{8\sqrt{3}\cdot\sqrt{3}+1\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}}$$

$$= \frac{24+\sqrt{3}}{3}$$

b) $\dfrac{3\sqrt{24}-4\sqrt{2}}{\sqrt{2}}$

$$= \frac{(6\sqrt{6}-4\sqrt{2})}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{6\sqrt{6}\cdot\sqrt{2}-4\sqrt{2}\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}}$$

$$= \frac{6\sqrt{12}-8}{2}$$

$$= \frac{12\sqrt{3}-8}{2}$$

$$= \frac{2(6\sqrt{3}-4)}{2}$$

$$= 6\sqrt{3}-4$$

7. Simplify.

a) $\dfrac{3\sqrt{2}}{2\sqrt{6}-5}$

$$= \frac{3\sqrt{2}}{(2\sqrt{6}-5)} \cdot \frac{(2\sqrt{6}+5)}{(2\sqrt{6}+5)}$$

$$= \frac{6\sqrt{12}+15\sqrt{2}}{(2\sqrt{6})^2-(5)^2}$$

$$= \frac{12\sqrt{3}+15\sqrt{2}}{24-25}$$

$$= \frac{12\sqrt{3}+15\sqrt{2}}{-1}$$

$$= -12\sqrt{3}-15\sqrt{2}$$

b) $\dfrac{3\sqrt{8}+2\sqrt{5}}{\sqrt{2}+\sqrt{20}}$

$$= \frac{6\sqrt{2}+2\sqrt{5}}{\sqrt{2}+2\sqrt{5}}$$

$$= \frac{(6\sqrt{2}+2\sqrt{5})}{(\sqrt{2}+2\sqrt{5})} \cdot \frac{(\sqrt{2}-2\sqrt{5})}{(\sqrt{2}-2\sqrt{5})}$$

$$= \frac{6\sqrt{2}(\sqrt{2}-2\sqrt{5})+2\sqrt{5}(\sqrt{2}-2\sqrt{5})}{(\sqrt{2})^2-(2\sqrt{5})^2}$$

$$= \frac{12-12\sqrt{10}+2\sqrt{10}-20}{2-20}$$

$$= \frac{-8-10\sqrt{10}}{-18}$$

$$= \frac{4+5\sqrt{10}}{9}$$