PRACTICE TEST, pages 246–248

1. Multiple Choice Which polynomial has $x - 4$ as a factor?

A. $16x^2 - 8x + 1$ B. $4x^2 - 8x - 96$

C. $x^2 - 16y^2$ D. $3x^2 - 8x - 16$

2. Multiple Choice Which equation has two real roots?

A. $x^2 + 9 = 6x$ B. $4x^2 - 8x + 5 = 0$

C. $3x^2 - 10x + 5 = 0$ D. $8x^2 - x + 4 = 0$

3. Factor each polynomial expression.

a) $3x^2 - 108$

$= 3(x^2 - 36)$
$= 3(x + 6)(x - 6)$

b) $6x^2 - 13x - 8$

Guess and test factors of 6 with factors of -8.
$= (3x - 8)(2x + 1)$

c) $4(4x - 3)^2 - 9(3y - 2)^2$

$= [2(4x - 3)^2] - [3(3y - 2)]^2$
$= [2(4x - 3) + 3(3y - 2)][2(4x - 3) - 3(3y - 2)]$
$= (8x + 9y - 12)(8x - 9y)$

d) $10(3x - 4)^2 + 13(3x - 4) - 3$

Guess and test factors of 10 with factors of -3.
$= [2(3x - 4) + 3][5(3x - 4) - 1]$
$= (6x - 5)(15x - 21)$
$= 3(6x - 5)(5x - 7)$

4. Solve each quadratic equation. Use a different strategy each time. Verify each solution.

a) $3x^2 - 10x + 6 = 0$

Use completing the square.

$$x^2 - \frac{10}{3}x = -2$$
$$x^2 - \frac{10}{3}x + \frac{25}{9} = -2 + \frac{25}{9}$$
$$\left(x - \frac{5}{3}\right)^2 = \frac{7}{9}$$
$$x - \frac{5}{3} = \pm\sqrt{\frac{7}{9}}$$
$$x = \frac{5}{3} \pm \frac{\sqrt{7}}{3}$$

b) $4x^2 - 1 = 11$

Use square roots.

$4x^2 = 12$
$x^2 = 3$
$x = \pm\sqrt{3}$

c) $(x + 3)(2x - 1) = 9$

$2x^2 + 5x - 12 = 0$
Use factoring.
$(2x - 3)(x + 4) = 0$
$x = 1.5$ or $x = -4$

d) $2.5x^2 + 1.5x - 5 = 0$

Use the quadratic formula.
Substitute:
$a = 2.5, b = 1.5, c = -5$
in: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$$x = \frac{-1.5 \pm \sqrt{1.5^2 - 4(2.5)(-5)}}{2(2.5)}$$
$$x = \frac{-1.5 \pm \sqrt{52.25}}{5}$$

5. Is it possible to construct a rectangle whose length is 1 cm less than twice its width, and whose area is 40 cm^2? If your answer is yes, determine the dimensions of the rectangle to the nearest tenth of a centimetre. If your answer is no, explain why it is not possible.

Let the width of the rectangle be x centimetres.
Then its length is $(2x - 1)$ centimetres.
And its area is $x(2x - 1)$ square centimetres.
If this rectangle can have area 40 cm^2, then the equation:
$x(2x - 1) = 40$ has real roots.
Solve the equation: $2x^2 - x - 40 = 0$

Substitute: $a = 2, b = -1, c = -40$ in: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-40)}}{2(2)}$$
$$x = \frac{1 \pm \sqrt{321}}{4}$$
Ignore the negative root because the width cannot be negative.
$$x = \frac{1 + \sqrt{321}}{4}$$
$x = 4.7291...$
So, the length is: $2(4.7291...) - 1 = 8.4582...$
A rectangle can be constructed and its dimensions are approximately 4.7 cm by 8.5 cm.

6. Consider the equation $6x^2 - 10x + k = 0$. Determine the value of k when the equation has exactly one root.

$6x^2 - 10x + k = 0$
In $b^2 - 4ac$, substitute: $a = 6, b = -10, c = k$
$$\begin{aligned} b^2 - 4ac &= (-10)^2 - 4(6)(k) \\ &= 100 - 24k \end{aligned}$$
For exactly 1 real root, $100 - 24k = 0$
$$k = \frac{100}{24}, \text{ or } \frac{25}{6}$$

7. A model rocket is launched with an initial speed of 32 m/s. After t seconds, its height, h metres, is given by this formula:
$h = 0.5 + 32t - 4.9t^2$

a) When will the rocket hit the ground?

When the rocket hits the ground, its height is 0.
So, in $h = 0.5 + 32t - 4.9t^2$, substitute $h = 0$, then solve for t.
$0 = 0.5 + 32t - 4.9t^2$

Substitute: $a = -4.9$, $b = 32$, $c = 0.5$ in: $t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$t = \frac{-32 \pm \sqrt{32^2 - 4(-4.9)(0.5)}}{2(-4.9)}$$

$$t = \frac{-32 \pm \sqrt{1033.8}}{-9.8}, \text{ or } \frac{32 \pm \sqrt{1033.8}}{9.8}$$

Since t cannot be negative, ignore the negative root.

$$t = \frac{32 + \sqrt{1033.8}}{9.8}$$

$t = 6.5462\ldots$
So, the rocket hits the ground after approximately 6.5 s.

b) Without solving an equation, show that the rocket will not reach a height of 60 m.

If the rocket does not reach a height of 60 m, then the equation $60 = 0.5 + 32t - 4.9t^2$ has no real roots.
Write the equation as $0 = -59.5 + 32t - 4.9t^2$
In $b^2 - 4ac$, substitute: $a = -4.9$, $b = 32$, $c = -59.5$

$$\begin{aligned} b^2 - 4ac &= 32^2 - 4(-4.9)(-59.5) \\ &= -142.2 \end{aligned}$$

Since the discriminant is negative, the equation has no real roots.