REVIEW, pages 242–245

3.1

1. Is $x - 5$ a factor of each trinomial? Justify your answer.

a) $3x^2 + 3x - 60$

Write the trinomial as:
$(x - 5)(3x + b)$
$= 3x^2 + (b - 15)x - 5b$
Equate constant terms.
$-5b = -60$, so $b = 12$
Check:
$(x - 5)(3x + 12)$
$= 3x^2 - 3x - 60$
So, $x - 5$ is not a factor.

b) $3x^2 - 13x - 10$

Write the trinomial as:
$(x - 5)(3x + b)$
$= 3x^2 + (b - 15)x - 5b$
Equate constant terms.
$-5b = -10$, so $b = 2$
Check:
$(x - 5)(3x + 2)$
$= 3x^2 - 13x - 10$
So, $x - 5$ is a factor.

2. Factor.

a) $0.5x^2 - 0.4x - 1.2$

$= 0.1(5x^2 - 4x - 12)$
Guess and test factors of 5 with factors of -12.
$= 0.1(x - 2)(5x + 6)$

b) $3(x - 3)^2 + 2(x - 3) - 5$

Guess and test factors of 3 with factors of -5.
$= [3(x - 3) + 5][(x - 3) - 1]$
$= (3x - 4)(x - 4)$

3. Factor.

a) $81x^2 - 4y^2$

$= (9x)^2 - (2y)^2$
$= (9x + 2y)(9x - 2y)$

b) $49(x - 4)^2 - 9(5y - 2)^2$

$= [7(x - 4)]^2 - [3(5y - 2)]^2$
$= [7(x - 4) + 3(5y - 2)][7(x - 4) - 3(5y - 2)]$
$= (7x + 15y - 34)(7x - 15y - 22)$

3.2

4. Solve by factoring. Verify the solutions.

a) $20x^2 + 3x - 2 = 0$

$(4x - 1)(5x + 2) = 0$
Either $4x - 1 = 0$;
then $x = 0.25$;
or $5x + 2 = 0$, then $x = -0.4$

b) $6x^2 - 21x + 18 = 0$

$3(2x^2 - 7x + 6) = 0$
$3(2x - 3)(x - 2) = 0$
Either $2x - 3 = 0$, then $x = 1.5$;
or $x - 2 = 0$, then $x = 2$

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c) $(x - 5)(x + 8) = 14$

$x^2 + 3x - 54 = 0$
$(x - 6)(x + 9) = 0$
Either $x - 6 = 0$, then $x = 6$;
or $x + 9 = 0$, then $x = -9$
I used mental math to verify the solutions.

d) $6x^2 = 8x$

$6x^2 - 8x = 0$
$2x(3x - 4) = 0$
Either $2x = 0$, then $x = 0$;
or $3x - 4 = 0$, then $x = \frac{4}{3}$

5. Two numbers have a sum of 20 and a product of 84. Use a quadratic equation to determine the numbers.

Let one number be x. Then the other number is $20 - x$.
An equation is: $x(20 - x) = 84$
$20x - x^2 - 84 = 0$
$x^2 - 20x + 84 = 0$
$(x - 14)(x - 6) = 0$
$x = 14$ or $x = 6$
The numbers are 14 and 6.

3.3

6. Solve each equation.

a) $(2x + 1)^2 + 4 = 49$

$$(2x + 1)^2 = 45$$
$$2x + 1 = \pm\sqrt{45}$$
$$2x = -1 \pm \sqrt{45}$$
$$x = \frac{-1 \pm \sqrt{45}}{2}$$

b) $-3 + (3 - 2x)^2 = 5$

$$(3 - 2x)^2 = 8$$
$$3 - 2x = \pm\sqrt{8}$$
$$2x = 3 \pm \sqrt{8}$$
$$x = \frac{3 \pm \sqrt{8}}{2}$$

7. Solve each equation by completing the square.

a) $x^2 + 4x + 2 = 0$

$$x^2 + 4x = -2$$
$$x^2 + 4x + 4 = -2 + 4$$
$$(x + 2)^2 = 2$$
$$x + 2 = \pm\sqrt{2}$$
$$x = -2 \pm \sqrt{2}$$

b) $3x^2 - 2x - 1 = 0$

$$x^2 - \frac{2}{3}x = \frac{1}{3}$$
$$x^2 - \frac{2}{3}x + \frac{1}{9} = \frac{1}{3} + \frac{1}{9}$$
$$\left(x - \frac{1}{3}\right)^2 = \frac{4}{9}$$
$$x - \frac{1}{3} = \pm\sqrt{\frac{4}{9}}$$
$$x = \frac{1}{3} \pm \frac{2}{3}$$
$$x = 1 \text{ or } x = -\frac{1}{3}$$

3.4

8. Solve each quadratic equation.

a) $2x^2 - 6x + 1 = 0$

Substitute:
$a = 2, b = -6, c = 1$

in: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(2)(1)}}{2(2)}$$

$$x = \frac{6 \pm \sqrt{28}}{4}$$

$$x = \frac{6 \pm 2\sqrt{7}}{4}$$

$$x = \frac{3 \pm \sqrt{7}}{2}$$

b) $(x + 1)(x + 2) = x$

$x^2 + 2x + 2 = 0$
Substitute: $a = 1, b = 2, c = 2$

in: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{-4}}{2}$$

There are no real roots.

9. A truck was travelling at 23 m/s. It decelerated for 15 s. The distance travelled by the truck, d metres, during this time is given by the formula $d = 23t - 0.6t^2$, where t is the time in seconds. How long did it take the truck to travel 60 m? Give the answer to the nearest tenth of a second.

In $d = 23t - 0.6t^2$, substitute: $d = 60$, then solve for t.

$$60 = 23t - 0.6t^2$$

$$0.6t^2 - 23t + 60 = 0$$

Substitute: $a = 0.6, b = -23, c = 60$ in: $t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$t = \frac{23 \pm \sqrt{(-23)^2 - 4(0.6)(60)}}{2(0.6)}$$

$$t = \frac{23 \pm \sqrt{385}}{1.2}$$

Either $t = \dfrac{23 + \sqrt{385}}{1.2}$, so $t \doteq 35.5$

Or $t = \dfrac{23 - \sqrt{385}}{1.2}$, so $t \doteq 2.8$

Since the truck decelerated for only 15 s, $t \doteq 35.5$ s is not a solution.
So, the truck took approximately 2.8 s to travel 60 m.

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3.5

10. Without solving, determine whether each equation has one, two, or no real roots.

a) $2x^2 - 1.8x - 1.25 = 0$

In $b^2 - 4ac$, substitute:
$a = 2, b = -1.8, c = -1.25$
$b^2 - 4ac = (-1.8)^2 - 4(2)(-1.25)$
$= 13.24$
Since $b^2 - 4ac > 0$,
the equation has 2 real roots.

b) $-2x^2 + 3x - 10 = 0$

In $b^2 - 4ac$, substitute:
$a = -2, b = 3, c = -10$
$b^2 - 4ac = 3^2 - 4(-2)(-10)$
$= -71$
Since $b^2 - 4ac < 0$,
the equation has no real roots.

11. Consider the equation $8x^2 - 5x + k = 0$.
Determine the values of k in each case:

a) The equation has no real roots.

$8x^2 - 5x + k = 0$
In $b^2 - 4ac$, substitute: $a = 8, b = -5, c = k$
$b^2 - 4ac = (-5)^2 - 4(8)(k)$
$= 25 - 32k$
For no real roots, $25 - 32k < 0$
$k > \frac{25}{32}$

b) The equation has exactly one real root.

Use the value of $b^2 - 4ac$ from part a.
For exactly 1 real root, $25 - 32k = 0$
$k = \frac{25}{32}$

c) The equation has two real roots.

Use the value of $b^2 - 4ac$ from part a.
For 2 real roots, $25 - 32k > 0$
$k < \frac{25}{32}$