Lesson 4.5 Exercises, pages 295–299

A **Students should verify that two forms of an equation represent the same quadratic function, where necessary.**

3. Determine the number that would be added to each binomial to get a perfect square trinomial. Add the number, then factor the trinomial.

a) $x^2 + 12x$

$$\left(\frac{12}{2}\right)^2 = 36$$

$$x^2 + 12x + 36 = (x + 6)^2$$

b) $x^2 - 8x$

$$\left(\frac{-8}{2}\right)^2 = 16$$

$$x^2 - 8x + 16 = (x - 4)^2$$

c) $x^2 + 7x$

$$\left(\frac{7}{2}\right)^2 = \frac{49}{4}$$

$$x^2 + 7x + \frac{49}{4} = \left(x + \frac{7}{2}\right)^2$$

d) $x^2 + \frac{3}{2}x$

$$\left(\frac{\frac{3}{2}}{2}\right)^2 = \left(\frac{3}{4}\right)^2\text{, or }\frac{9}{16}$$

$$x^2 + \frac{3}{2}x + \frac{9}{16} = \left(x + \frac{3}{4}\right)^2$$

4. Do the equations in each pair represent the same quadratic function?

a) $y = x^2 + 4x - 1; y = (x + 2)^2 - 5$

Expand: $y = (x + 2)^2 - 5$

$y = x^2 + 4x + 4 - 5$

$y = x^2 + 4x - 1$

This matches the other equation. So, the equations represent the same quadratic function.

b) $y = -2x^2 - 6x + 1; y = -2(x + 3)^2 + 1$

Expand: $y = -2(x + 3)^2 + 1$

$y = -2(x^2 + 6x + 9) + 1$

$y = -2x^2 - 12x - 18 + 1$

$y = -2x^2 - 12x - 17$

This does not match the other equation. So, the equations do not represent the same quadratic function.

B

5. Write each equation in standard form. Verify algebraically that the two forms of the equation represent the same quadratic function.

a) $y = x^2 + 6x + 1$

Add and subtract: $\left(\frac{6}{2}\right)^2 = 9$

$y = (x^2 + 6x) + 1$

$= (x^2 + 6x + 9 - 9) + 1$

$= (x^2 + 6x + 9) - 9 + 1$

$= (x + 3)^2 - 8$

b) $y = x^2 - 2x - 4$

Add and subtract: $\left(\frac{-2}{2}\right)^2 = 1$

$y = (x^2 - 2x) - 4$

$= (x^2 - 2x + 1 - 1) - 4$

$= (x^2 - 2x + 1) - 1 - 4$

$= (x - 1)^2 - 5$

6. Write each equation in standard form. Use a graphing calculator to verify that the two forms of the equation represent the same quadratic function.

a) $y = 2x^2 + 8x - 4$

$y = 2(x^2 + 4x) - 4$

Add and subtract: $\left(\frac{4}{2}\right)^2 = 4$

$y = 2(x^2 + 4x + 4 - 4) - 4$

$= 2(x^2 + 4x + 4) - 2(4) - 4$

$= 2(x + 2)^2 - 12$

b) $y = 3x^2 - 12x - 1$

$y = 3(x^2 - 4x) - 1$

Add and subtract: $\left(\frac{-4}{2}\right)^2 = 4$

$y = 3(x^2 - 4x + 4 - 4) - 1$

$= 3(x^2 - 4x + 4) - 3(4) - 1$

$= 3(x - 2)^2 - 13$

7. Write each equation in standard form.

a) $y = \frac{3}{4}x^2 - 6x + 2$

$y = \frac{3}{4}(x^2 - 8x) + 2$

Add and subtract: $\left(\frac{-8}{2}\right)^2 = 16$

$$\begin{aligned} y &= \tfrac{3}{4}(x^2 - 8x + 16 - 16) + 2 \\ &= \tfrac{3}{4}(x^2 - 8x + 16) - \tfrac{3}{4}(16) + 2 \\ &= 0.75(x - 4)^2 - 12 + 2 \\ &= 0.75(x - 4)^2 - 10 \end{aligned}$$

b) $y = -\frac{1}{2}x^2 + 5x + 1$

$y = -\frac{1}{2}(x^2 - 10x) + 1$

Add and subtract: $\left(\frac{-10}{2}\right)^2 = 25$

$$\begin{aligned} y &= -\tfrac{1}{2}(x^2 - 10x + 25 - 25) + 1 \\ &= -\tfrac{1}{2}(x^2 - 10x + 25) - \tfrac{1}{2}(-25) + 1 \\ &= -0.5(x - 5)^2 + 12.5 + 1 \\ &= -0.5(x - 5)^2 + 13.5 \end{aligned}$$

8. Write each equation in standard form, then identify the given characteristic of the graph of the function.

a) $y = 2x^2 + 5x - 3$; the coordinates of the vertex

$y = 2\left(x^2 + \frac{5}{2}x\right) - 3$

Add and subtract: $\left(\frac{\frac{5}{2}}{2}\right)^2 = \left(\frac{5}{4}\right)^2$, or $\frac{25}{16}$

$$\begin{aligned} y &= 2\left(x^2 + \tfrac{5}{2}x + \tfrac{25}{16} - \tfrac{25}{16}\right) - 3 \\ &= 2\left(x^2 + \tfrac{5}{2}x + \tfrac{25}{16}\right) - 2\left(\tfrac{25}{16}\right) - 3 \\ &= 2\left(x + \tfrac{5}{4}\right)^2 - \tfrac{25}{8} - 3 \\ &= 2(x + 1.25)^2 - 6.125 \end{aligned}$$

Compare this with $y = a(x - p)^2 + q$.

The vertex of the parabola has coordinates $(-1.25, -6.125)$.

b) $y = -4x^2 + 11x + 12$; the y-coordinate of the vertex

$$y = -4\left(x^2 - \frac{11}{4}x\right) + 12$$

Add and subtract: $\left(\frac{-\frac{11}{4}}{2}\right)^2 = \left(-\frac{11}{8}\right)^2$, or $\frac{121}{64}$

$$\begin{aligned} y &= -4\left(x^2 - \frac{11}{4}x + \frac{121}{64} - \frac{121}{64}\right) + 12 \\ &= -4\left(x^2 - \frac{11}{4}x + \frac{121}{64}\right) - 4\left(-\frac{121}{64}\right) + 12 \\ &= -4\left(x - \frac{11}{8}\right)^2 + \frac{121}{16} + 12 \\ &= -4\left(x - \frac{11}{8}\right)^2 + \frac{313}{16} \end{aligned}$$

Compare this with $y = a(x - p)^2 + q$.

The vertex of the parabola has coordinates $\left(\frac{11}{8}, \frac{313}{16}\right)$.

So, the y-coordinate of the vertex is $\frac{313}{16}$.

9. Compare the two solutions for completing the square. Identify the error, then explain each step for the correct solution.

	Solution A	Solution B
	$y = -\frac{2}{3}x^2 - 4x - 10$	$y = -\frac{2}{3}x^2 - 4x - 10$
Step 1	$y = -\frac{2}{3}(x^2 + 6x) - 10$	$y = -\frac{2}{3}(x^2 + 6x) - 10$
Step 2	$y = -\frac{2}{3}(x^2 + 6x + 9 - 9) - 10$	$y = -\frac{2}{3}(x^2 + 6x + 9 - 9) - 10$
Step 3	$y = -\frac{2}{3}(x^2 + 6x + 9) + 6 - 10$	$y = -\frac{2}{3}(x^2 + 6x + 9) - 9 - 10$
Step 4	$y = -\frac{2}{3}(x + 3)^2 - 4$	$y = -\frac{2}{3}(x + 3)^2 - 19$

Solution A is correct.

Solution B has an error in the 4th line. When -9 was taken out of the brackets, it should have been multiplied by $-\frac{2}{3}$.

Step 1: Remove $-\frac{2}{3}$ as a common factor from the first 2 terms.

Step 2: Add and subtract the square of one-half of 6, the coefficient of x.

Step 3: Take -9 outside of the brackets by multiplying it by $-\frac{2}{3}$.

Step 4: Write the terms in the brackets as a perfect square. Simplify the terms outside of the brackets.

10. Identify the errors in this solution of completing the square. Write the correct solution.

$y = -3x^2 - 6x + 4$
$y = -3(x^2 - 2x) + 4$
$y = -3(x^2 - 2x + 1) + 4 + 3$
$y = -3(x - 1)^2 + 7$

There is an error in line 2 of the solution: when a factor of -3 was removed from $-6x$, the result should have been $2x$, not $-2x$.
A correct solution is:
$y = -3x^2 - 6x + 4$
$y = -3(x^2 + 2x) + 4$
$y = -3(x^2 + 2x + 1 - 1) + 4$
$y = -3(x^2 + 2x + 1) - 3(-1) + 4$
$y = -3(x + 1)^2 + 7$

C

11. What are the coordinates of the vertex of the graph of $y = ax^2 + bx + c$?

Complete the square.
$y = ax^2 + bx + c$
$y = a\left(x^2 + \frac{b}{a}x\right) + c$ **Add and subtract:** $\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$
$y = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right) + c$
$y = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) - a\left(\frac{b^2}{4a^2}\right) + c$
$y = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$
$y = a\left(x + \frac{b}{2a}\right)^2 + \frac{-b^2 + 4ac}{4a}$
Compare this with $y = a(x - p)^2 + q$.
The vertex of the parabola has coordinates $\left(-\frac{b}{2a}, \frac{-b^2 + 4ac}{4a}\right)$.