Lesson 4.7 Exercises, pages 323–326

A

3. Identify which quadratic functions have a maximum value and which have a minimum value. What is that value?

a) $y = -35(x + 100)^2 - 1200$

Compare each equation to $y = a(x - p)^2 + q$.
a is negative, so the function has a maximum value.
The coordinates of the vertex are: $(-100, -1200)$
So, the maximum value is -1200.

b) $R(x) = \frac{1}{2}(x - 37)^2 + 37$

a is positive, so the function has a minimum value.
The coordinates of the vertex are: (37, 37)
So, the minimum value is 37.

B

4. Two numbers have a difference of 6 and their product is a minimum. Determine the numbers.

Let one number be represented by x. Then the other number is: $6 + x$
The product, P, of the numbers is: $x(6 + x)$
An equation is: $P = x(6 + x)$, or $p = x^2 + 6x$
The coefficient of x^2 is positive, so the graph has a minimum value.
From the equation $P = x(6 + x)$, the x-intercepts are: 0, -6
So, the x-coordinate of the vertex is -3. So, one number is -3.
The other number is: $6 - 3 = 3$
The numbers are -3 and 3.

5. The sum of the length and width of a rectangle is 20 cm. Determine the dimensions that produce the maximum area.

Let the length of the rectangle be l centimetres. Then the width is: $(20 - l)$ centimetres
So, the area, A square centimetres, is given by the equation:
$A = l(20 - l)$, or $A = -l^2 + 20l$
The coefficient of l^2 is negative, so the graph has a maximum value.
From the equation $A = l(20 - l)$, the l-intercepts are: 0, 20
So, the l-coordinate of the vertex is 10. So, the length is 10 cm.
The width is: $20 - 10 = 10$
The dimensions that produce the maximum area are 10 cm by 10 cm.

6. Two numbers have a difference of 18. The sum of their squares is a minimum. Determine the numbers.

Let one number be represented by x. Then the other number is: $18 + x$

The sum of the squares, S, of the numbers is: $x^2 + (18 + x)^2$

An equation is: $S = x^2 + (18 + x)^2$

$= x^2 + 324 + 36x + x^2$

$= 2x^2 + 36x + 324$

The coefficient of x^2 is positive, so the graph has a minimum value.

$S = 2(x^2 + 18x) + 324$

$S = 2(x^2 + 18x + 81 - 81) + 324$

$S = 2(x^2 + 18x + 81) - 162 + 324$

$S = 2(x + 9)^2 + 162$

The coordinates of the vertex are: $(-9, 162)$

The x-coordinate of the vertex is -9. So, one number is -9.

The other number is: $18 - 9 = 9$.

7. A rectangular play area is to be bounded by 120 m of fencing. Determine the maximum area and the dimensions of this rectangle.

Let the length of the rectangle be l metres and its width be w metres.

The area, A square metres, is given by the equation: $A = lw$

The perimeter of the rectangle is 120 m. So, $120 = 2l + 2w$

$120 - 2l = 2w$

$w = 60 - l$

So, $A = l(60 - l)$

$A = -l^2 + 60l$

The coefficient of l^2 is negative, so the graph has a maximum value.

From the equation, the l-intercepts are: 0, 60

The l-coordinate of the vertex is 30. So, $l = 30$

Then, $A = 30(60 - 30)$

$= 900$

The maximum area is 900 m^2.

$w = 60 - 30$

$= 30$

The dimensions of the rectangle are 30 m by 30 m.

8. A rectangular lot is bordered on one side by a building and the other 3 sides by 800 m of fencing. Determine the area of the largest lot possible.

Let the length of the rectangle be l metres and its width be w metres.

The area, A square metres, is given by the equation: $A = lw$

The perimeter of 3 sides of the rectangle is 800 m.

So, $800 = 2l + w$

$800 - 2l = w$

So, $A = l(800 - 2l)$

$A = -2l^2 + 800l$

The coefficient of l^2 is negative, so the graph has a maximum value.

From the equation, the l-intercepts are: 0, 400

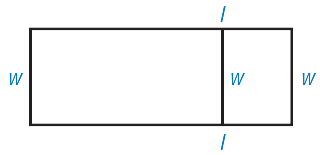
The l-coordinate of the vertex is 200. So, $l = 200$

Then, $A = 200(800 - 2 \cdot 200)$

$A = 80\,000$

The maximum area is 80 000 m^2.

9. A rectangular area is divided into 2 rectangles with 450 m of fencing used for the perimeter and the divider. What are the dimensions of the total maximum area?



Let the width of each rectangle be w metres. Let the length of the rectangle made up of the two smaller rectangles be l metres.
The area, A square metres, is given by the equation: $A = lw$
The perimeter of the rectangular areas is 450 m.

So, $450 = 2l + 3w$

$450 - 2l = 3w$

$w = \frac{1}{3}(450 - 2l)$

So, $A = \frac{1}{3}l(450 - 2l)$

$A = -\frac{2}{3}l^2 + \frac{450}{3}l$

The coefficient of l^2 is negative, so the graph has a maximum value.
From the equation, the l-intercepts are: 0, 225
The l-coordinate of the vertex is 112.5. So, $l = 112.5$

Then, $A = \frac{1}{3}(112.5)(450 - 2 \cdot 112.5)$

$A = 8437.5$

The maximum area is 8437.5 m^2.

$w = \frac{1}{3}(450 - 2(112.5))$

$w = 75$

The dimensions of the maximum area are 75 m by 112.5 m.

10. The Nuts and Bolts Company sells one type of bolt for 95¢. At this price, the company sells approximately 10 000 bolts per month. Market research indicates that for every 15¢ increase in price, the company will sell 500 fewer bolts. Determine the price of a bolt that will maximize the revenue.

Let x represent the number of \$0.15 increases in the price of a bolt.
When the cost is \$0.95, 10 000 are sold for a revenue of:
\$0.95(10 000) = \$9500
When the cost is \$$(0.95 + 0.15x)$, $(10\,000 - 500x)$ are sold for a revenue of \$$(0.95 + 0.15x)(10\,000 - 500x)$.
Let the revenue be R dollars.
An equation is: $R = (0.95 + 0.15x)(10\,000 - 500x)$
Use a graphing calculator to graph the equation.
From the graph, the maximum revenue is about \$13 002.08 when the number of \$0.15 increases is 6.83.
The number of increases is a whole number, so round 6.83 to 7.
Seven increases of \$0.15 in the cost of a bolt mean that the bolt will now cost: 7(\$0.15) + \$0.95 = \$2.00

C

11. This problem was solved in *Example 2*, page 321.

A student parking pass costs \$20. At this price, 150 students will purchase passes. For every \$5 increase in price, 20 fewer students will purchase passes. What is the price of a parking pass that will maximize the revenue?

a) Solve the problem by letting x dollars represent the increase in price.

Let x dollars represent the increase in the price of a parking pass.
When the cost is \$20, 150 are sold for a revenue of: \$3000
When the cost is \$20 + x, $\left(150 - 20\left(\frac{x}{5}\right)\right)$, or $(150 - 4x)$ are sold for a revenue of \$$(20 + x)(150 - 4x)$.
Let the revenue be R dollars.
An equation is: $R = (20 + x)(150 - 4x)$
Use a graphing calculator to graph the equation.
From the graph, the maximum revenue is about \$3306.25 when the price increase is \$8.75.
The cost of a parking pass will now be \$8.75 + \$20.00 = \$28.75
Since \$28.75 is not a multiple of \$5, round up.
The parking pass will now cost \$30.00.

b) Solve the problem by letting x dollars represent the price of a parking pass.

Let x dollars represent the new cost of a parking pass.
When the cost is \$20, 150 are sold for a revenue of: \$3000
When the cost is \$$x$, $\left(150 - 20\left(\frac{x - 20}{5}\right)\right)$, or $(230 - 4x)$ are sold for a revenue of \$$x(230 - 4x)$.
Let the revenue be R dollars.
An equation is: $R = x(230 - 4x)$
Use a graphing calculator to graph the equation.
From the graph, the maximum revenue is about \$3306.25 when the price of the parking pass is \$28.75.
Since \$28.75 is not a multiple of \$5, round up.
The parking pass will now cost \$30.00.

c) Which of the 3 strategies do you think is most efficient? Justify your answer.

I think the method shown in Example 2 is most efficient because the solution does not involve fractions.