Checkpoint 2: Assess Your Understanding, pages 313–317

4.4

1. **Multiple Choice** What are the coordinates of the vertex of the graph of $y = -3(x + 1)^2 - 4$?

   A. $(1, -4)$ **B.** $(-1, -4)$ **C.** $(-1, 4)$ **D.** $(1, 4)$

2. For this quadratic function:

$y = \frac{1}{2}(x + 3)^2 - 2$

a) Identify the coordinates of the vertex, the domain, the range, the direction of opening, the equation of the axis of symmetry, and the intercepts.

a is positive, so the graph opens up.
$p = -3$ and $q = -2$, so the coordinates of the vertex are: $(-3, -2)$
The equation of the axis of symmetry is $x = p$; that is, $x = -3$.
To determine the y-intercept, substitute $x = 0$:

$y = \frac{1}{2}(0 + 3)^2 - 2$

$y = \frac{5}{2}$

The y-intercept is $\frac{5}{2}$, or 2.5.
To determine the x-intercepts, substitute $y = 0$:

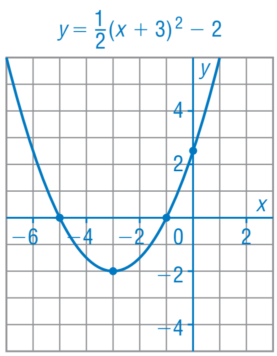
$0 = \frac{1}{2}(x + 3)^2 - 2$

$2 = \frac{1}{2}(x + 3)^2$

$4 = (x + 3)^2$

$x = -1$ or -5

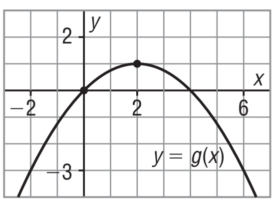
The x-intercepts are -5 and -1. The domain is: $x \in \mathbb{R}$
The graph opens up, so the vertex is a minimum point with y-coordinate -2. The range is: $y \geq -2, y \in \mathbb{R}$



b) Sketch a graph.

The graph is congruent to the graph of $y = \frac{1}{2}x^2$.
On a grid, mark a point at the vertex $(-3, -2)$.
Use the step pattern. Multiply each vertical step by $\frac{1}{2}$.

3. Use the given information to write an equation for the graph of this quadratic function.



Use: $y = a(x - p)^2 + q$.
The coordinates of the vertex are $(p, q) = (2, 1)$
Substitute the values of p and q.
The equation becomes: $y = a(x - 2)^2 + 1$.
Substitute the coordinates of a point on the graph: (0, 0)

$0 = a(0 - 2)^2 + 1$

$0 = 4a + 1$

$-1 = 4a$

$a = -\frac{1}{4}$

The equation is: $y = -\frac{1}{4}(x - 2)^2 + 1$

4.5

4. Multiple Choice Which pair of equations represents the same quadratic function?

A. $y = -3x^2 - 30x - 79$; $y = -3(x + 5)^2 - 4$

B. $y = 0.5x^2 + 4x + 5$; $y = 0.5(x + 3)^2 - 3$

C. $y = 2x^2 - 12x + 22$; $y = 2(x - 3)^2 + 3$

D. $y = -\frac{3}{4}x^2 + \frac{3}{4}x + \frac{13}{16}$; $y = -\frac{3}{4}\left(x - \frac{1}{2}\right)^2 - 1$

5. Write the reason for each step in completing the square below.

$$y = -3x^2 + 8x - 7$$

Step 1 $y = -3\left(x^2 - \frac{8}{3}x\right) - 7$

Step 2 $y = -3\left(x^2 - \frac{8}{3}x + \left(-\frac{4}{3}\right)^2 - \left(-\frac{4}{3}\right)^2\right) - 7$

Step 3 $y = -3\left(x^2 - \frac{8}{3}x + \frac{16}{9} - \frac{16}{9}\right) - 7$

Step 4 $y = -3\left(x^2 - \frac{8}{3}x + \frac{16}{9}\right) - 3\left(-\frac{16}{9}\right) - 7$

Step 5 $y = -3\left(x - \frac{4}{3}\right)^2 + \frac{16}{3} - \frac{21}{3}$

Step 6 $y = -3\left(x - \frac{4}{3}\right)^2 - \frac{5}{3}$

Step 1: Remove -3 as a common factor from the first 2 terms.

Step 2: Add and subtract the square of one-half of $-\frac{8}{3}$, the coefficient of x.

Step 3: Simplify by squaring.

Step 4: Take $-\frac{16}{9}$ outside of the brackets by multiplying it by -3.

Step 5: Write the terms in the first set of brackets as a perfect square. Simplify by multiplying -3 by the term in the second set of brackets.

Step 6: Simplify by adding the terms outside the brackets.

6. Complete the square to write each equation in standard form. Verify, with or without technology, that the two forms of the quadratic function are equivalent.

a) $y = 2x^2 - 11x + 4$

$y = 2\left(x^2 - \frac{11}{2}x\right) + 4$

Add and subtract: $\left(\frac{-11}{4}\right)^2 = \frac{121}{16}$

$y = 2\left(x^2 - \frac{11}{2}x + \frac{121}{16} - \frac{121}{16}\right) + 4$

$y = 2\left(x^2 - \frac{11}{2}x + \frac{121}{16}\right) - 2\left(\frac{121}{16}\right) + 4$

$y = 2\left(x - \frac{11}{4}\right)^2 - \frac{89}{8}$

b) $y = -4x^2 - 24x - 7$

$y = -4(x^2 + 6x) - 7$

Add and subtract: $\left(\frac{6}{2}\right)^2 = 9$

$y = -4(x^2 + 6x + 9 - 9) - 7$

$y = -4(x^2 + 6x + 9) - 4(-9) - 7$

$y = -4(x + 3)^2 + 29$

4.6

7. **Multiple Choice** What are the x-intercepts and the coordinates of the vertex of the graph of the quadratic function $y = -4x^2 + 24x + 64$?

A. -8 and 2; $(-3, 100)$

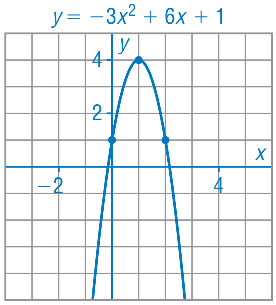
B. -8 and 2; $(-3, 28)$

C. -2 and 8; $(3, 55)$

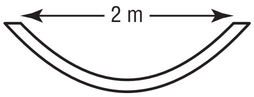
D. -2 and 8; $(3, 100)$

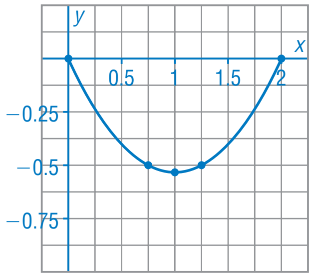
8. For the quadratic function: $y = -3x^2 + 6x + 1$
Determine the x- and y-intercepts of its graph and the coordinates of its vertex.
Use the characteristics to sketch a graph of the function.

Discriminant is: $6^2 - 4(-3)(1) = 48$
Since 48 is not a perfect square, the equation does not factor.
Complete the square.
$y = -3x^2 + 6x + 1$
$y = -3(x^2 - 2x) + 1$
$y = -3(x^2 - 2x + 1 - 1) + 1$
$y = -3(x^2 - 2x + 1) - 3(-1) + 1$
$y = -3(x - 1)^2 + 4$
The coordinates of the vertex are: (1, 4)
The y-intercept of the graph is 1.
The graph of $y = -3x^2 + 6x + 1$ opens down and is congruent to the graph of $y = -3x^2$. On a grid, mark points at the vertex and the y-intercept. Use the step pattern. Multiply each vertical step by -3.



9. A trough has a parabolic cross section. It is 2 m wide at the top. At a point 0.75 m from its top edge, the trough is 0.5 m deep. Determine an equation that represents the cross section of the trough. What is the greatest depth of the trough?





Sample response: Place one end of the cross section at the origin. Since the dish is 2 m wide, the other end of the cross section has coordinates (2, 0). At a point 0.75 m from the origin, the trough is 0.5 m deep. So, the parabola passes through the point (0.75, −0.5).

Use the factored form: $y = a(x - x_1)(x - x_2)$

Substitute: $x_1 = 0$ and $x_2 = 2$

The equation becomes: $y = ax(x - 2)$

Substitute: $x = 0.75$ and $y = -0.5$

$$-0.5 = a(0.75)(0.75 - 2)$$

$$-0.5 = -\frac{15}{16}a$$

$$a = \frac{8}{15}$$

So, an equation that represents the cross section of the trough is:

$$y = \frac{8}{15}x(x - 2)$$

The centre of the cross section is at the vertex of the parabola. The depth of the trough is the y-coordinate of the vertex.

The x-coordinate of the vertex is halfway between 0 and 2; that is, $x = 1$. Substitute $x = 1$ in $y = \frac{8}{15}x(x - 2)$

$$y = \frac{8}{15}(1)(1 - 2)$$

$$y = -\frac{8}{15}$$

The greatest depth of the trough is $\frac{8}{15}$ m.