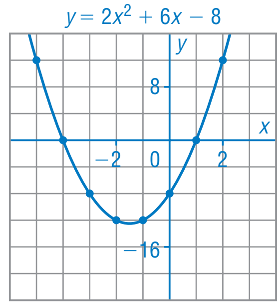
REVIEW, pages 330–335

4.1

1. a) Use a table of values to graph $y = 2x^2 + 6x - 8$.

x	–5	–4	–3	–2	–1	0	1	2
y	12	0	–8	–12	–12	–8	0	12



b) Determine:

i) the intercepts **ii)** the coordinates of the vertex

iii) the equation of the axis of symmetry

iv) the domain of the function **v)** the range of the function

Give the values to the nearest tenth where necessary.

i) ***x*-intercepts: $-4, 1$**
***y*-intercept: -8**

ii) **From the graph, the axis of symmetry is midway between $x = -1$ and $x = -2$. So, the equation of the axis of symmetry is $x = -1.5$.**
When $x = -1.5$, $y = 2(-1.5)^2 + 6(-1.5) - 8$, or -12.5
The coordinates of the vertex are: $(-1.5, -12.5)$

iii) **axis of symmetry: $x = -1.5$**

iv) **domain: $x \in \mathbb{R}$**

v) **range: $y \geq -12.5$, $y \in \mathbb{R}$**

2. Which of these tables of values represents a quadratic function? Justify your response.

a)

x	0	1	2	3
y	3	–3	–13	–27

The *x*-coordinates increase by 1 each time.
First differences:
$-3 - 3 = -6$
$-13 - (-3) = -10$
$-27 - (-13) = -14$
The first differences decrease by 4 each time. So, the function is quadratic.

b)

x	0	1	2	3
y	1	3	5	7

The *x*-coordinates increase by 1 each time.
First differences:
$3 - 1 = 2$
$5 - 3 = 2$
$7 - 5 = 2$
The first differences are constant. So, the function is linear.

4.2

3. Use graphing technology to approximate the solution of the equation below. Write the roots to 3 decimal places.

$3x^2 + 6x - 70 = 0$

Graph $y = 3x^2 + 6x - 70$. Use the CALC feature to display $X = -5.932883$ and $X = 3.9328829$. The roots are approximately $x = -5.933$ and $x = 3.933$.

4.3

4. For each pair of quadratic functions, describe how their graphs are related.

a) $y = (x - 3)^2$; $y = (x + 2)^2$

Compare the equations with $y = (x - p)^2$.
$y = (x - 3)^2$: Its graph is the graph of $y = x^2$ translated 3 units to the right.
$y = (x + 2)^2$: Its graph is the graph of $y = x^2$ translated 2 units to the left.
So, the graph of $y = (x - 3)^2$ is translated 5 units left to get the graph of $y = (x + 2)^2$.

b) $y = x^2 + 5$; $y = x^2 - 1$

Compare the equations with $y = x^2 + q$.
$y = x^2 + 5$: Its graph is the graph of $y = x^2$ translated 5 units up.
$y = x^2 - 1$: Its graph is the graph of $y = x^2$ translated 1 unit down.
So, the graph of $y = x^2 + 5$ is translated 6 units down to get the graph of $y = x^2 - 1$.

c) $y = -\frac{1}{2}x^2$; $y = \frac{1}{2}x^2$

Compare the equations with $y = ax^2$.
$y = -\frac{1}{2}x^2$: Its graph is the graph of $y = x^2$ compressed by a vertical factor of $\frac{1}{2}$, then reflected in the x-axis.
$y = \frac{1}{2}x^2$: Its graph is the graph of $y = x^2$ compressed by a vertical factor of $\frac{1}{2}$.
So, the graph of $y = -\frac{1}{2}x^2$ is reflected in the x-axis to get the graph of $y = \frac{1}{2}x^2$.

4.4

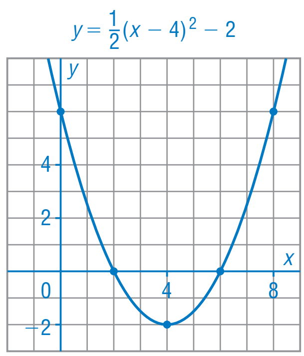
5. For this quadratic function: $y = \frac{1}{2}(x - 4)^2 - 2$

a) Identify the coordinates of the vertex, the domain, the range, the direction of opening, the equation of the axis of symmetry, and the intercepts.

a is positive, so the graph opens up.
$p = 4$ and $q = -2$, so the coordinates of the vertex are: $(4, -2)$
The equation of the axis of symmetry is $x = p$; that is, $x = 4$.
To determine the y-intercept, substitute $x = 0$:
$y = \frac{1}{2}(0 - 4)^2 - 2$
$y = 6$
The y-intercept is 6.
To determine the x-intercepts, substitute $y = 0$:
$0 = \frac{1}{2}(x - 4)^2 - 2$
$0 = \frac{x^2}{2} - 4x + 6$
$0 = x^2 - 8x + 12$
$0 = (x - 6)(x - 2)$
$x = 6$ or $x = 2$
The x-intercepts are 2 and 6.
The domain is: $x \in \mathbb{R}$
The graph opens up, so the vertex is a minimum point with y-coordinate -2. The range is: $y \geq -2, y \in \mathbb{R}$

b) Sketch a graph.

The graph is congruent to the graph of $y = \frac{1}{2}x^2$.
On a grid, mark a point at the vertex $(4, -2)$. Use the step pattern.
Multiply each vertical step by $\frac{1}{2}$.



6. Determine an equation of the quadratic function for each set of data given.

a) The coordinates of the vertex are V(4, 12) and the graph passes through A(7, 6).

An equation has the form $y = a(x - p)^2 + q$.
The vertex is at V(4, 12), so $p = 4$ and $q = 12$.
The equation becomes $y = a(x - 4)^2 + 12$.
Substitute the given coordinates for point A: $x = 7$, $y = 6$

$$6 = a(7 - 4)^2 + 12$$
$$6 = 9a + 12$$
$$-6 = 9a$$
$$a = -\frac{2}{3}$$

So, the equation of the function is:

$$y = -\frac{2}{3}(x - 4)^2 + 12$$

b) The graph passes through B(2, −5) and has x-intercepts −3 and 4.

Use $y = a(x - x_1)(x - x_2)$ — Substitute: $x_1 = -3$ and $x_2 = 4$

$y = a(x + 3)(x - 4)$ — Substitute for B(2, −5).

$$-5 = a(2 + 3)(2 - 4)$$
$$-5 = -10a$$
$$a = 0.5$$

In factored form, the equation is: $y = 0.5(x + 3)(x - 4)$

4.5

7. Write this equation in standard form.

$y = -3x^2 + 24x - 45$

$y = -3(x^2 - 8x) - 45$

Add and subtract: $\left(\frac{-8}{2}\right)^2 = 16$

$$\begin{aligned} y &= -3(x^2 - 8x) - 45 \\ &= -3(x^2 - 8x + 16 - 16) - 45 \\ &= -3(x^2 - 8x + 16) + 48 - 45 \\ &= -3(x - 4)^2 + 3 \end{aligned}$$

4.6

8. Determine the intercepts, the equation of the axis of symmetry, and the coordinates of the vertex of the graph of each quadratic function, then sketch the graph.

a) $y = 2x^2 + 2x - 24$

The y-intercept is -24.
Factor the equation.

$$\begin{aligned} y &= 2x^2 + 2x - 24 \\ &= 2(x^2 + x - 12) \\ &= 2(x + 4)(x - 3) \end{aligned}$$

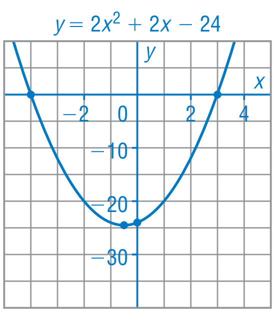
The x-intercepts are: -4 and 3
The mean of the intercepts is:

$\frac{-4 + 3}{2} = -0.5$

So, the equation of the axis of symmetry is: $x = -0.5$
Substitute $x = -0.5$ in

$$\begin{aligned} y &= 2x^2 + 2x - 24 \\ &= 2(-0.5)^2 + 2(-0.5) - 24 \\ &= -24.5 \end{aligned}$$

The coordinates of the vertex are: $(-0.5, -24.5)$



b) $y = -\frac{1}{2}x^2 - x + 4$

The y-intercept is 4.
Factor the equation.

$$\begin{aligned} y &= -\frac{1}{2}x^2 - x + 4 \\ &= -\frac{1}{2}(x^2 + 2x - 8) \\ &= -\frac{1}{2}(x + 4)(x - 2) \end{aligned}$$

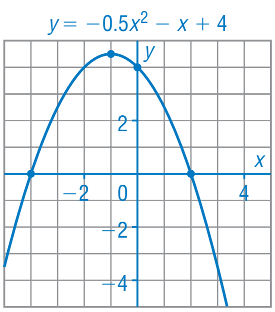
The x-intercepts are: -4 and 2
The mean of the intercepts is:

$\frac{-4 + 2}{2} = -1$

So, the equation of the axis of symmetry is: $x = -1$
Substitute $x = -1$ in

$$\begin{aligned} y &= -\frac{1}{2}x^2 - x + 4 \\ &= -\frac{1}{2}(-1)^2 + 1 + 4 \\ &= 4.5 \end{aligned}$$

The coordinates of the vertex are: $(-1, 4.5)$



4.7

9. Select Audio Company sells an MP3 player for \$75. At that price, the company sells approximately 1000 players per week. The company predicts that for every \$5 increase in price, it will sell 50 fewer MP3 players. Which price for an MP3 player will maximize the revenue?

Let x represent the number of \$5 increases in the price of an MP3 player.
When the cost is \$75, 1000 are sold for a revenue of:
\$75(1000) = \$75 000
When the cost is \$(75 + 5x), (1000 − 50x) are sold for a revenue of \$(75 + 5x)(1000 − 50x).
Let the revenue be R dollars.
An equation is: $R = (75 + 5x)(1000 - 50x)$
Use a graphing calculator to graph the equation.
From the graph, the maximum revenue is about \$76 562.50 when the number of \$5 increases is 2.5.
The number of increases is a whole number, so round 2.5 to 2 or to 3.
Two increases of \$5 mean that the MP3 player will now cost:
2(\$5) + \$75 = \$85
Three increases of \$5 mean that the MP3 player will now cost:
3(\$5) + \$75 = \$90
To maximize the revenue, the MP3 player should sell for \$85 or \$90.