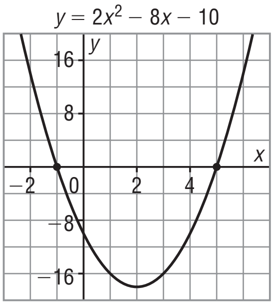
Lesson 5.1 Exercises, pages 346–352

A

4. Use the given graphs to write the solutions of the corresponding quadratic inequalities.

a) $2x^2 - 8x - 10 < 0$

The solution is the values of x for which $y < 0$; that is, $-1 < x < 5, x \in \mathbb{R}$

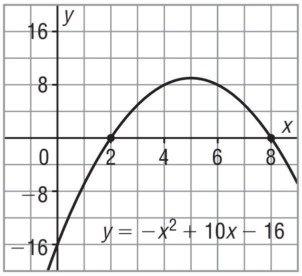


b) $2x^2 - 8x - 10 \geq 0$

The solution is the values of x for which $y \geq 0$; that is, $x \leq -1$ or $x \geq 5, x \in \mathbb{R}$

c) $-x^2 + 10x - 16 > 0$

The solution is the values of x for which $y > 0$; that is, $2 < x < 8, x \in \mathbb{R}$



d) $-x^2 + 10x - 16 \leq 0$

The solution is the values of x for which $y \leq 0$; that is, $x \leq 2$ or $x \geq 8, x \in \mathbb{R}$

5. Solve this quadratic inequality.

$(x - 2)(x - 6) \leq 0$

Solve: $(x - 2)(x - 6) = 0$
$x = 2$ or $x = 6$
When $x \leq 2$, such as $x = 0$, L.S. = 12; R.S. = 0;
so $x = 0$ does not satisfy the inequality.
When $2 \leq x \leq 6$, such as $x = 4$, L.S. = −4; R.S. = 0;
so $x = 4$ does satisfy the inequality.
The solution is: $2 \leq x \leq 6, x \in \mathbb{R}$

B

6. Solve each quadratic inequality. Represent each solution on a number line.

a) $x^2 - x - 12 \leq 0$

Solve: $x^2 - x - 12 = 0$

$(x - 4)(x + 3) = 0$

$x = 4$ or $x = -3$

When $x \leq -3$, such as $x = -4$, L.S. = 8; R.S. = 0;

so $x = -4$ does not satisfy the inequality.

When $-3 \leq x \leq 4$, such as $x = 0$, L.S. = -12; R.S. = 0;

so $x = 0$ does satisfy the inequality.

The solution is: $-3 \leq x \leq 4, x \in \mathbb{R}$

b) $4x^2 + 8x + 3 > 0$

Solve: $4x^2 + 8x + 3 = 0$

$(2x + 3)(2x + 1) = 0$

$x = -1.5$ or $x = -0.5$

When $x < -1.5$, such as $x = -2$, L.S. = 3; R.S. = 0;

so $x = -2$ does satisfy the inequality.

When $x > -0.5$, such as $x = 0$, L.S. = 3; R.S. = 0;

so $x = 0$ does satisfy the inequality.

The solution is: $x < -1.5$ or $x > -0.5, x \in \mathbb{R}$

c) $-2x^2 + 5x + 3 \geq 0$

Solve: $-2x^2 + 5x + 3 = 0$

$2x^2 - 5x - 3 = 0$

$(2x + 1)(x - 3) = 0$

$x = -0.5$ or $x = 3$

When $x \leq -0.5$, such as $x = -1$, L.S. = -4; R.S. = 0;

so $x = -1$ does not satisfy the inequality.

When $-0.5 \leq x \leq 3$, such as $x = 0$, L.S. = 3; R.S. = 0;

so $x = 0$ does satisfy the inequality.

The solution is: $-0.5 \leq x \leq 3, x \in \mathbb{R}$

7. Solve each quadratic inequality. Represent each solution on a number line.

a) $-5x^2 > 17x - 12$

Solve: $-5x^2 = 17x - 12$
$5x^2 + 17x - 12 = 0$
$(5x - 3)(x + 4) = 0$
$x = 0.6$ or $x = -4$
When $x < -4$, such as $x = -5$, L.S. $= -125$; R.S. $= -97$;
so $x = -5$ does not satisfy the inequality.
When $-4 < x < 0.6$, such as $x = 0$, L.S. $= 0$; R.S. $= -12$;
so $x = 0$ does satisfy the inequality.
The solution is: $-4 < x < 0.6$, $x \in \mathbb{R}$

b) $4x^2 + 15x > -14$

Solve: $4x^2 + 15x = -14$
$4x^2 + 15x + 14 = 0$
$(x + 2)(4x + 7) = 0$
$x = -2$ or $x = -1.75$
When $x < -2$, such as $x = -3$, L.S. $= -9$; R.S. $= -14$;
so $x = -3$ does satisfy the inequality.
When $x > -1.75$, such as $x = 0$, L.S. $= 0$; R.S. $= -14$;
so $x = 0$ does satisfy the inequality.
The solution is: $x < -2$ or $x > -1.75$, $x \in \mathbb{R}$

8. Solve each quadratic inequality by graphing. Give the solutions to the nearest tenth.

a) $1.2x^2 + 3.5x \leq 4.8$

Rearrange the inequality.
$1.2x^2 + 3.5x - 4.8 \leq 0$
Graph: $y = 1.2x^2 + 3.5x - 4.8$
The critical values are approximately -3.9 and 1.0.
The solution of the inequality is these points and the values of x for which $y \leq 0$; that is, $-3.9 \leq x \leq 1.0$, $x \in \mathbb{R}$.

b) $0 < 13.8 + 12.6x - 0.4x^2$

Graph: $y = 13.8 + 12.6x - 0.4x^2$
The critical values are approximately -1.1 and 32.6.
The solution of the inequality is the values of x for which $y > 0$; that is, $-1.1 < x < 32.6$, $x \in \mathbb{R}$.

9. Use the quadratic formula to solve each quadratic inequality. Give the solutions to the nearest tenth.

a) $2x^2 - 3x - 4 < 0$

Solve: $2x^2 - 3x - 4 = 0$
Substitute:
$a = 2, b = -3, c = -4$
in: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \frac{3 \pm \sqrt{(-3)^2 - 4(2)(-4)}}{2(2)}$
$x = \frac{3 \pm \sqrt{41}}{4}$
$x \doteq 2.4$ or $x \doteq -0.9$
When $x < -0.9$, such as $x = -1$, L.S. = 1; R.S. = 0; so $x = 1$ does not satisfy the inequality.
When $-0.9 < x < 2.4$, such as $x = 0$, L.S. = -4; R.S. = 0; so $x = 0$ does satisfy the inequality.
The solution is:
$-0.9 < x < 2.4, x \in \mathbb{R}$

b) $\frac{x^2}{3} + \frac{2x}{5} > 1$

Solve: $5x^2 + 6x - 15 = 0$
Substitute:
$a = 5, b = 6, c = -15$
in: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \frac{-6 \pm \sqrt{6^2 - 4(5)(-15)}}{2(5)}$
$x = \frac{-6 \pm \sqrt{336}}{10}$
$x \doteq -2.4$ or $x \doteq 1.2$
When $x < -2.4$, such as $x = -3$, L.S. = 1.8; R.S. = 1; so $x = -3$ does satisfy the inequality.
When $x > 1.2$, such as $x = 3$, L.S. = 4.2; R.S. = 1; so $x = 3$ does satisfy the inequality.
The solution is:
$x < -2.4$ or $x > 1.2, x \in \mathbb{R}$

10. Solve each quadratic inequality. Give the solutions to the nearest tenth where necessary. Use a different strategy each time. Explain each strategy.

a) $3x^2 < 21x$

Use intervals and test points.
Solve: $3x^2 < 21x$
$3x^2 - 21x = 0$
$3x(x - 7) = 0$
$x = 0$ or $x = 7$
When $x < 0$, such as $x = -1$, L.S. = 3; R.S. = -21;
so $x = -1$ does not satisfy the inequality.
When $0 < x < 7$, such as $x = 1$, L.S. = 3; R.S. = 21;
so $x = 1$ does satisfy the inequality.
The solution is: $0 < x < 7, x \in \mathbb{R}$

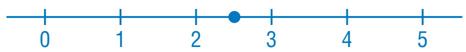
b) $4x^2 - 1 > 3x + 25$

Rearrange the inequality.
$4x^2 - 3x - 26 > 0$
Use a graphing calculator.
Graph: $y = 4x^2 - 3x - 26$
The critical values are approximately -2.2 and 3.0.
The solution of the inequality is the values of x for which $y > 0$; that is,
$x < -2.2$ or $x > 3.0$, $x \in \mathbb{R}$.

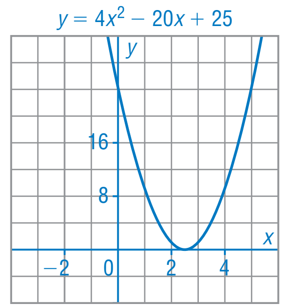
11. Consider this inequality: $4x^2 - 20x + 25 \leq 0$

a) Solve the inequality by factoring.
Illustrate the solution on a number line.

Solve: $4x^2 - 20x + 25 = 0$
$(2x - 5)(2x - 5) = 0$
$x = 2.5$
When $x \leq 2.5$, such as $x = 2$, L.S. = 1; R.S. = 0;
so $x = 2$ does not satisfy the inequality.
When $x \geq 2.5$, such as $x = 3$, L.S. = 1; R.S. = 0;
so $x = 3$ does not satisfy the inequality.
The solution is: $x = 2.5$



b) Solve the inequality by graphing.
Sketch the graph.



The graph of $y = 4x^2 - 20x + 25$ opens up, has x-intercept 2.5 and is congruent to $y = 4x^2$.
The only point where $4x^2 - 20x + 25 \leq 0$ is at $x = 2.5$.

c) **i)** What do you notice about the solution of the inequality?

The solution is one number.

ii) Write a different inequality that has the same number of solutions. What strategy did you use?

Sample response: Write the equation of a parabola that touches the x-axis; for example, $y = -(x - 3)^2$. Another inequality with exactly one solution is: $-x^2 + 6x - 9 \geq 0$.

12. The product of two consecutive even integers is at least 48. What might the integers be?

Let the lesser integer be x, then the greater integer is $x + 2$.
An inequality is: $x(x + 2) \geq 48$
Solve the equation: $x^2 + 2x - 48 = 0$
$(x - 6)(x + 8) = 0$
$x = 6$ or $x = -8$
When $x = 6$, $x + 2 = 8$
When $x = -8$, $x + 2 = -6$
Any pair of consecutive even integers greater than or equal to 6 and 8, or less than or equal to −8 and −6 are solutions to the problem; such as 8 and 10, 10 and 12, −8 and −10, −10 and −12.

13. A tennis ball is thrown upward at an initial speed of 15 m/s. The approximate height of the ball, h metres, after t seconds, is given by the equation $h = 15t - 5t^2$. Determine the time period for which the ball is higher than 10 m.

An inequality that represents this situation is: $15t - 5t^2 > 10$
A related quadratic equation is:
$5t^2 - 15t + 10 = 0$**, or**
$t^2 - 3t + 2 = 0$ **Solve the equation.**
$(t - 1)(t - 2) = 0$
$t = 1$ **or** $t = 2$
When $t < 1$, such as $t = 0$, L.S. = 0; R.S. = 10;
so $t = 0$ does not satisfy the inequality.
When $1 < t < 2$, such as $t = 1.5$, L.S. = 11.25; R.S. = 10;
so $t = 1.5$ does satisfy the inequality.
The solution is: $1 < t < 2$
So, the tennis ball is higher than 10 m between 1 and 2 s after it is thrown.

14. $2x^2 + bx + 7 = 0$ is a quadratic equation. For which values of b does the quadratic equation have:

a) two real roots?

For $2x^2 + bx + 7 = 0$ to have 2 real roots, its discriminant is greater than 0.
$b^2 - 4ac > 0$ **Substitute: $a = 2$, $c = 7$**
$b^2 - 4(2)(7) > 0$
$b^2 > 56$
$b > \sqrt{56}$ **or** $b < -\sqrt{56}$

b) no real roots?

For $2x^2 + bx + 7 = 0$ to have no real roots, its discriminant is negative.
$b^2 - 4(2)(7) < 0$
$b^2 < 56$
$b < \sqrt{56}$ **and** $b > -\sqrt{56}$**,**
which is written $-\sqrt{56} < b < \sqrt{56}$

C

15. Create an inequality that has each solution.

a) $-13 \leq x \leq -3$

Work backward.
The critical points are -13 and -3.
So a related quadratic equation is:
$(x + 13)(x + 3) = 0$
$x^2 + 16x + 39 = 0$
On a graphing calculator, graph the related function.
The solution is the x-intercepts and all real numbers between them. An inequality is:
$x^2 + 16x + 39 \leq 0$

b) $x < -1.1$ or $x > 0.4$

The critical points are -1.1 and 0.4.
A related quadratic equation is:
$(x + 1.1)(x - 0.4) = 0$
$x^2 + 0.7x - 0.44 = 0$
On a graphing calculator, graph the related function.
The solution is all real numbers to the left and to the right of the x-intercepts.
An inequality is:
$x^2 + 0.7x - 0.44 > 0$