Lesson 5.2 Exercises, pages 360–368

A

3. Determine whether each point is a solution of the given inequality.

a) $3x - 2y \geq -16$ A(−3, 4)

In the inequality, substitute: $x = -3$, $y = 4$
L.S.: $3(-3) - 2(4) = -17$ R.S. $= -16$
Since the L.S. $<$ R.S., the point is not a solution.

b) $4x - y \leq 5$ B(−1, 1)

In the inequality, substitute: $x = -1$, $y = 1$
L.S.: $4(-1) - 1 = -5$ R.S. $= 5$
Since the L.S. $<$ R.S., the point is a solution.

c) $3y > 2x - 7$ C(−2, −5)

In the inequality, substitute: $x = -2$, $y = -5$
L.S.: $3(-5) = -15$ R.S.: $2(-2) - 7 = -11$
Since the L.S. $<$ R.S., the point is not a solution.

d) $5x - 2y + 8 < 0$ D(6, 7)

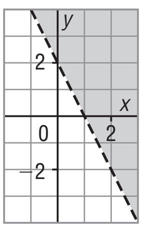
In the inequality, substitute: $x = 6$, $y = 7$
L.S.: $5(6) - 2(7) + 8 = 24$ R.S. $= 0$
Since the L.S. $>$ R.S., the point is not a solution.

4. Match each graph with an inequality below.

i) $2x + y \leq -2$ **ii)** $2x + y > 2$

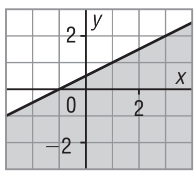
iii) $x - 2y < 2$ **iv)** $x - 2y \geq -1$

a)

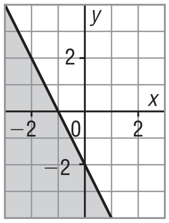


The line has slope −2 and y-intercept 2, so its equation is:
$y = -2x + 2$, or $2x + y = 2$
The inequality is:
$2x + y > 2$

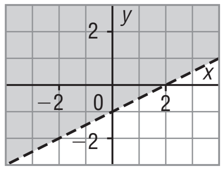
b)



The line has slope 0.5 and y-intercept 0.5, so its equation is:
$y = 0.5x + 0.5$, or $x - 2y = -1$
The inequality is:
$x - 2y \geq -1$

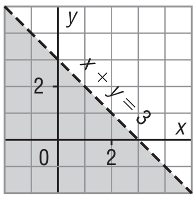
c) 

The line has slope −2 and y-intercept −2, so its equation is:
$y = -2x - 2$, or $2x + y = -2$
The inequality is:
$2x + y \leq -2$

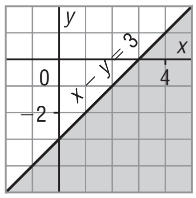
d) 

The line has slope 0.5 and y-intercept −1, so its equation is:
$y = 0.5x - 1$, or $x - 2y = 2$
The inequality is:
$x - 2y < 2$

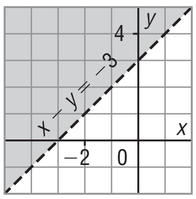
5. Write an inequality to describe each graph.

a) 

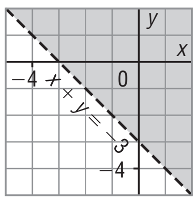
The equation can be written as: $y = -x + 3$
The line is broken, and the shaded region is below the line so an inequality is: $y < -x + 3$, or $x + y < 3$

b) 

The equation can be written as: $y = x - 3$
The line is solid, and the shaded region is below the line so an inequality is: $y \leq x - 3$, or $x - y \geq 3$

c) 

The equation can be written as: $y = x + 3$
The line is broken, and the shaded region is above the line so an inequality is: $y > x + 3$, or $x - y < -3$

d) 

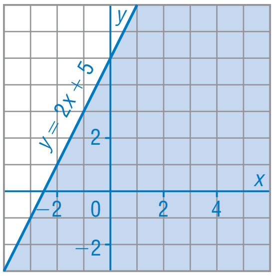
The equation can be written as: $y = -x - 3$
The line is broken, and the shaded region is above the line so the inequality is: $y > -x - 3$, or $x + y > -3$

B

6. Graph each linear inequality.

a) $y \leq 2x + 5$

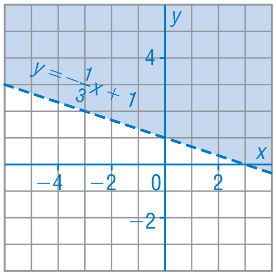
Graph of $y \leq 2x + 5$



Use intercepts to graph the related functions.
When $x = 0$, $y = 5$
When $y = 0$, $x = -2.5$
Draw a solid line. Shade the region below the line.

b) $y > -\frac{1}{3}x + 1$

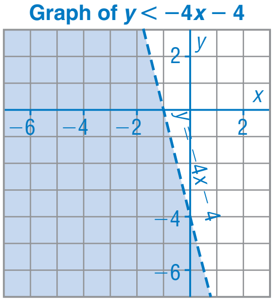
Graph of $y > -\frac{1}{3}x + 1$



When $x = 0$, $y = 1$
When $y = 0$, $x = 3$
Draw a broken line. Shade the region above the line.

c) $y < -4x - 4$

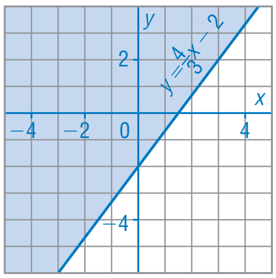
Graph of $y < -4x - 4$



Use intercepts to graph the related functions.
When $x = 0$, $y = -4$
When $y = 0$, $x = -1$
Draw a broken line. Shade the region below the line.

d) $y \geq \frac{4}{3}x - 2$

Graph of $y \geq \frac{4}{3}x - 2$

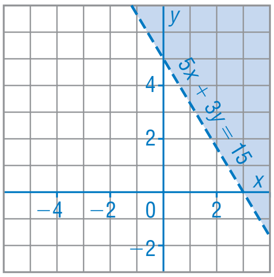


When $x = 0$, $y = -2$
When $y = 0$, $x = 1.5$
Draw a solid line. Shade the region above the line.

7. Graph each linear inequality. Give the coordinates of 3 points that satisfy the inequality.

a) $5x + 3y > 15$

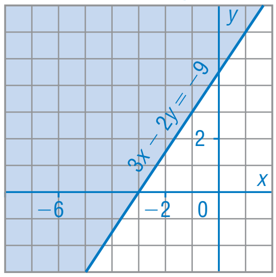
Graph of $5x + 3y > 15$



Use intercepts to graph the related functions.
When $x = 0$, $y = 5$
When $y = 0$, $x = 3$
Use (0, 0) as a test point.
L.S. = 0; R.S. = 15
Since $0 < 15$, the origin does not lie in the shaded region.
Draw a broken line. Shade the region above the line.
From the graph, 3 points that satisfy the inequality are: (2, 3), (1, 5), (3, 2)

b) $3x - 2y \leq -9$

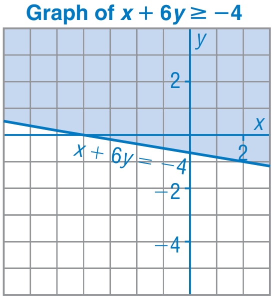
Graph of $3x - 2y \leq -9$



When $x = 0$, $y = 4.5$
When $y = 0$, $x = -3$
Use (0, 0) as a test point.
L.S. = 0; R.S. = −9
Since $0 > -9$, the origin does not lie in the shaded region.
Draw a solid line. Shade the region above the line.
From the graph, 3 points that satisfy the inequality are: (−2, 3), (−1, 4), (−1, 6)

c) $x + 6y \geq -4$

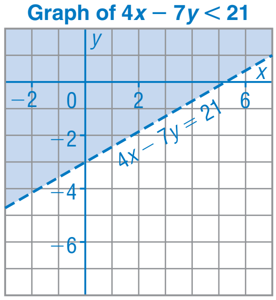
Graph of $x + 6y \geq -4$



Graph the related functions.
When $y = 0$, $x = -4$
When $y = -1$, $x = 2$
Use (0, 0) as a test point.
L.S. = 0; R.S. = −4
Since $0 > -4$, the origin lies in the shaded region.
Draw a solid line. Shade the region above the line.
From the graph, 3 points that satisfy the inequality are: (2, 1), (1, 2), (3, 3)

d) $4x - 7y < 21$

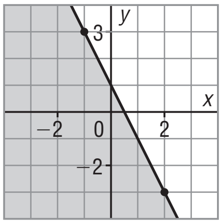
Graph of $4x - 7y < 21$



When $x = 0$, $y = -3$
When $y = 1$, $x = 7$
Use (0, 0) as a test point.
L.S. = 0; R.S. = 21
Since $0 < 21$, the origin lies in the shaded region.
Draw a broken line. Shade the region above the line.
From the graph, 3 points that satisfy the inequality are: (−1, 3), (1, −1), (2, 3)

8. Write an inequality to describe each graph.

a)



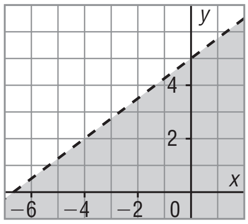
The line has slope −2 and y-intercept 1, so its equation is: $y = -2x + 1$

The line is solid and the region below is shaded.

An inequality is:

$y \leq -2x + 1$

b)



The line has slope $\frac{3}{4}$ and y-intercept 5, so its equation is: $y = \frac{3}{4}x + 5$,

The line is broken and the region below is shaded.

An inequality is:

$y < \frac{3}{4}x + 5$

9. A student graphed the inequality $2x - y < 0$ and used the origin as a test point. Could the student then shade the correct region of the graph? Explain your answer.

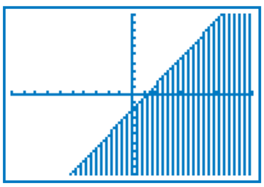
No, the line passes through the origin, so it cannot be used as a test point. The test point must not lie on the line that divides the region.

10. Use technology to graph each linear inequality. Sketch the graph.

a) $y < 1.6x - 1.95$

Graph: $y = 1.6x - 1.95$

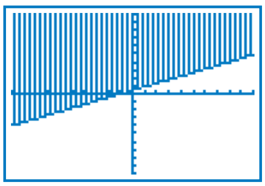
The boundary is not part of the graph.



b) $y > \frac{4}{9}x + \frac{3}{7}$

Graph: $y = \frac{4}{9}x + \frac{3}{7}$

The boundary is not part of the graph.



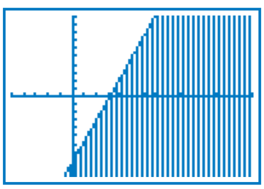
c) $8x - 3y - 25 \geq 0$

$3y \leq 8x - 25$

$y \leq \frac{8}{3}x - \frac{25}{3}$

Graph: $y = \frac{8}{3}x - \frac{25}{3}$

The boundary is part of the graph.



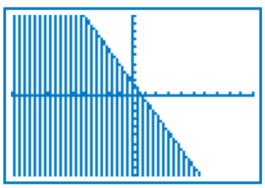
d) $4.8x + 2.3y - 3.7 \leq 0$

$2.3y \leq -4.8x + 3.7$

$y \leq \frac{-4.8}{2.3}x + \frac{3.7}{2.3}$

Graph: $y = \frac{-4.8}{2.3}x + \frac{3.7}{2.3}$

The boundary is part of the graph.

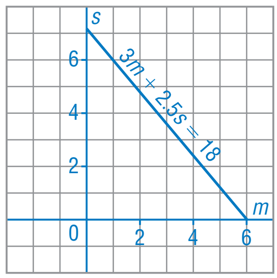


11. Nina takes her friends to an ice cream store. A milkshake costs \$3 and a chocolate sundae costs \$2.50. Nina has \$18 in her purse.

a) Write an inequality to describe how Nina can spend her money.

Let m represent the number of milkshakes and s represent the number of sundaes.
An inequality is: $3m + 2.5s \leq 18$

b) Determine 3 possible ways Nina can spend up to \$18.



Determine the coordinates of 2 points that satisfy the related function.
When $s = 0$, $m = 6$
When $s = 6$, $m = 1$
Join the points with a solid line.
The solution is the points, with whole-number coordinates, on and below the line.
Three ways are: 4 milkshakes, 2 sundaes; 3 milkshakes, 3 sundaes; 2 milkshakes, 4 sundaes

c) What is the most money Nina can spend and still have change from \$18?

The point, with whole-number coordinates, that is closest to the line has coordinates (5, 1); the cost, in dollars, is:
$(5)(3) + 1(2.50) = 17.50$
Nina can spend \$17.50 and still have change.

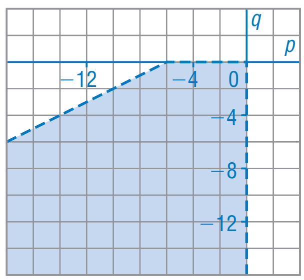
12. The relationship between two negative numbers p and q is described by the inequality $p - 2q > -6$.

a) What are the restrictions on the variables?

Since the numbers are negative, $p < 0$ and $q < 0$

b) Graph the inequality.

Graph of $p - 2q > -6$



Determine the coordinates of 2 points that satisfy the related function.
When $p = -10$, $q = -2$
When $p = -6$, $q = 0$
Draw a broken line through the points.
The solution is the points below the line in Quadrant 3.

c) Write the coordinates of 2 points that satisfy the inequality.

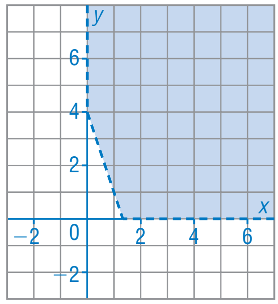
Sample response: Two points are: $(-4, -4)$ and $(-12, -4)$

13. Graph each inequality for the given restrictions on the variables.

a) $y > -3x + 4$; for $x > 0, y > 0$

Since $x > 0, y > 0$, the graph is in Quadrant 1.
The graph of the related function has slope -3 and y-intercept 4.
Draw a broken line to represent the related function in Quadrant 1.
Shade the region above the line.
The axes bounding the graph are broken lines.

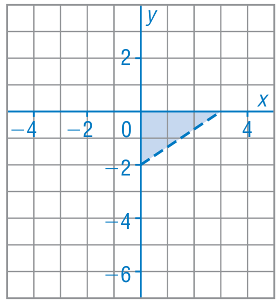
Graph of $y > -3x + 4$, $x > 0, y > 0$



b) $2x - 3y < 6$; for $x \geq 0, y \leq 0$

Since $x \geq 0, y \leq 0$, the graph is in Quadrant 4.
Graph the related function.
When $y = 0, x = 3$
When $x = 0, y = -2$
Draw a broken line in Quadrant 4.
Use (0, 0) as a test point.
L.S. = 0; R.S. = 6
Since $0 < 6$, the origin lies in the shaded region.
Shade the region above the line.

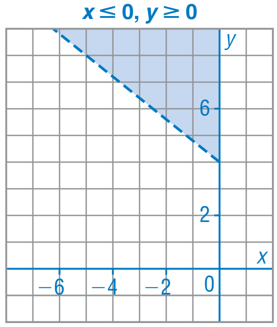
Graph of $2x - 3y < 6$, $x \geq 0, y \leq 0$



c) $4x + 5y - 20 > 0$; for $x \leq 0, y \geq 0$

Since $x \leq 0, y \geq 0$, the graph is in Quadrant 2.
Graph the related function.
When $x = 0, y = 4$
When $x = -5, y = 8$
Draw a broken line in Quadrant 2.
Use (0, 0) as a test point.
L.S. = -20; R.S. = 0
Since $-20 < 0$, the origin does not lie in the shaded region.
Shade the region above the line.

Graph of $4x + 5y - 20 > 0$, $x \leq 0, y \geq 0$



14. a) For A(9, a) to be a solution of $3x - 2y < 5$, what must be true about a?

Substitute the coordinates of A in the inequality.

$3(9) - 2(a) < 5$ Solve for a.

$2a > 22$

$a > 11$

b) For B(b, −3) to be a solution of $3x + 4y \geq -12$, what must be true about b?

Substitute the coordinates of B in the inequality.

$3(b) + 4(-3) \geq -12$ Solve for b.

$3b \geq 0$

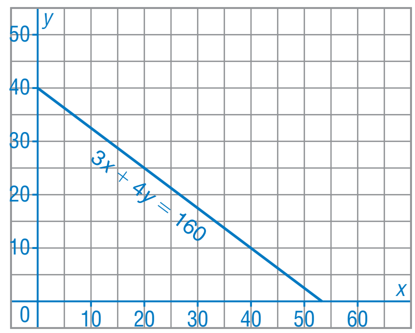
$b \geq 0$

15. A personal trainer books clients for either 45-min or 60-min appointments. He meets with clients a maximum of 40 h each week.

a) Write an inequality that represents the trainer's weekly appointments.

Let x represent the number of 45-min appointments and y represent the number of 60-min appointments.
An inequality is: $45x + 60y \leq 2400$
Divide by 15.
$3x + 4y \leq 160$

b) Graph the related equation, then describe the graph of the inequality.



Determine the coordinates of 2 points that satisfy the related function.
When $x = 0$, $y = 40$
When $x = 20$, $y = 25$
Join the points with a solid line.
The solution is the points, with whole-number coordinates, on and below the line.

c) How many 45-min appointments are possible if no 60-min appointments are scheduled? Where is the point that represents this situation located on the graph?

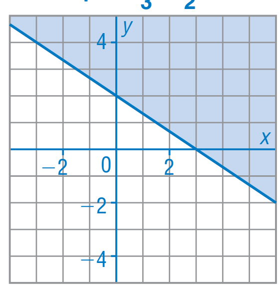
For no 60-min appointments, $y = 0$, so the point is on the x-axis; it is the point with whole-number coordinates that is closest to the x-intercept of the graph of the related equation. When $y = 0$,
$x = \frac{2400}{45}$, or $53.\overline{3}$
Fifty-three 45-min appointments are possible.

16. Graph this inequality. Identify the strategy you used and explain why you chose that strategy.

$$\frac{x}{3} + \frac{y}{2} \geq 1$$

Graph the related function.
Determine the intercepts.
When $y = 0$, $x = 3$
When $x = 0$, $y = 2$
Draw a solid line.
Use (0, 0) as a test point.
L.S. = 0; R.S. = 1
Since $0 < 1$, the origin is not in the shaded region.
Shade the region above the line.

Graph of $\frac{x}{3} + \frac{y}{2} \geq 1$



C

17. How is a linear inequality in two variables similar to a linear inequality in one variable? How are the inequalities different?

The solutions of both inequalities are usually sets of values. A linear inequality in one variable is a set of numbers that can be represented on a number line. A linear inequality in two variables is a set of ordered pairs that can be represented on a coordinate plane.