

Lesson 5.3 Exercises, pages 374–380

A

4. Determine whether each ordered pair is a solution of the quadratic inequality: $y \geq 3x^2 - 4$

a) $(-3, 8)$

b) $(0, 5)$

Substitute each ordered pair in $y \geq 3x^2 - 4$.

L.S. = 8; R.S.: $3(-3)^2 - 4 = 23$

Since $8 < 23$, the ordered pair is not a solution.

L.S. = 5; R.S.: $3(0)^2 - 4 = -4$

Since $5 > -4$, the ordered pair is a solution.

c) $(4, 44)$

d) $(-5, 12)$

L.S. = 44; R.S.: $3(4)^2 - 4 = 44$

Since $44 = 44$, the ordered pair is a solution.

L.S. = 12; R.S.: $3(-5)^2 - 4 = 71$

Since $12 < 71$, the ordered pair is not a solution.

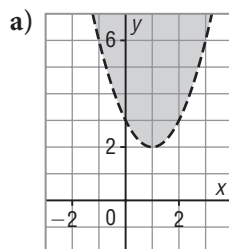
5. Match each inequality with a graph below.

i) $y < (x + 1)^2 - 2$

ii) $y \geq (x - 2)^2 - 1$

iii) $y > (x - 1)^2 + 2$

iv) $y \leq (x - 2)^2 + 1$



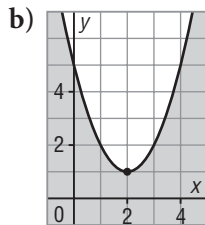
The parabola is congruent to $y = x^2$, and its vertex is $(1, 2)$.

Its equation is:

$y = (x - 1)^2 + 2$

The inequality is:

$y > (x - 1)^2 + 2$



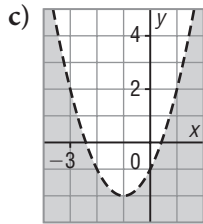
The parabola is congruent to $y = x^2$, and its vertex is $(2, 1)$.

Its equation is:

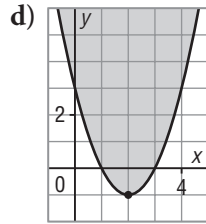
$y = (x - 2)^2 + 1$

The inequality is:

$y \leq (x - 2)^2 + 1$

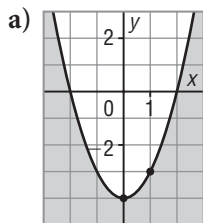


The parabola is congruent to $y = x^2$, and its vertex is $(-1, -2)$.
 Its equation is:
 $y = (x + 1)^2 - 2$
 The inequality is:
 $y < (x + 1)^2 - 2$

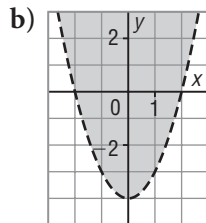


The parabola is congruent to $y = x^2$, and its vertex is $(2, -1)$.
 Its equation is:
 $y = (x - 2)^2 - 1$
 The inequality is:
 $y \geq (x - 2)^2 - 1$

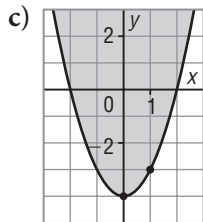
6. Write an inequality to describe each graph.



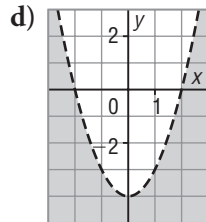
All 4 parabolas are congruent to $y = x^2$, and have vertex $(0, -4)$.
 The equation is: $y = x^2 - 4$
 The curve is solid and the shaded region is below.
 An inequality is: $y \leq x^2 - 4$



The curve is broken and the shaded region is above.
 An inequality is: $y > x^2 - 4$



The curve is solid and the shaded region is above.
 An inequality is: $y \geq x^2 - 4$



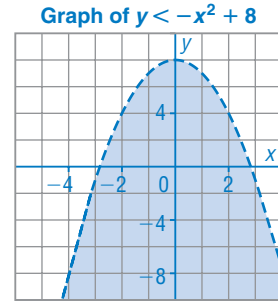
The curve is broken and the shaded region is below.
 An inequality is: $y < x^2 - 4$

B

7. Graph each inequality. Write the coordinates of 3 points that satisfy the inequality.

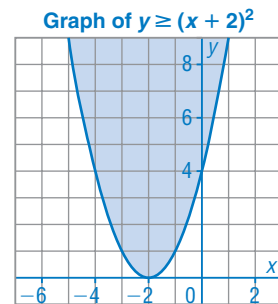
a) $y < -x^2 + 8$

The graph of the related quadratic function is congruent to $y = -x^2$ and has vertex $(0, 8)$. The curve is broken and the region below is shaded. 3 points that satisfy the inequality have coordinates: $(1, 4)$, $(2, 1)$, $(-1, 1)$



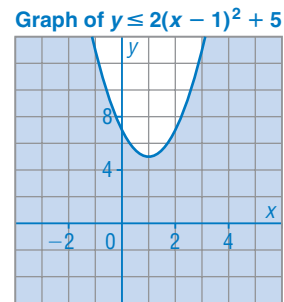
b) $y \geq (x + 2)^2$

The graph of the related quadratic function is congruent to $y = x^2$ and has vertex $(-2, 0)$. The curve is solid and the region above is shaded. 3 points that satisfy the inequality have coordinates: $(-2, 2)$, $(-1, 3)$, $(0, 5)$



c) $y \leq 2(x - 1)^2 + 5$

The graph of the related quadratic function is congruent to $y = 2x^2$ and has vertex $(1, 5)$. The curve is solid and the region below is shaded. 3 points that satisfy the inequality have coordinates: $(-1, 4)$, $(1, 2)$, $(2, 4)$



8. Are the points on the graph of $y = 4x^2 - 3x + 7$ part of the solution of the inequality $y > 4x^2 - 3x + 7$? Explain your answer.

No, the points are not part of the solution, because the inequality indicates that y is greater than $4x^2 - 3x + 7$, and, on the line, $y = 4x^2 - 3x + 7$.

9. Use technology to graph each inequality. Sketch or print the graph.

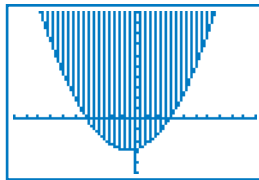
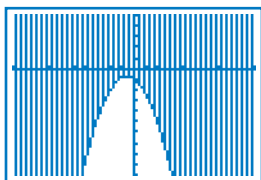
a) $y \geq -\frac{2}{3}x^2 - \frac{4}{5}x - 1$

b) $y > 0.25x^2 + 0.3x - 2.8$

Graph the related quadratic functions.

The boundary is part of the graph.

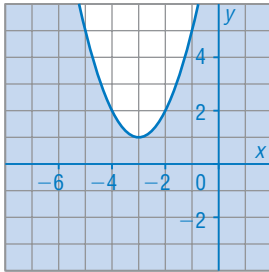
The boundary is not part of the graph.



10. Graph each inequality. Write the coordinates of 3 points that satisfy the inequality.

a) $y \leq x^2 + 6x + 10$

Graph of $y \leq x^2 + 6x + 10$



Complete the square for:

$$y = x^2 + 6x + 10$$

$$y = x^2 + 6x + 9 - 9 + 10$$

$$y = (x + 3)^2 + 1$$

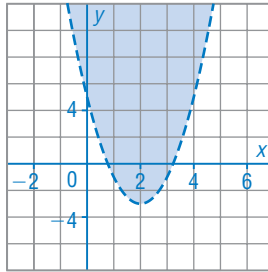
The graph of this function is congruent to $y = x^2$ and has vertex $(-3, 1)$.

The curve is solid and the region below is shaded.

3 points that satisfy the inequality have coordinates: $(-4, 1)$, $(-2, 1)$, $(0, 3)$

b) $y > 2x^2 - 8x + 5$

Graph of $y > 2x^2 - 8x + 5$



Complete the square for:

$$y = 2x^2 - 8x + 5$$

$$y = 2(x^2 - 4x + 4 - 4) + 5$$

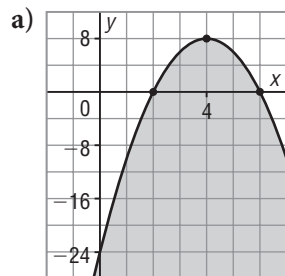
$$y = 2(x - 2)^2 - 3$$

The graph of this function is congruent to $y = 2x^2$ and has vertex $(2, -3)$.

The curve is broken and the region above is shaded.

3 points that satisfy the inequality have coordinates: $(1, 4)$, $(2, 8)$, $(3, 1)$

11. Write an inequality to describe each graph.



The parabola is congruent to

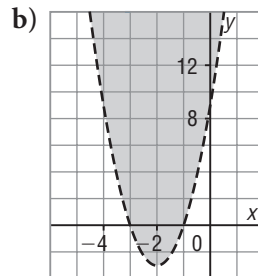
$y = -2x^2$, and has vertex $(4, 8)$. Its equation is:

$$y = -2(x - 4)^2 + 8$$

The curve is solid and the region below it is shaded.

An inequality is:

$$y \leq -2(x - 4)^2 + 8$$



The parabola is congruent to

$y = 3x^2$, and has vertex $(-2, -3)$. Its equation is:

$$y = 3(x + 2)^2 - 3$$

The curve is broken and the region above it is shaded.

An inequality is:

$$y > 3(x + 2)^2 - 3$$

12. a) For A(-1, a) to be a solution of $y > -2x^2 + 5$, what must be true about a?

In $y > -2x^2 + 5$, substitute: $x = -1, y = a$

$$a > -2(-1)^2 + 5$$

$$a > 3$$

- b) For B(b, 6) to be a solution of $y > x^2 - 5$, what must be true about b?

In $y > x^2 - 5$, substitute: $x = b, y = 6$

$$6 > b^2 - 5$$

$$b^2 < 11$$

$$b < \sqrt{11} \text{ or } b > -\sqrt{11}$$

$$\text{That is, } -\sqrt{11} < b < \sqrt{11}$$

13. Two numbers are related in this way: three times the square of one number is greater than or equal to the other number minus 4.

- a) Graph an inequality that represents this relationship.

Let the numbers be represented

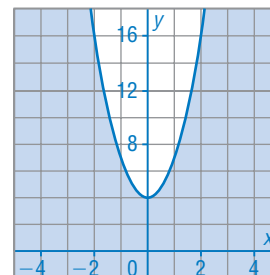
by x and y . An inequality is:

$$3x^2 \geq y - 4, \text{ or } y \leq 3x^2 + 4$$

The graph of the related function is congruent to $y = 3x^2$ and its vertex is (0, 4).

The curve is solid, with the region below it shaded.

Graph of $y \leq 3x^2 + 4$



- b) Use the graph to list three pairs of integer values for the two numbers.

Three pairs of integer values are:

$$(-2, 16), (-1, -2), (1, -10)$$

14. Graph this quadratic inequality: $\frac{(x + 1)^2}{2} - 3 > \frac{y}{3}$

$$\frac{(x + 1)^2}{2} - 3 > \frac{y}{3} \quad \text{Multiply by 3.}$$

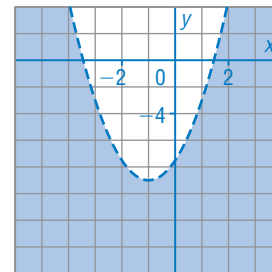
$$\frac{3(x + 1)^2}{2} - 3(3) > y$$

$$y < 1.5(x + 1)^2 - 9$$

The graph of the related function is congruent to $y = 1.5x^2$ and its vertex is (-1, -9).

The curve is broken, with the region below it shaded.

Graph of $y < 1.5(x + 1)^2 - 9$



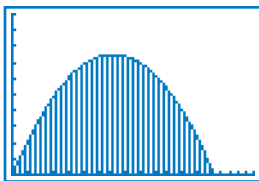
- 15.** An arch that supports a bridge over a river is parabolic and spans a horizontal distance of 250 m. An equation of the parabola is $y = -0.0048x^2 + 1.2x$, where y metres is the height of a point on the arch above the river, and x metres is the horizontal distance to that point measured from one end of the arch.

- a) Write an inequality to represent the cross-sectional region between the arch and the river.

Since both x and y are positive, the graph of the parabola is only in Quadrant 1. An inequality is: $y < -0.0048x^2 + 1.2x$, $x > 0$, $y > 0$

- b) Graph the inequality, then sketch it below.

Use a graphing calculator. Graph $y = -0.0048x^2 + 1.2x$, $x > 0$, $y > 0$ in Quadrant 1 with a broken curve. Shade the region between the curve and the x -axis.



- c) The tallest mast of a ship is 37 m above water level. Can the ship pass under the arch when it is 50 m from one end of the arch? Justify your answer.

Check whether the point (50, 37) satisfies the inequality.

In $y < -0.0048x^2 + 1.2x$, substitute: $x = 50$, $y = 37$

L.S. = 37 R.S.: $-0.0048(50)^2 + 1.2(50) = 48$

Since L.S. < R.S., the ship can pass under the arch.

C

- 16.** The length of a rectangle is 4 times a number. The width of the rectangle is 3 less than the square of another number. The length of the rectangle is greater than its width.

- a) Sketch a graph to represent this situation.

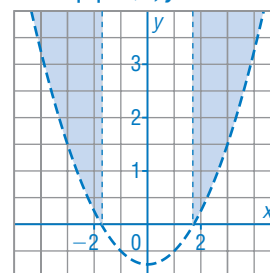
Let the length of the rectangle be represented by $4y$ units, and the width by $(x^2 - 3)$ units.

The length is greater than the width so an inequality is: $4y > x^2 - 3$, or $y > 0.25x^2 - 0.75$

The graph of the related function is congruent to $y = 0.25x^2$ and its vertex is $(0, -0.75)$. The curve is broken. Since y cannot be negative, only the region above the x -axis is shaded. Also, $x^2 - 3 > 0$, so $x > \sqrt{3}$ or $x < -\sqrt{3}$

Shade the region that satisfies these inequalities: $y > 0.25x^2 - 0.75$, $y > 0$, $|x| > \sqrt{3}$

Graph of $y > 0.25x^2 - 0.75$,
 $|x| > \sqrt{3}$, $y > 0$



b) Use the graph to list three possible sets of dimensions for the rectangle.

Three sets of coordinates are:

$(-2, 1)$, $(2, 2)$, $(3, 3)$

So, three possible sets of dimensions are:

width: $(-2)^2 - 3 = 1$; length: $4(1) = 4$

width: $2^2 - 3 = 1$; length: $4(2) = 8$

width: $3^2 - 3 = 6$; length: $4(3) = 12$

Possible dimensions are: 1 unit by 4 units; 1 unit by 8 units, 6 units by 12 units

17. Write a quadratic inequality in 2 variables that has these 3 points as solutions: $A(0, -3)$, $B(-3, 0)$, and $C(3, 3)$

Sample response: Visualize the points on a grid. The highest point is $(3, 3)$. Visualize a parabola that lies above the points; for example, the parabola opens up, has vertex $(4, 4)$, and is congruent to $y = x^2$. Its equation is: $y = (x - 4)^2 + 4$

So, an inequality is: $y < (x - 4)^2 + 4$