REVIEW, pages 408–414

5.1

1. Solve each inequality. Give the answers to the nearest tenth when necessary.

a) $x^2 - 2x - 15 \geq 0$

Solve: $x^2 - 2x - 15 = 0$

$(x - 5)(x + 3) = 0$

$x = 5$ or $x = -3$

When $x \leq -3$, such as $x = -4$, L.S. = 9; R.S. = 0; so $x = -4$ does satisfy the inequality.

When $x \geq 5$, such as $x = 6$, L.S. = 9; R.S. = 0; so $x = 6$ does satisfy the inequality.

The solution is: $x \leq -3$ or $x \geq 5$, $x \in \mathbb{R}$

b) $0.3x^2 + 0.5x \leq 1.9$

$0.3x^2 + 0.5x - 1.9 \leq 0$

Use a graphing calculator.

Graph: $y = 0.3x^2 + 0.5x - 1.9$

The critical values are approximately -3.5 and 1.8.

The solution of the inequality is the values of x for which $y < 0$; that is, $-3.5 \leq x \leq 1.8$, $x \in \mathbb{R}$.

2. Graph the solution of the inequality $-2(x + 2)^2 + 2 \leq 0$ on a number line.

Solve: $-2(x + 2)^2 + 2 = 0$

$(x + 2)^2 = 1$

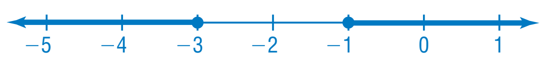
$x + 2 = \pm 1$

$x = -1$ or $x = -3$

When $x \leq -3$, such as $x = -4$, L.S. = -6; R.S. = 0; so $x = -4$ does satisfy the inequality.

When $x \geq -1$, such as $x = 0$, L.S. = -6; R.S. = 0; so $x = 0$ does satisfy the inequality.

The solution is: $x \leq -3$ or $x \geq -1$, $x \in \mathbb{R}$



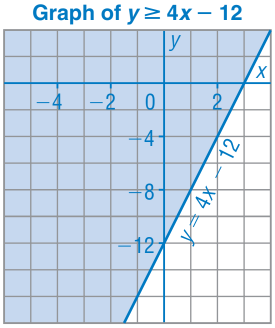
5.2

3. Graph each linear inequality. Write the coordinates of 3 points that satisfy the inequality.

a) $y \geq 4x - 12$

Graph the related functions.
When $x = 0$, $y = -12$
When $y = 0$, $x = 3$
Draw a solid line.
Use (0, 0) as a test point.
L.S. = 0; R.S. = −12
Since $0 > -12$, the origin is in the shaded region.
Shade the region above the line.
From the graph, 3 points that satisfy the inequality are: (2, 3), (1, 5), (3, 2)

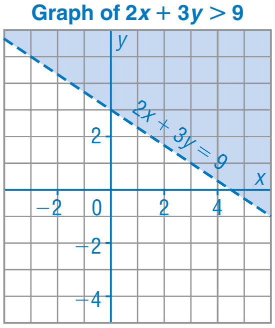
Graph of $y \geq 4x - 12$



b) $2x + 3y > 9$

When $x = 0$, $y = 3$
When $y = 1$, $x = 3$
Draw a broken line.
Use (0, 0) as a test point.
L.S. = 0; R.S. = 9
Since $0 < 9$, the origin is not in the shaded region.
Shade the region above the line.
From the graph, 3 points that satisfy the inequality are: (−1, 4), (0, 5), (3, 2)

Graph of $2x + 3y > 9$



4. Thomas sells plants at a local greenhouse. A rose plant costs $7.50 and a lily costs $4.25. Thomas hopes to sell more than $150 worth of these two plants in one day.

a) Write an inequality to represent this situation. What are the restrictions on the variables?

Let r represent the number of rose plants and l represent the number of lilies.
An inequality is: $7.5r + 4.25l > 150$
Both r and l are whole numbers.

b) Use technology to graph the related function. Describe the solution of the inequality.

Graph the related function.

$$7.5r + 4.25l = 150$$
$$4.25l = 150 - 7.5r$$
$$l = \frac{150}{4.25} - \frac{7.5}{4.25}r$$

Input: $y = \frac{150}{4.25} - \frac{7.5}{4.25}x$

The solution is the points, with whole-number coordinates, above the line.

c) What does each intercept of the graph of the related function represent?

The l-intercept represents the number of lilies Thomas hopes to sell if he sells no roses. Round the intercept to the nearest whole number; Thomas hopes to sell more than 35 lilies if no rose plants are sold. The r-intercept represents the number of roses Thomas hopes to sell if he sells no lilies. Thomas hopes to sell more than 20 roses when no lilies are sold.

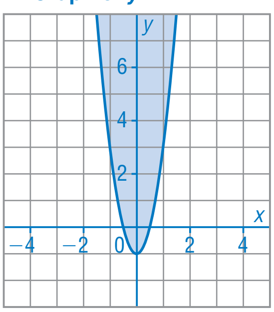
5.3

5. Graph each quadratic inequality. Write the coordinates of 3 points that satisfy the inequality.

a) $y \geq 4x^2 - 1$

The graph of the related quadratic function is congruent to $y = 4x^2$ and has vertex $(0, -1)$. The curve is solid and the region above is shaded. From the graph, 3 points that satisfy the inequality are: $(-1, 5)$, $(0, 1)$, $(1, 10)$

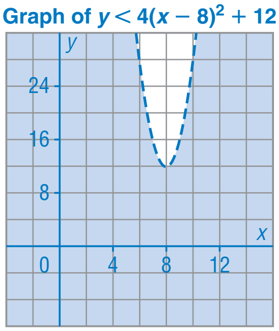
Graph of $y \geq 4x^2 - 1$



b) $y < 4(x - 8)^2 + 12$

The graph of the related quadratic function is congruent to $y = 4x^2$ and has vertex $(8, 12)$. The curve is broken and the region below is shaded. From the graph, 3 points that satisfy the inequality are: $(4, 8)$, $(6, 4)$, $(8, 2)$

Graph of $y < 4(x - 8)^2 + 12$

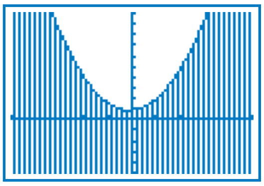


6. Use technology to graph each quadratic inequality. Sketch or print the graph.

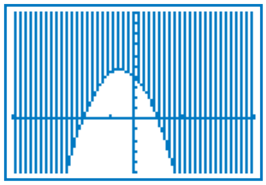
a) $y < 1.75x^2 + 0.4x + 1.8$ **b)** $y \geq -3.8x^2 - 4.2x + 8.1$

Graph the related quadratic functions.

The boundary is not part of the graph.



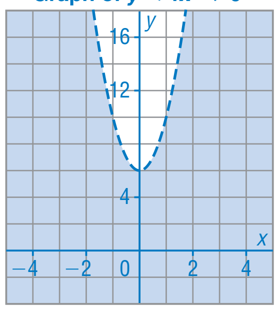
The boundary is part of the graph.



7. Two numbers are related in this way: three plus 2 times the square of one number is greater than one-half the other number. Graph an inequality that represents this relationship. Use the graph to identify three pairs of numbers that satisfy this relationship.

Let the numbers be x and y.
An inequality is: $3 + 2x^2 > 0.5y$, or $y < 4x^2 + 6$
Graph the related function. It is a parabola that is congruent to $y = 4x^2$ and has vertex (0, 6).
The curve is broken and the region below it is shaded.
From the graph, 3 points that satisfy the inequality are: (−1, 4), (0, 3), (1, 6)
So, 3 pairs of numbers are: −1 and 4, 0 and 3, 1 and 6

Graph of $y < 4x^2 + 6$



5.4

8. Use technology to graph each system of equations. Write the coordinates of the points of intersection to the nearest tenth.

a) $y = 2x - 6$
$y = x^2 + 3x - 10$

The coordinates of the points of intersection are approximately: (−2.6, −11.1) and (1.6, −2.9)

b) $y = -x^2 + 4$
$y = x + 5$

No solution

c) $y = -2x^2 + 1$
$y = 3x^2 + 2x - 1$

d) $y = 1.4(x - 2)^2 - 2.9$
$y = 0.2(x + 1)^2 + 3.4$

The coordinates of the points of intersection are approximately:

(−0.9, −0.5) and (0.5, 0.6) **(−0.1, 3.5) and (5.1, 11.0)**

9. From the roof of a building that is 50 m tall, a tennis ball is thrown downward with an initial speed of 10 m/s. The height of the tennis ball, h metres above the ground after t seconds, is given by the equation $h = -4.9t^2 - 10t + 50$. At the same time, a basketball is thrown upward from the ground with an initial speed of 15 m/s. The height of the basketball, h metres after t seconds, is given by the equation $h = -4.9t^2 + 15t$.

a) Determine the time at which the tennis ball and basketball reach the same height.

Solve this system of equations:
$h = -4.9t^2 - 10t + 50$ ①
$h = -4.9t^2 + 15t$ ②
Input each equation in a graphing calculator.
Use the intersect feature to display
$x = 2$ $y = 10.4$
The tennis ball and basketball reach the same height after 2 s.

b) What is this height?

The height is the value of h when $t = 2$.
From the calculator screen, $h = 10.4$
The balls are at the same height of 10.4 m.

5.5

10. a) Solve each system of equations. Give the solutions to the nearest tenth where necessary.

i) $y = 2x + 7$ ①
$y = (x - 2)^2 + 3$ ②

From equation ①, substitute $y = 2x + 7$ in equation ②.
$2x + 7 = (x - 2)^2 + 3$
$2x + 7 = x^2 - 4x + 4 + 3$
$x^2 - 6x = 0$
$x(x - 6) = 0$
So, $x = 0$ or $x = 6$
Substitute each value of x in equation ①.

When $x = 0$:	When $x = 6$:
$y = 2(0) + 7$	$y = 2(6) + 7$
$y = 7$	$y = 19$

The solutions are: (0, 7) and (6, 19)

ii) $y = 2x^2 + 11x + 12$ ①

$y = -(x + 3)^2$ ②

From equation ①, substitute $y = 2x^2 + 11x + 12$ in equation ②.

$2x^2 + 11x + 12 = -(x + 3)^2$

$2x^2 + 11x + 12 = -x^2 - 6x - 9$

$3x^2 + 17x + 21 = 0$

Use the quadratic formula.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Substitute: $a = 3$, $b = 17$, $c = 21$

$x = \frac{-17 \pm \sqrt{17^2 - 4(3)(21)}}{2(3)}$

$x = \frac{-17 \pm \sqrt{37}}{6}$

$x = -3.8471...$ or $x = -1.8195...$

Substitute each value of x in equation ②.

When $x = -3.8471...$

$y = -(-3.8471... + 3)^2$

$y = -0.7176...$

When $x = -1.8195...$

$y = -(-1.8195... + 3)^2$

$y = -1.3934...$

The solutions are approximately: $(-3.8, -0.7)$ and $(-1.8, -1.4)$

b) Choose one system from part a. Explain the meaning of the points of intersection of the graphs.

The coordinates of the points where the graphs intersect are the solution of the system. For the system in part a) i, the graphs of the equations intersect at the points with coordinates (0, 7) and (6, 19).

11. a) How many solutions are possible for a linear-quadratic system of equations?

A linear-quadratic system of equations has: no solution when the line does not intersect the parabola; 1 solution when the line is a tangent to the parabola; and 2 solutions when the line intersects the parabola in 2 points.

b) How many solutions are possible for a quadratic-quadratic system of equations?

Justify your answers.

A quadratic-quadratic system of equations has: no solution when the parabolas do not intersect; 1 solution when the parabolas are tangent to each other; 2 solutions when the parabolas intersect in 2 points; and infinite solutions when the parabolas coincide.

12. A stunt person jumped from a high tower and was in free fall before she opened her parachute.
Her height, h metres above the ground, t seconds after jumping is modelled by these two equations:
$h = -4.9t^2 + t + 350$, before the parachute was opened
$h = -4t + 141$, after the parachute was opened

a) How many seconds after the stunt person jumped did she open her parachute?

Solve this system:
$h = -4.9t^2 + t + 350$ ①
$h = -4t + 141$ ②
From equation ①, substitute $h = -4.9t^2 + t + 350$ in equation ②.
$-4.9t^2 + t + 350 = -4t + 141$
$4.9t^2 - 5t - 209 = 0$
Use the quadratic formula.

$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Substitute: $a = 4.9$, $b = -5$, $c = -209$

$t = \dfrac{5 \pm \sqrt{(-5)^2 - 4(4.9)(-209)}}{2(4.9)}$

$t = \dfrac{5 \pm \sqrt{4121.4}}{9.8}$

Ignore the negative root because time cannot be negative.
$t = 7.0610\ldots$
The stunt person opened her parachute after approximately 7.1 s.

b) How high was she when she opened her parachute?

Give the answers to the nearest tenth.

The height is the value of h when $t = 7.0610\ldots$
Substitute $t = 7.0610\ldots$ in equation ②.
$h = -4(7.0610\ldots) + 141$
$h = 112.7558\ldots$
The stunt person opened her parachute when she was approximately 112.8 m high.