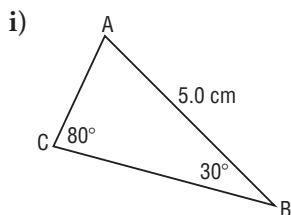


## Lesson 6.4 Exercises, pages 478–489

### A

3. a) For each triangle, write the Sine Law equation you would use to determine the length of AC.

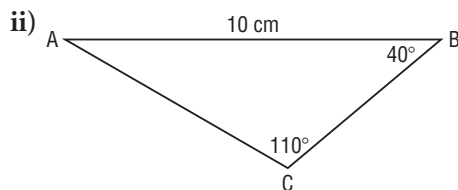


Use:  $\frac{b}{\sin B} = \frac{c}{\sin C}$

Substitute:

$\angle B = 30^\circ$ ,  $\angle C = 80^\circ$ ,  $c = 5$

$$\frac{b}{\sin 30^\circ} = \frac{5}{\sin 80^\circ}$$



Use:  $\frac{b}{\sin B} = \frac{c}{\sin C}$

Substitute:

$\angle B = 40^\circ$ ,  $\angle C = 110^\circ$ ,  $c = 10$

$$\frac{b}{\sin 40^\circ} = \frac{10}{\sin 110^\circ}$$

b) For each triangle in part a, determine the length of AC to the nearest tenth of a centimetre.

$$\text{i) } b = \frac{5 \sin 30^\circ}{\sin 80^\circ}$$

$$b = 2.5385 \dots$$

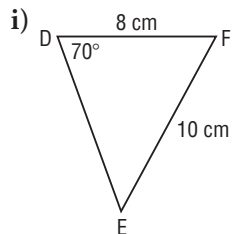
AC is approximately  
2.5 cm.

$$\text{ii) } b = \frac{10 \sin 40^\circ}{\sin 110^\circ}$$

$$b = 6.8404 \dots$$

AC is approximately  
6.8 cm.

4. a) For each triangle, write the Sine Law equation you would use to determine the measure of  $\angle E$ .

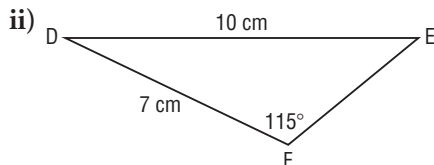


$$\text{Use: } \frac{\sin E}{e} = \frac{\sin D}{d}$$

Substitute:

$$\angle D = 70^\circ, e = 8, d = 10$$

$$\frac{\sin E}{8} = \frac{\sin 70^\circ}{10}$$



$$\text{Use: } \frac{\sin E}{e} = \frac{\sin F}{f}$$

Substitute:

$$\angle F = 115^\circ, e = 7, f = 10$$

$$\frac{\sin E}{7} = \frac{\sin 115^\circ}{10}$$

b) For each triangle in part a, determine the measure of  $\angle E$  to the nearest degree.

$$\text{i) } \sin E = \frac{8 \sin 70^\circ}{10}$$

Since  $\angle E$  is acute:

$$\angle E = \sin^{-1}\left(\frac{8 \sin 70^\circ}{10}\right)$$

$$\angle E = 48.7425 \dots^\circ$$

$$\angle E \doteq 49^\circ$$

$$\text{ii) } \sin E = \frac{7 \sin 115^\circ}{10}$$

Since  $\angle E$  is acute:

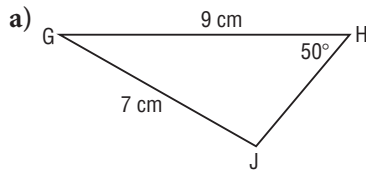
$$\angle E = \sin^{-1}\left(\frac{7 \sin 115^\circ}{10}\right)$$

$$\angle E = 39.3766 \dots^\circ$$

$$\angle E \doteq 39^\circ$$

**B**

5. For each triangle, determine the measure of  $\angle J$  to the nearest degree.



$$\text{Use: } \frac{\sin J}{j} = \frac{\sin H}{h}$$

Substitute:

$$\angle H = 50^\circ, j = 9, h = 7$$

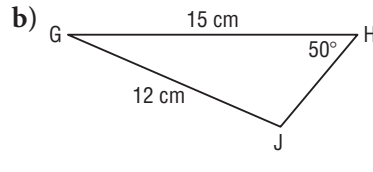
$$\frac{\sin J}{9} = \frac{\sin 50^\circ}{7}$$

$$\sin J = \frac{9 \sin 50^\circ}{7}$$

$$\sin^{-1}\left(\frac{9 \sin 50^\circ}{7}\right) \doteq 80^\circ$$

Since  $\angle J$  is obtuse:

$$\angle J \doteq 180^\circ - 80^\circ, \text{ or } 100^\circ$$



$$\text{Use: } \frac{\sin J}{j} = \frac{\sin H}{h}$$

Substitute:

$$\angle H = 50^\circ, j = 15, h = 12$$

$$\frac{\sin J}{15} = \frac{\sin 50^\circ}{12}$$

$$\sin J = \frac{15 \sin 50^\circ}{12}$$

$$\sin^{-1}\left(\frac{15 \sin 50^\circ}{12}\right) \doteq 73^\circ$$

Since  $\angle J$  is obtuse:

$$\angle J \doteq 180^\circ - 73^\circ, \text{ or } 107^\circ$$

6. Given the following information about each possible  $\triangle ABC$ , determine how many triangles can be constructed.

a)  $c = 10$  cm,  $a = 12$  cm,  $\angle A = 20^\circ$

The ratio of the side opposite the angle to the side adjacent to the angle is:  $\frac{a}{c} = \frac{12}{10}$ , which is 1.2

Since  $\frac{a}{c} > 1$ , only 1 triangle can be constructed

b)  $c = 18$  cm,  $a = 12$  cm,  $\angle A = 20^\circ$

The ratio of the side opposite the angle to the side adjacent to the angle is:  $\frac{a}{c} = \frac{12}{18}$ , which is  $0.\bar{6}$

$$\sin 20^\circ = 0.3420 \dots$$

Since  $\sin 20^\circ < 0.\bar{6} < 1$ , two triangles can be constructed

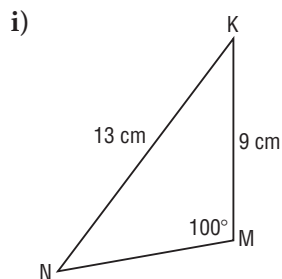
c)  $c = 18$  cm,  $a = 12$  cm,  $\angle A = 50^\circ$

The ratio of the side opposite the angle to the side adjacent to the angle is:  $\frac{a}{c} = \frac{12}{18}$ , which is  $0.\bar{6}$

$$\sin 50^\circ = 0.7660 \dots$$

Since  $\frac{a}{c} < \sin 50^\circ$ , no triangle can be constructed

7. a) For each triangle, determine the length of MN to the nearest tenth of a centimetre.



Determine  $\angle N$ .

Use:  $\frac{\sin N}{n} = \frac{\sin M}{m}$

Substitute:  $\angle M = 100^\circ$ ,  
 $n = 9$ ,  $m = 13$

$$\frac{\sin N}{9} = \frac{\sin 100^\circ}{13}$$

$$\sin N = \frac{9 \sin 100^\circ}{13}$$

Since  $\angle N$  is acute:

$$\angle N = \sin^{-1}\left(\frac{9 \sin 100^\circ}{13}\right)$$

$$\angle N = 42.9836 \dots^\circ$$

So,  $\angle K$

$$= 180^\circ - (100^\circ + 42.9836 \dots^\circ)$$

$$= 37.0163 \dots^\circ$$

Use:  $\frac{k}{\sin K} = \frac{m}{\sin M}$

Substitute:  $\angle K = 37.0163 \dots^\circ$

$$\angle M = 100^\circ, m = 13$$

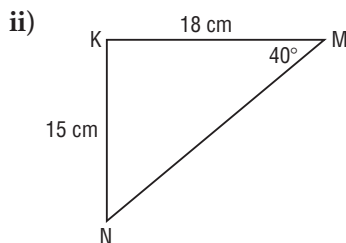
$$\frac{k}{\sin 37.0163 \dots^\circ} = \frac{13}{\sin 100^\circ}$$

$$k = \frac{13 \sin 37.0163 \dots^\circ}{\sin 100^\circ}$$

$$k = 7.9472 \dots$$

MN is approximately

7.9 cm.



Determine  $\angle N$ .

Use:  $\frac{\sin N}{n} = \frac{\sin M}{m}$

Substitute:  $\angle M = 40^\circ$ ,  
 $n = 18$ ,  $m = 15$

$$\frac{\sin N}{18} = \frac{\sin 40^\circ}{15}$$

$$\sin N = \frac{18 \sin 40^\circ}{15}$$

Since  $\angle N$  is acute:

$$\angle N = \sin^{-1}\left(\frac{18 \sin 40^\circ}{15}\right)$$

$$\angle N = 50.4748 \dots^\circ$$

So,  $\angle K$

$$= 180^\circ - (40^\circ + 50.4748 \dots^\circ)$$

$$= 89.5251 \dots^\circ$$

Use:  $\frac{k}{\sin K} = \frac{m}{\sin M}$

Substitute:  $\angle K = 89.5251 \dots^\circ$

$$\angle M = 40^\circ, m = 15$$

$$\frac{k}{\sin 89.5251 \dots^\circ} = \frac{15}{\sin 40^\circ}$$

$$k = \frac{15 \sin 89.5251 \dots^\circ}{\sin 40^\circ}$$

$$k = 23.3350 \dots$$

MN is approximately

23.3 cm.

b) Suppose you had been given the measures for the triangles in part a and not the diagrams. In which triangle would there have been an ambiguous case? Justify your choice.

i) Angle M is obtuse, so there is only 1 triangle

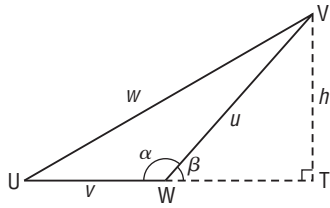
ii) The ratio of the side opposite the angle to the side adjacent to the angle is:

$$\frac{NK}{KM} = \frac{15}{18}, \text{ which is } 0.8\bar{3}$$

$$\sin 40^\circ = 0.6427 \dots$$

since  $\sin 40^\circ < 0.8\bar{3} < 1$ , then there is an ambiguous case

8. Here is a part of a proof of the Sine Law for  $\triangle UVW$  with obtuse  $\angle W$ . Explain each step.



In  $\triangle VUT$ ,

$$\sin U = \frac{h}{w} \quad \text{Using the sine ratio in a right triangle}$$

$$h = w \sin U \quad \text{Solving for } h$$

In  $\triangle VWT$ ,

$$\sin \beta = \frac{h}{u} \quad \text{Using the sine ratio in a right triangle}$$

$$\beta = 180^\circ - \alpha$$

$$\sin \beta = \sin(180^\circ - \alpha) \quad \text{Supplementary angles on a line}$$

$$= \sin \alpha, \text{ or } \sin W \quad \text{The sine of an angle is equal to the sine of its supplement.}$$

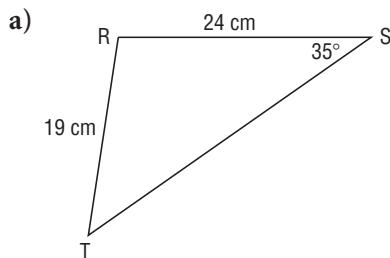
$$\text{So, } \sin W = \frac{h}{u} \quad \sin UWV = \sin VWT$$

$$h = u \sin W \quad \text{Solving for } h$$

$$u \sin W = w \sin U \quad \text{Equating expressions for } h$$

$$\frac{u}{\sin U} = \frac{w}{\sin W} \quad \text{Dividing each side by } \sin W \sin U$$

9. Solve each triangle. Give the angle measures to the nearest degree and side lengths to the nearest tenth of a centimetre.



Determine  $\angle T$ .

$$\text{Use: } \frac{\sin T}{t} = \frac{\sin S}{s}$$

$$\text{Substitute: } \angle S = 35^\circ,$$

$$t = 24, s = 19$$

$$\frac{\sin T}{24} = \frac{\sin 35^\circ}{19}$$

$$\sin T = \frac{24 \sin 35^\circ}{19}$$

Since  $\angle T$  is acute:

$$\angle T = \sin^{-1}\left(\frac{24 \sin 35^\circ}{19}\right)$$

$$\angle T \doteq 46.4287 \dots^\circ$$

$$\text{So, } \angle R = 180^\circ - (35^\circ + 46.4287 \dots^\circ)$$

$$= 98.5712 \dots^\circ$$

$$\text{Use: } \frac{r}{\sin R} = \frac{s}{\sin S}$$

$$\text{Substitute: } \angle R = 98.5712 \dots^\circ$$

$$\angle S = 35^\circ, s = 19$$

$$\frac{r}{\sin 98.5712 \dots^\circ} = \frac{19}{\sin 35^\circ}$$

$$r = \frac{19 \sin 98.5712 \dots^\circ}{\sin 35^\circ}$$

$$r = 32.7555 \dots$$

$$\angle T \doteq 46^\circ, \angle R \doteq 99^\circ,$$

$$ST \doteq 32.8 \text{ cm}$$

b) In  $\triangle KMN$ ,  $\angle M = 70^\circ$ ,  $KN = 14.1$  cm, and  $MK = 14.5$  cm

Check how many triangles can be drawn.

The ratio of the side opposite  $\angle M$  to the side adjacent to  $\angle M$  is:

$$\frac{KN}{KM} = \frac{14.1}{14.5}, \text{ which is } 0.9724 \dots$$

$$\sin 70^\circ = 0.9396 \dots$$

Since  $\sin 70^\circ < 0.9724 \dots < 1$ , two triangles can be constructed:

$\triangle MKN_1$  is acute;  $\triangle MKN_2$  is obtuse.

In  $\triangle MKN_1$

Determine  $\angle N_1$ .

$$\text{Use: } \frac{\sin N_1}{n} = \frac{\sin M}{m}$$

Substitute:  $\angle M = 70^\circ$ ,

$$n = 14.5, m = 14.1$$

$$\frac{\sin N_1}{14.5} = \frac{\sin 70^\circ}{14.1}$$

$$\sin N_1 = \frac{14.5 \sin 70^\circ}{14.1}$$

$$\angle N_1 = \sin^{-1}\left(\frac{14.5 \sin 70^\circ}{14.1}\right)$$

$$\angle N_1 = 75.0943 \dots^\circ$$

So,  $\angle K$

$$= 180^\circ - (70^\circ + 75.0943 \dots^\circ)$$

$$= 34.9056 \dots^\circ$$

$$\text{Use: } \frac{k}{\sin K} = \frac{m}{\sin M}$$

Substitute:  $\angle K = 34.9056 \dots^\circ$

$$\angle M = 70^\circ, m = 14.1$$

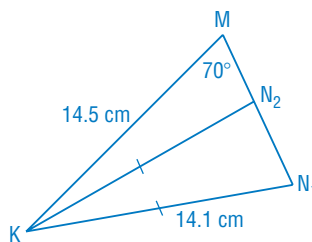
$$\frac{k}{\sin 34.9056 \dots^\circ} = \frac{14.1}{\sin 70^\circ}$$

$$k = \frac{14.1 \sin 34.9056 \dots^\circ}{\sin 70^\circ}$$

$$k = 8.5862 \dots$$

$$\angle N_1 \doteq 75^\circ, \angle K \doteq 35^\circ,$$

$$MN_1 \doteq 8.6 \text{ cm}$$



This diagram is not drawn to scale.

In  $\triangle MKN_2$

$$\angle N_2 = 180^\circ - 75.0943 \dots^\circ$$

$$= 104.9056 \dots^\circ$$

So,  $\angle K$

$$= 180^\circ - (70^\circ + 104.9056 \dots^\circ)$$

$$= 5.0943 \dots^\circ$$

$$\text{Use: } \frac{k}{\sin K} = \frac{m}{\sin M}$$

Substitute:  $\angle K = 5.0943 \dots^\circ$

$$\angle M = 70^\circ, m = 14.1$$

$$\frac{k}{\sin 5.0943 \dots^\circ} = \frac{14.1}{\sin 70^\circ}$$

$$k = \frac{14.1 \sin 5.0943 \dots^\circ}{\sin 70^\circ}$$

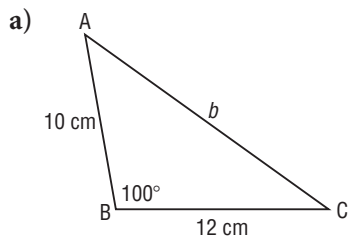
$$k = 1.3323 \dots$$

$$\angle N_2 \doteq 105^\circ, \angle K \doteq 5^\circ,$$

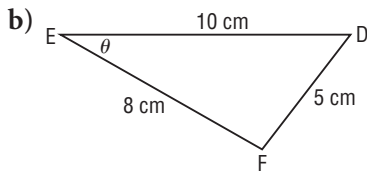
$$MN_2 \doteq 1.3 \text{ cm}$$

10. For each triangle below, can you use the Sine Law to determine the indicated measure?

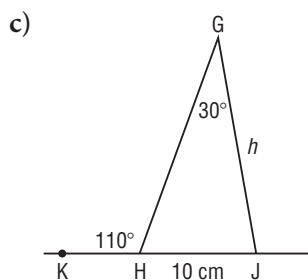
If your answer is yes, determine the measure to the nearest tenth of a unit. If your answer is no, explain why.



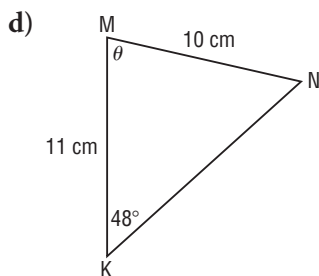
No, because only one angle is given and it is a contained angle.



No, because no angles are given.

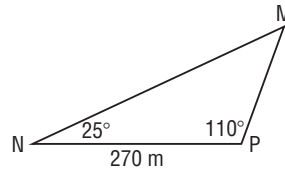


Yes; in  $\triangle GHJ$ ,  
 $\angle H = 180^\circ - 110^\circ$   
 $= 70^\circ$   
 Use:  $\frac{h}{\sin H} = \frac{g}{\sin G}$   
 Substitute:  $\angle H = 70^\circ$ ,  
 $\angle G = 30^\circ$ ,  $g = 10$   
 $\frac{h}{\sin 70^\circ} = \frac{10}{\sin 30^\circ}$   
 $h = \frac{10 \sin 70^\circ}{\sin 30^\circ}$   
 $h = 18.7938\dots$   
 $h \doteq 18.8 \text{ cm}$



Yes  
 Use:  $\frac{\sin N}{n} = \frac{\sin K}{k}$   
 Substitute:  $\angle K = 48^\circ$ ,  
 $n = 11$ ,  $k = 10$   
 $\frac{\sin N}{11} = \frac{\sin 48^\circ}{10}$   
 $\sin N = \frac{11 \sin 48^\circ}{10}$   
 Since  $\angle N$  is acute:  
 $\angle N = \sin^{-1}\left(\frac{11 \sin 48^\circ}{10}\right)$   
 $\angle N = 54.8312\dots^\circ$   
 $\theta = 180^\circ - (48^\circ + 54.8312\dots^\circ)$   
 $= 77.1687\dots^\circ$   
 $\theta \doteq 77.2^\circ$

11. A surveyor constructed this drawing of a triangular lot.



- a) Determine the unknown side lengths.

$$\begin{aligned}\angle M &= 180^\circ - (25^\circ + 110^\circ) \\ &= 45^\circ\end{aligned}$$

$$\text{Use: } \frac{p}{\sin P} = \frac{m}{\sin M}$$

$$\begin{aligned}\text{Substitute: } \angle P &= 110^\circ, \\ \angle M &= 45^\circ, m = 270\end{aligned}$$

$$\frac{p}{\sin 110^\circ} = \frac{270}{\sin 45^\circ}$$

$$p = \frac{270 \sin 110^\circ}{\sin 45^\circ}$$

$$p = 358.8100 \dots$$

MN is approximately  
359 m.

$$\text{Use: } \frac{n}{\sin N} = \frac{m}{\sin M}$$

$$\begin{aligned}\text{Substitute: } \angle N &= 25^\circ, \\ \angle M &= 45^\circ, m = 270\end{aligned}$$

$$\frac{n}{\sin 25^\circ} = \frac{270}{\sin 45^\circ}$$

$$n = \frac{270 \sin 25^\circ}{\sin 45^\circ}$$

$$n = 161.3715 \dots$$

PM is approximately  
161 m.

- b) Determine the total length of fencing needed to enclose the lot.

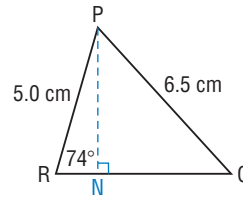
Give the answers to the nearest metre.

The total length of fencing is:

$$359 \text{ m} + 161 \text{ m} + 270 \text{ m} = 790 \text{ m}$$

12. a) Solve  $\triangle PQR$  by drawing a perpendicular from P to QR, then use primary trigonometric ratios in each right triangle formed.

Give the angle measures to the nearest degree and the side lengths to the nearest tenth of a centimetre.



In  $\triangle PRN$ ,

$$\cos 74^\circ = \frac{RN}{5}$$

$$RN = 5 \cos 74^\circ$$

$$RN = 1.3781 \dots$$

$$\sin 74^\circ = \frac{PN}{5}$$

$$PN = 5 \sin 74^\circ$$

$$PN = 4.8063 \dots$$

In  $\triangle PQN$ ,

$$\sin Q = \frac{4.8063 \dots}{6.5}$$

$$\angle Q = 47.6830 \dots^\circ$$

$$\angle Q \doteq 48^\circ$$

$$\cos Q = \frac{QN}{6.5}$$

$$QN = 6.5 \cos 47.6830 \dots^\circ$$

$$QN = 4.3760 \dots$$

In  $\triangle PQR$ ,

$$\angle P \doteq 180^\circ - (48^\circ + 74^\circ)$$

$$\doteq 58^\circ$$

$$RQ = (1.3781 \dots + 4.3760 \dots) \text{ cm}$$

$$RQ \doteq 5.8 \text{ cm}$$



b) Use the Sine Law to solve  $\triangle PQR$ .

$$\text{In } \triangle PQR, \text{ use: } \frac{\sin Q}{q} = \frac{\sin R}{r}$$

$$\frac{\sin Q}{5} = \frac{\sin 74^\circ}{6.5}$$

$$\sin Q = \frac{5 \sin 74^\circ}{6.5}$$

Since  $\angle Q$  is acute:

$$\angle Q = \sin^{-1}\left(\frac{5 \sin 74^\circ}{6.5}\right)$$

$$\angle Q = 47.6830\dots^\circ$$

$$\angle P = 180^\circ - (47.6830\dots^\circ + 74^\circ)$$

$$\angle P = 58.3169\dots^\circ$$

$$\text{Use: } \frac{p}{\sin P} = \frac{r}{\sin R}$$

$$\frac{p}{\sin 58.3169\dots^\circ} = \frac{6.5}{\sin 74^\circ}$$

$$p = \frac{6.5 \sin 58.3169\dots^\circ}{\sin 74^\circ}$$

$$p = 5.7541\dots$$

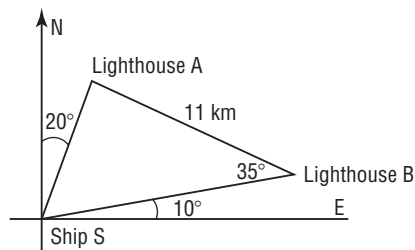
$$\angle Q \doteq 48^\circ, \angle P \doteq 58^\circ, RQ \doteq 5.8 \text{ cm}$$

c) Which strategy for solving  $\triangle PQR$  was more efficient?

Justify your answer.

The Sine Law needed 3 calculations, while the primary trigonometric ratios needed 6 calculations; so the Sine Law is more efficient.

13. A sailor made this sketch on her navigation chart. How much closer is the ship at S to lighthouse A than to lighthouse B?



In  $\triangle SAB$ ,

$$\angle ASB = 90^\circ - (10^\circ + 20^\circ)$$

$$= 60^\circ$$

$$\text{So, } \angle SAB = 180^\circ - (60^\circ + 35^\circ)$$

$$= 85^\circ$$

$$\text{Use: } \frac{b}{\sin B} = \frac{s}{\sin S}$$

$$\text{Substitute: } \angle B = 35^\circ,$$

$$\angle S = 60^\circ, s = 11$$

$$\frac{b}{\sin 35^\circ} = \frac{11}{\sin 60^\circ}$$

$$b = \frac{11 \sin 35^\circ}{\sin 60^\circ}$$

$$b = 7.2853\dots$$

$$\text{Use: } \frac{a}{\sin A} = \frac{11}{\sin 60^\circ}$$

$$\text{Substitute: } \angle A = 85^\circ$$

$$\frac{a}{\sin 85^\circ} = \frac{11}{\sin 60^\circ}$$

$$a = \frac{11 \sin 85^\circ}{\sin 60^\circ}$$

$$a = 12.6533\dots$$

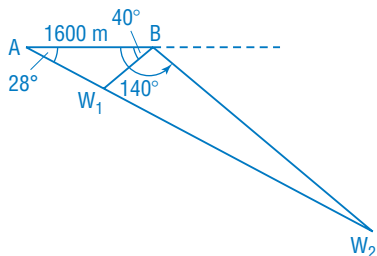
The required distance is:  $12.6533\dots - 7.2853\dots = 5.3679\dots$

The ship is approximately 5 km closer to lighthouse A.

14. Two ships are 1600 m apart. Each ship detects a wreck on the ocean floor. The wreck is vertically below the line through the ships. From the ships, the angles of depression to the wreck are  $40^\circ$  and  $28^\circ$ .

a) To the nearest metre, how far is the wreck from each ship?

The wreck could be between the ships or on one side of both ships. Sketch a diagram to show both positions.



Case 1 –  $\triangle ABW_1$   
 $\angle W_1 = 180^\circ - (28^\circ + 40^\circ)$   
 $= 112^\circ$

Use:  $\frac{a}{\sin A} = \frac{w}{\sin W_1}$

Substitute:  $\angle A = 28^\circ$ ,  
 $\angle W_1 = 112^\circ$ ,  $w = 1600$

$$\frac{a}{\sin 28^\circ} = \frac{1600}{\sin 112^\circ}$$

$$a = \frac{1600 \sin 28^\circ}{\sin 112^\circ}$$

$$a = 810.1462 \dots$$

Use:  $\frac{b}{\sin B} = \frac{1600}{\sin 112^\circ}$

Substitute:  $\angle B = 40^\circ$

$$b = \frac{1600 \sin 40^\circ}{\sin 112^\circ}$$

$$b = 1109.2300 \dots$$

The wreck is approximately 810 m and 1109 m from the ships.

Case 2 –  $\triangle ABW_2$   
 $\angle W_2 = 180^\circ - (28^\circ + 140^\circ)$   
 $= 12^\circ$

Use:  $\frac{a}{\sin A} = \frac{w}{\sin W_2}$

Substitute:  $\angle A = 28^\circ$ ,  
 $\angle W_2 = 12^\circ$ ,  $w = 1600$

$$\frac{a}{\sin 28^\circ} = \frac{1600}{\sin 12^\circ}$$

$$a = \frac{1600 \sin 28^\circ}{\sin 12^\circ}$$

$$a = 3612.8535 \dots$$

Use:  $\frac{b}{\sin B} = \frac{1600}{\sin 12^\circ}$

Substitute:  $\angle B = 140^\circ$

$$b = \frac{1600 \sin 140^\circ}{\sin 12^\circ}$$

$$b = 4946.6202 \dots$$

The wreck is approximately 3613 m and 4947 m from the ships.

b) To the nearest metre, what is the depth of the wreck?

In  $\triangle ABW_1$ , draw the perpendicular from  $W_1$  to  $AB$  at  $N$ . Then,  $W_1N$  is the depth of the wreck.

In right  $\triangle ANW_1$ :

$$\sin 28^\circ = \frac{W_1N}{AW_1}$$

$$W_1N = 1109.2300 \dots (\sin 28^\circ)$$

$$W_1N = 520.7519 \dots$$

The wreck is approximately 521 m deep or 2322 m deep.

In  $\triangle ABW_2$ , draw the perpendicular from  $W_2$  to  $AB$  extended to  $M$ . Then,  $W_2M$  is the depth of the wreck.

In right  $\triangle AMW_2$ :

$$\sin 28^\circ = \frac{W_2M}{AW_2}$$

$$W_2M = 4946.6202 \dots (\sin 28^\circ)$$

$$W_2M = 2322.2975 \dots$$

15. Two people use transits to sight the top of a pole that is on the line through the bases of the transits. The distances to the top of the pole are: 19.8 m at an angle of elevation of  $38.0^\circ$ ; and 14.4 m at an angle of elevation of  $57.8^\circ$ . To the nearest tenth of a metre, determine the distance between the people.

Sketch a diagram. There are 2 solutions.

In  $\triangle ABC_1$ ,

$$\begin{aligned}\angle A &= 180^\circ - (38.0^\circ + 57.8^\circ) \\ &= 84.2^\circ\end{aligned}$$

$$\text{Use: } \frac{a}{\sin A} = \frac{c}{\sin C_1}$$

Substitute:  $\angle A = 84.2^\circ$ ,

$$\angle C_1 = 57.8^\circ, c = 19.8$$

$$\frac{a}{\sin 84.2^\circ} = \frac{19.8}{\sin 57.8^\circ}$$

$$a = \frac{19.8 \sin 84.2^\circ}{\sin 57.8^\circ}$$

$$a = 23.2791\dots$$

In  $\triangle ABC_2$ ,

$$\begin{aligned}\angle C_2 &= 180^\circ - 57.8^\circ \\ &= 122.2^\circ\end{aligned}$$

$$\text{So, } \angle A = 180^\circ - (38.0^\circ + 122.2^\circ) = 19.8^\circ$$

$$\text{Use: } \frac{a}{\sin A} = \frac{c}{\sin C_2}$$

Substitute:  $\angle A = 19.8^\circ$ ,

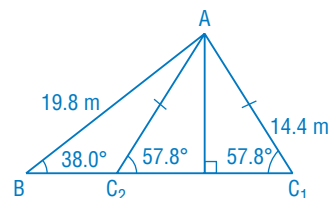
$$\angle C_2 = 122.2^\circ, c = 19.8$$

$$\frac{a}{\sin 19.8^\circ} = \frac{19.8}{\sin 122.2^\circ}$$

$$a = \frac{19.8 \sin 19.8^\circ}{\sin 122.2^\circ}$$

$$a = 7.9260\dots$$

The people are approximately 23.3 m or 7.9 m apart.



### C

16. Two angles in a triangle measure  $60^\circ$  and  $45^\circ$ . The longest side is 10 cm longer than the shortest side. Determine the perimeter of the triangle to the nearest tenth of a centimetre.

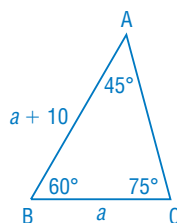
Sketch a diagram.

The third angle in the triangle is:

$$180^\circ - (45^\circ + 60^\circ) = 75^\circ$$

The shortest side is opposite the least angle, so let  $BC = a$ .

The longest side is opposite the greatest angle, so  $AB = a + 10$ .



$$\text{Use: } \frac{a}{\sin A} = \frac{c}{\sin C}$$

Substitute:  $\angle A = 45^\circ$ ,  $\angle C = 75^\circ$ ,  $c = a + 10$

$$\frac{a}{\sin 45^\circ} = \frac{a + 10}{\sin 75^\circ}$$

$$a \sin 75^\circ = a \sin 45^\circ + 10 \sin 45^\circ$$

$$a(\sin 75^\circ - \sin 45^\circ) = 10 \sin 45^\circ$$

$$a = \frac{10 \sin 45^\circ}{\sin 75^\circ - \sin 45^\circ}$$

$$a = 27.3205\dots$$

$$\text{Use: } \frac{b}{\sin B} = \frac{a}{\sin A}$$

Substitute:  $\angle B = 60^\circ$ ,  $\angle A = 45^\circ$ ,  $a = 27.3205\dots$

$$\frac{b}{\sin 60^\circ} = \frac{27.3205\dots}{\sin 45^\circ}$$

$$b = \frac{27.3205\dots \sin 60^\circ}{\sin 45^\circ}$$

$$b = 33.4606\dots$$

The perimeter is:

$$(27.3205\dots + 27.3205\dots + 10 + 33.4606\dots) \text{ cm} \doteq 98.1 \text{ cm}$$

17. A hiker plans a trip in two sections. Her destination is 15 km away on a bearing of N70°E from her starting position. The first leg of the trip is on a bearing of N10°E. The second leg of the trip is 14 km. How long is the first leg? Give the answer to the nearest tenth of a kilometre.

Sketch a diagram.

$$\begin{aligned}\angle DCB &= 70^\circ - 10^\circ \\ &= 60^\circ\end{aligned}$$

Check how many triangles can be drawn.

The ratio of the side opposite  $\angle C$  to the side adjacent to  $\angle C$  is:

$$\frac{BD}{BC} = \frac{14}{15}, \text{ which is } 0.9\bar{3}$$

$$\sin 60^\circ = 0.8660\dots$$

Since  $\sin 60^\circ < 0.9\bar{3} < 1$ , two triangles can be constructed:

$\Delta D_1CB$  is acute;  $\Delta D_2CB$  is obtuse

In  $\Delta D_1CB$

Determine  $\angle D_1$ .

$$\text{Use: } \frac{\sin D_1}{d} = \frac{\sin C}{c}$$

$$\text{Substitute: } \angle C = 60^\circ, d = 15, c = 14$$

$$\frac{\sin D_1}{15} = \frac{\sin 60^\circ}{14}$$

$$\sin D_1 = \frac{15 \sin 60^\circ}{14}$$

$$\angle D_1 = 68.1073\dots^\circ$$

$$\begin{aligned}\angle B &= 180^\circ - (60^\circ + 68.1073\dots^\circ) \\ &= 51.8926\dots^\circ\end{aligned}$$

To determine  $CD_1$ ,

$$\text{use: } \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{Substitute: } \angle B = 51.8926\dots^\circ,$$

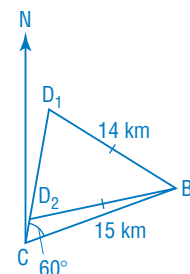
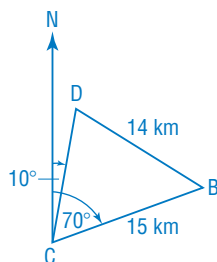
$$\angle C = 60^\circ, c = 14$$

$$\frac{b}{\sin 51.8926\dots^\circ} = \frac{14}{\sin 60^\circ}$$

$$b = \frac{14 \sin 51.8926\dots^\circ}{\sin 60^\circ}$$

$$b = 12.7201\dots$$

The first leg is approximately 12.7 km or 2.3 km.



In  $\Delta D_2CB$

$$\begin{aligned}\angle D_2 &= 180^\circ - 68.1073\dots^\circ \\ &= 111.8926\dots^\circ\end{aligned}$$

$$\begin{aligned}\text{So, } \angle B &= 180^\circ - (60^\circ + 111.8926\dots^\circ) \\ &= 8.1073\dots^\circ\end{aligned}$$

To determine  $CD_2$ ,

$$\text{use: } \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{Substitute: } \angle B = 8.1073\dots^\circ,$$

$$\angle C = 60^\circ, c = 14$$

$$\frac{b}{\sin 8.1073\dots^\circ} = \frac{14}{\sin 60^\circ}$$

$$b = \frac{14 \sin 8.1073\dots^\circ}{\sin 60^\circ}$$

$$b = 2.2798\dots$$