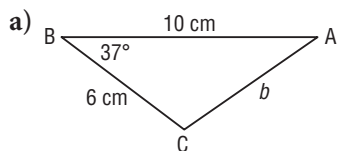


Lesson 6.5 Exercises, pages 498–506

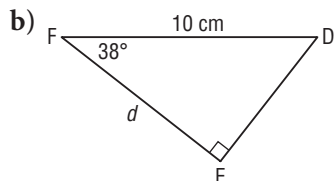
A

3. Which strategy would you use to determine the indicated measure in each triangle?

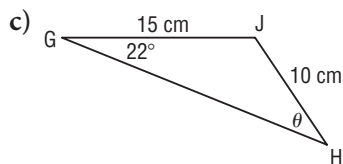
- a primary trigonometric ratio
- the Cosine Law
- the Sine Law



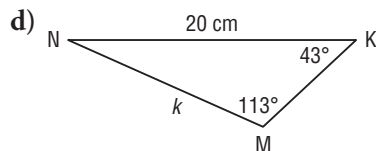
Since 2 sides and the contained angle are given, use the Cosine Law.



Since it is a right triangle, use a primary trigonometric ratio.



Since 2 sides and a non-contained angle are given, use the Sine Law.



Since 2 angles and a side are given, use the Sine Law.

B

4. Determine each measure in question 3. Give the angles to the nearest degree and the side lengths to the nearest tenth of a unit.

a) Use: $b^2 = a^2 + c^2 - 2ac \cos B$

Substitute: $a = 6, c = 10, \angle B = 37^\circ$

$$b^2 = 6^2 + 10^2 - 2(6)(10) \cos 37^\circ$$

$$b = \sqrt{6^2 + 10^2 - 2(6)(10) \cos 37^\circ}$$

$$b = 6.3374 \dots$$

$$b \doteq 6.3 \text{ cm}$$

c) Use: $\frac{\sin H}{h} = \frac{\sin G}{g}$

Substitute: $\angle G = 22^\circ,$

$$h = 15, g = 10$$

$$\frac{\sin H}{15} = \frac{\sin 22^\circ}{10}$$

$$\sin H = \frac{15 \sin 22^\circ}{10}$$

$$\angle H = \sin^{-1}\left(\frac{15 \sin 22^\circ}{10}\right)$$

$$\angle H = 34.1879 \dots^\circ$$

$$\theta \doteq 34^\circ$$

b) Use: $\cos 38^\circ = \frac{d}{10}$

$$d = 10 \cos 38^\circ$$

$$d = 7.8801 \dots$$

$$d \doteq 7.9 \text{ cm}$$

d) Use: $\frac{k}{\sin K} = \frac{m}{\sin M}$

Substitute: $\angle K = 43^\circ,$

$$\angle M = 113^\circ, m = 20$$

$$\frac{k}{\sin 43^\circ} = \frac{20}{\sin 113^\circ}$$

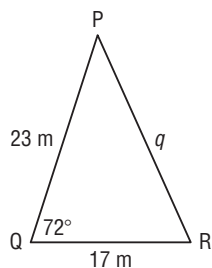
$$k = \frac{20 \sin 43^\circ}{\sin 113^\circ}$$

$$k = 14.8179 \dots$$

$$k \doteq 14.8 \text{ cm}$$

5. Determine the indicated measure in each triangle. Give the angles to the nearest degree and the side lengths to the nearest tenth of a unit.

a)



Use: $q^2 = p^2 + r^2 - 2pr \cos Q$

Substitute: $p = 17, r = 23, \angle Q = 72^\circ$

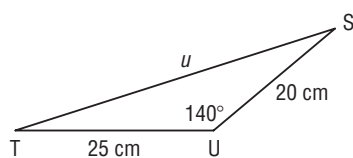
$$q^2 = 17^2 + 23^2 - 2(17)(23) \cos 72^\circ$$

$$q = \sqrt{17^2 + 23^2 - 2(17)(23) \cos 72^\circ}$$

$$q = 24.0072 \dots$$

$$q \doteq 24.0 \text{ m}$$

b)



Use: $u^2 = s^2 + t^2 - 2st \cos U$

Substitute: $s = 25, t = 20, \angle U = 140^\circ$

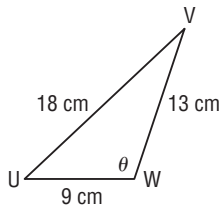
$$u^2 = 25^2 + 20^2 - 2(25)(20) \cos 140^\circ$$

$$u = \sqrt{25^2 + 20^2 - 2(25)(20) \cos 140^\circ}$$

$$u = 42.3207 \dots$$

$$u \doteq 42.3 \text{ cm}$$

c)



$$\text{Use: } w^2 = u^2 + v^2 - 2uv \cos W$$

$$\text{Substitute: } w = 18, u = 13, v = 9$$

$$18^2 = 13^2 + 9^2 - 2(13)(9) \cos W$$

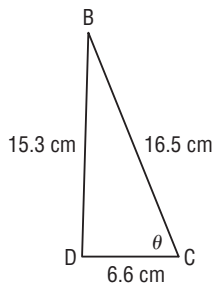
$$\cos W = \frac{13^2 + 9^2 - 18^2}{2(13)(9)}$$

$$\angle W = \cos^{-1}\left(\frac{13^2 + 9^2 - 18^2}{2(13)(9)}\right)$$

$$\angle W = 108.4356 \dots^\circ$$

$$\theta \doteq 108^\circ$$

d)



$$\text{Use: } c^2 = b^2 + d^2 - 2bd \cos C$$

$$\text{Substitute: } c = 15.3, b = 6.6, d = 16.5$$

$$15.3^2 = 6.6^2 + 16.5^2 - 2(6.6)(16.5) \cos C$$

$$\cos C = \frac{6.6^2 + 16.5^2 - 15.3^2}{2(6.6)(16.5)}$$

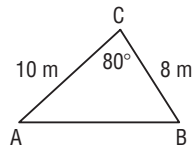
$$\angle C = \cos^{-1}\left(\frac{6.6^2 + 16.5^2 - 15.3^2}{2(6.6)(16.5)}\right)$$

$$\angle C = 67.9629 \dots^\circ$$

$$\theta \doteq 68^\circ$$

6. A security camera, C, rotates through 80° .

The camera is 10 m and 8 m from two doors, A and B, on the same wall of a building. To the nearest metre, how far apart are the doors?



$$\text{Use: } c^2 = a^2 + b^2 - 2ab \cos C$$

$$\text{Substitute: } a = 8, b = 10, \angle C = 80^\circ$$

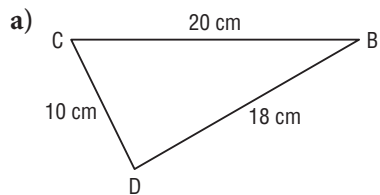
$$c^2 = 8^2 + 10^2 - 2(8)(10) \cos 80^\circ$$

$$c = \sqrt{8^2 + 10^2 - 2(8)(10) \cos 80^\circ}$$

$$c = 11.6711 \dots$$

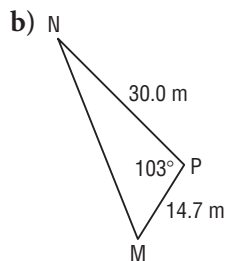
The doors are about 12 m apart.

7. Solve each triangle. Give the side lengths to the nearest tenth of a unit and the angle measures to the nearest degree.



Use: $b^2 = c^2 + d^2 - 2cd \cos B$
 Substitute: $b = 10, c = 18, d = 20$
 $10^2 = 18^2 + 20^2 - 2(18)(20) \cos B$
 $\cos B = \frac{18^2 + 20^2 - 10^2}{2(18)(20)}$
 $\angle B = \cos^{-1}\left(\frac{18^2 + 20^2 - 10^2}{2(18)(20)}\right)$
 $\angle B = 29.9264 \dots^\circ$
 $\angle B \doteq 30^\circ$

Use: $c^2 = b^2 + d^2 - 2bd \cos C$
 Substitute: $c = 18, b = 10, d = 20$
 $18^2 = 10^2 + 20^2 - 2(10)(20) \cos C$
 $\cos C = \frac{10^2 + 20^2 - 18^2}{2(10)(20)}$
 $\angle C = \cos^{-1}\left(\frac{10^2 + 20^2 - 18^2}{2(10)(20)}\right)$
 $\angle C = 63.8961 \dots^\circ$
 $\angle C \doteq 64^\circ$
 $\angle D \doteq 180^\circ - (64^\circ + 30^\circ)$
 $\angle D \doteq 86^\circ$



Use: $p^2 = m^2 + n^2 - 2mn \cos P$
 Substitute: $m = 30, n = 14.7, \angle P = 103^\circ$
 $p^2 = 30^2 + 14.7^2 - 2(30)(14.7) \cos 103^\circ$
 $p = \sqrt{30^2 + 14.7^2 - 2(30)(14.7) \cos 103^\circ}$
 $p = 36.2559 \dots$

Use: $\frac{\sin M}{m} = \frac{\sin P}{p}$

Substitute: $\angle P = 103^\circ, m = 30, p = 36.2559 \dots$
 $\frac{\sin M}{30} = \frac{\sin 103^\circ}{36.2559 \dots}$
 $\sin M = \frac{30 \sin 103^\circ}{36.2559 \dots}$
 Since $\angle M$ is acute:
 $\angle M = \sin^{-1}\left(\frac{30 \sin 103^\circ}{36.2559 \dots}\right)$
 $\angle M = 53.7303 \dots^\circ$
 $\angle N = 180^\circ - (103^\circ + 53.7303 \dots^\circ)$
 $\angle N = 23.2696 \dots^\circ$
 So, $MN \doteq 36.3 \text{ m}, \angle M \doteq 54^\circ, \angle N \doteq 23^\circ$

8. a) Use the Cosine Law to determine the length of DF to the nearest tenth of a centimetre.

Use: $e^2 = d^2 + f^2 - 2df \cos E$

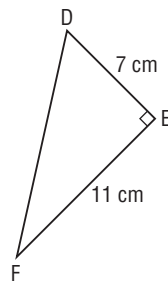
Substitute: $d = 11, f = 7, \angle E = 90^\circ$

$e^2 = 11^2 + 7^2 - 2(11)(7) \cos 90^\circ$

$e = \sqrt{11^2 + 7^2}$

$e = 13.0384 \dots$

$DF \approx 13.0 \text{ cm}$



- b) How is the Pythagorean Theorem a special case of the Cosine Law?

When the angle in the statement of the Cosine Law is 90° , since $\cos 90^\circ = 0$, the Cosine Law reduces to the Pythagorean Theorem.

9. A fire spotter sees smoke on a bearing of 060° . At a point 20 km due east of the fire spotter, a ranger sees the same smoke on a bearing of 320° .

- a) How far is the smoke from each location?

Sketch a diagram.

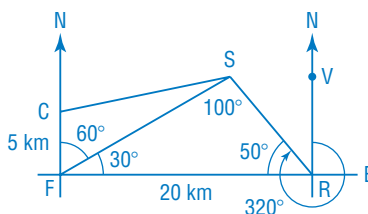
$\angle SRV$ is: $360^\circ - 320^\circ = 40^\circ$

In $\triangle SFR$,

$\angle F$ is: $90^\circ - 60^\circ = 30^\circ$

$\angle R$ is: $90^\circ - 40^\circ = 50^\circ$

$\angle S$ is: $180^\circ - (30^\circ + 50^\circ) = 100^\circ$



Use: $\frac{f}{\sin F} = \frac{s}{\sin S}$

Substitute: $\angle F = 30^\circ$,

$\angle S = 100^\circ, s = 20$

$\frac{f}{\sin 30^\circ} = \frac{20}{\sin 100^\circ}$

$f = \frac{20 \sin 30^\circ}{\sin 100^\circ}$

$f = 10.1542 \dots$

Use: $\frac{r}{\sin R} = \frac{20}{\sin 100^\circ}$

Substitute: $\angle R = 50^\circ$

$\frac{r}{\sin 50^\circ} = \frac{20}{\sin 100^\circ}$

$r = \frac{20 \sin 50^\circ}{\sin 100^\circ}$

$r = 15.5572 \dots$

The smoke is approximately 10 km and 16 km from each location.

- b) A fire crew is 5 km due north of the fire spotter. How far is the crew from the smoke?

Give the answers to the nearest kilometre.

In $\triangle SFC$,

Use: $f^2 = c^2 + s^2 - 2cs \cos F$

Substitute: $c = 15.5572 \dots, s = 5, \angle F = 60^\circ$

$f^2 = 15.5572 \dots^2 + 5^2 - 2(15.5572 \dots)(5) \cos 60^\circ$

$f = \sqrt{15.5572 \dots^2 + 5^2 - 2(15.5572 \dots)(5) \cos 60^\circ}$

$f = 13.7565 \dots$

The crew is approximately 14 km from the fire.

10. In $\triangle ABC$, $BC = 2AB$, $\angle B = 120^\circ$, and $AC = 14$ cm; determine the exact lengths of AB and BC .

Sketch a diagram.

Let $AB = x$ centimetres

Then $BC = 2x$ centimetres

Use: $b^2 = a^2 + c^2 - 2ac \cos B$

Substitute:

$$b = 14, a = 2x, c = x, \angle B = 120^\circ$$

$$14^2 = (2x)^2 + x^2 - 2(2x)(x) \cos 120^\circ$$

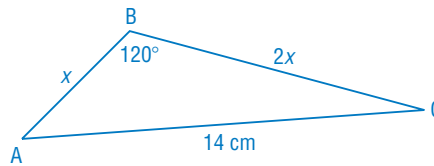
$$196 = 5x^2 - 4x^2(-0.5)$$

$$196 = 7x^2$$

$$x^2 = 28$$

$$x = \sqrt{28}, \text{ or } 2\sqrt{7}$$

So, $AB = 2\sqrt{7}$ cm and $AC = 4\sqrt{7}$ cm



11. Three circles have radii 3 cm, 4 cm, and 5 cm. Each circle just touches the other 2 circles externally. To the nearest tenth of a square centimetre, determine the area of the triangle formed by the centres of the circles.

Sketch a diagram.

The centres of the circle form $\triangle ABC$, with:

$$AB: 3 \text{ cm} + 4 \text{ cm} = 7 \text{ cm}$$

$$BC: 4 \text{ cm} + 5 \text{ cm} = 9 \text{ cm}$$

$$AC: 3 \text{ cm} + 5 \text{ cm} = 8 \text{ cm}$$

Draw the perpendicular from A to meet BC at D .

In $\triangle ABC$, use:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Substitute: $c = 7, a = 9, b = 8$

$$7^2 = 9^2 + 8^2 - 2(9)(8) \cos C$$

$$144 \cos C = 96$$

$$\cos C = \frac{96}{144}$$

$$\angle C = 48.1896 \dots^\circ$$

In $\triangle ACD$, use:

$$\sin C = \frac{AD}{AC}$$

$$AD = 8 \sin 48.1896 \dots^\circ$$

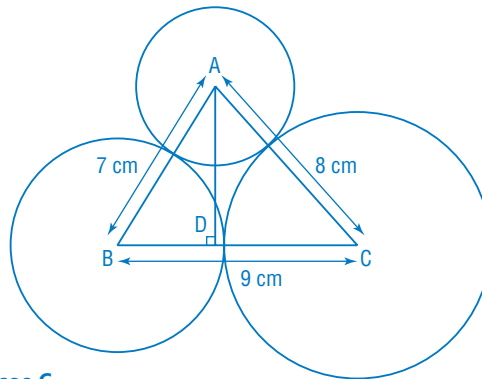
$$AD = 5.9628 \dots$$

The area of $\triangle ABC$ is:

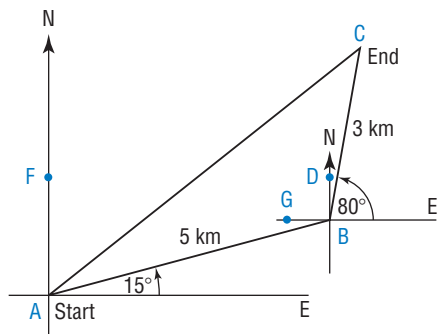
$$0.5(BC)(AD) = 0.5(9)(5.9628 \dots)$$

$$= 26.8328 \dots$$

So, the area of the triangle formed by the centres of the circles is approximately 26.8 cm^2 .



12. Here is a sketch of the route taken by an orienteering group on a one-day trek.



- a) To the nearest tenth of a kilometre, what is the straight-line distance from the start to the end?

$$\angle DBC \text{ is: } 90^\circ - 80^\circ = 10^\circ$$

$$\angle GBA = 15^\circ$$

$$\text{So, } \angle ABC \text{ is: } 15^\circ + 90^\circ + 10^\circ = 115^\circ$$

$$\text{In } \triangle ABC, \text{ use: } b^2 = a^2 + c^2 - 2ac \cos B$$

$$\text{Substitute: } a = 3, c = 5, \angle B = 115^\circ$$

$$b^2 = 3^2 + 5^2 - 2(3)(5) \cos 115^\circ$$

$$b = \sqrt{3^2 + 5^2 - 2(3)(5) \cos 115^\circ}$$

$$b = 6.8321 \dots$$

The straight-line distance is approximately 6.8 km.

- b) To the nearest degree, what is the bearing of the end point from the start point?

In $\triangle ABC$, determine $\angle A$; use:

$$\frac{\sin A}{a} = \frac{\sin B}{b} \quad \text{Substitute: } \angle B = 115^\circ, a = 3, b = 6.8321 \dots$$

$$\frac{\sin A}{3} = \frac{\sin 115^\circ}{6.8321 \dots}$$

$$\sin A = \frac{3 \sin 115^\circ}{6.8321 \dots}$$

Since $\angle A$ is acute:

$$\angle A = \sin^{-1}\left(\frac{3 \sin 115^\circ}{6.8321 \dots}\right)$$

$$\angle A = 23.4506 \dots^\circ$$

$$\angle A \doteq 23^\circ$$

$$\text{So, } \angle FAC \text{ is approximately } 90^\circ - (23^\circ + 15^\circ) = 52^\circ$$

The bearing of the end point from the start point is approximately 052° .

C

- 13.** Observers on two ships see an aircraft flying at 10 000 m.
 Observer A reports the bearing of the aircraft as 048° and its angle of elevation as 70° . Observer B reports the bearing of the aircraft as 285° and its angle of elevation as 15° . The bearing of observer B from observer A is 096° .
 To the nearest tenth of a kilometre, how far apart are the ships?

Sketch a diagram.

In $\triangle ACD$,

$$\begin{aligned}\angle C &= 90^\circ - 70^\circ \\ &= 20^\circ\end{aligned}$$

Determine AD; use:

$$\begin{aligned}\tan 20^\circ &= \frac{AD}{10} \\ AD &= 10 \tan 20^\circ\end{aligned}$$

In $\triangle BCD$,

$$\begin{aligned}\angle C &= 90^\circ - 15^\circ \\ &= 75^\circ\end{aligned}$$

Determine BD; use:

$$\begin{aligned}\tan 75^\circ &= \frac{BD}{10} \\ BD &= 10 \tan 75^\circ\end{aligned}$$

$$\begin{aligned}\angle FBA &= 180^\circ - 96^\circ \\ &= 84^\circ\end{aligned}$$

$$\begin{aligned}\angle FBD &= 360^\circ - 285^\circ \\ &= 75^\circ\end{aligned}$$

$$\begin{aligned}\text{So, } \angle ABD &= 84^\circ - 75^\circ \\ &= 9^\circ\end{aligned}$$

In $\triangle ABD$,

$$\begin{aligned}\angle A &= 96^\circ - 48^\circ \\ &= 48^\circ\end{aligned}$$

$$\angle B = 9^\circ$$

$$\begin{aligned}\text{So, } \angle D &= 180^\circ - (48^\circ + 9^\circ) \\ &= 123^\circ\end{aligned}$$

Use: $d^2 = a^2 + b^2 - 2ab \cos D$

Substitute: $a = 10 \tan 75^\circ$, $b = 10 \tan 20^\circ$, $\angle D = 123^\circ$

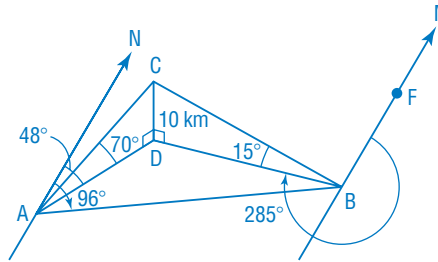
$$d^2 = (10 \tan 75^\circ)^2 + (10 \tan 20^\circ)^2 - 2(10 \tan 75^\circ)(10 \tan 20^\circ) \cos 123^\circ$$

$$d^2 = 10^2[(\tan 75^\circ)^2 + (\tan 20^\circ)^2 - 2(\tan 75^\circ)(\tan 20^\circ) \cos 123^\circ]$$

$$d = 10\sqrt{(\tan 75^\circ)^2 + (\tan 20^\circ)^2 - 2(\tan 75^\circ)(\tan 20^\circ) \cos 123^\circ}$$

$$d = 39.4211\dots$$

The ships are approximately 39.4 km apart.



The diagram is not drawn to scale.

- 14.** The lengths of the sides of a parallelogram are 9 cm and 10 cm. The shorter diagonal is 12 cm long. To the nearest tenth of a centimetre, what is the length of the longer diagonal?

Sketch a diagram.

In $\triangle PQR$, determine $\angle Q$.

Use: $q^2 = p^2 + r^2 - 2pr \cos Q$

Substitute: $q = 12, p = 9, r = 10$

$$12^2 = 9^2 + 10^2 - 2(9)(10) \cos Q$$

$$180 \cos Q = 37$$

$$\cos Q = \frac{37}{180}$$

$$\angle Q = 78.1379\dots^\circ$$

$$\text{So, } \angle QPS = 180^\circ - 78.1379\dots^\circ$$

$$= 101.8620\dots^\circ$$

In $\triangle PQS$, determine QS .

Use: $p^2 = q^2 + s^2 - 2qs \cos P$

Substitute: $q = 9, s = 10, \angle P = 101.8620\dots^\circ$

$$p^2 = 9^2 + 10^2 - 2(9)(10) \cos 101.8620\dots^\circ$$

$$p = \sqrt{9^2 + 10^2 - 2(9)(10) \cos 101.8620\dots^\circ}$$

$$p = 14.7648\dots$$

The longer diagonal is approximately 14.8 cm long.

