

Checkpoint: Assess Your Understanding, pages 461–464

6.1

1. A 15-m ladder leans against a wall at an angle of 55° to the horizontal ground. To the nearest tenth of a metre, how far from the wall is the base of the ladder and how far up the wall does it reach?

Sketch a diagram.

The length of the ladder is OP .

The x -coordinate of P is the distance of the ladder from the base of the wall.

$$x = r \cos \theta \quad \text{Substitute: } r = 15, \theta = 55^\circ$$

$$x = 15 \cos 55^\circ$$

$$x = 8.6036 \dots$$

The base of the ladder is approximately 8.6 m from the wall.

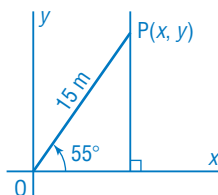
The y -coordinate of P is the distance that the ladder reaches up the wall.

$$y = r \sin \theta \quad \text{Substitute: } r = 15, \theta = 55^\circ$$

$$y = 15 \sin 55^\circ$$

$$y = 12.2872 \dots$$

The ladder reaches approximately 12.3 m up the wall.



2. **Multiple Choice** Which statements are correct?

I. $\cos 60^\circ + \sin 30^\circ = 1$

II. $\sin 60^\circ + \cos 30^\circ = \sqrt{3}$

III. $\cos 45^\circ + \sin 45^\circ = \sqrt{2}$

IV. $\frac{\cos 45^\circ}{\sin 45^\circ} = 1$

A. only I and II

B. only III and IV

C. no statements

D. all statements

6.2

3. For each angle below:

i) Sketch it in standard position.

ii) Determine its reference angle.

iii) Identify the other angles from 0° to 360° that have the same reference angle.

a) 205°

i) Since the angle is between 180° and 270° , the terminal arm lies in Quadrant 3.

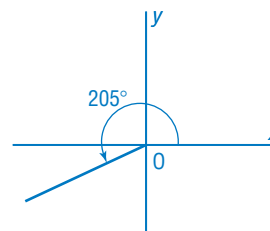
ii) Its reference angle is: $205^\circ - 180^\circ = 25^\circ$

iii) In Quadrant 2, the angle with the same reference angle is:

$$180^\circ - 25^\circ = 155^\circ$$

In Quadrant 4, the angle with the same reference angle is:

$$360^\circ - 25^\circ = 335^\circ$$



b) 12°

i) Since the angle is between 0° and 90° , the terminal arm is in Quadrant 1.

ii) Its reference angle is: 12°

iii) In Quadrant 2, the angle with the same reference angle is:

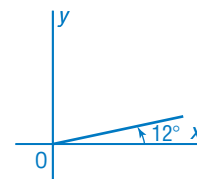
$$180^\circ - 12^\circ = 168^\circ$$

In Quadrant 3, the angle with the same reference angle is:

$$180^\circ + 12^\circ = 192^\circ$$

In Quadrant 4, the angle with the same reference angle is:

$$360^\circ - 12^\circ = 348^\circ$$

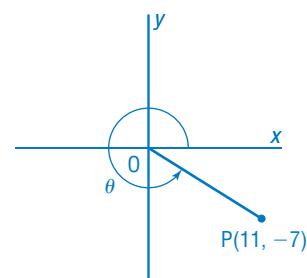


4. The point $P(11, -7)$ is on the terminal arm of angle θ in standard position.

a) Sketch the angle.

Plot $P(11, -7)$; draw a line through OP .

Label θ .



b) Determine the primary trigonometric ratios of θ .

Let the length of $OP = r$.

Use: $r = \sqrt{x^2 + y^2}$ Substitute: $x = 11, y = -7$

$$r = \sqrt{(11)^2 + (-7)^2}$$

$$r = \sqrt{170}$$

$$x = 11, y = -7, r = \sqrt{170}$$

$$\sin \theta = \frac{y}{r} \qquad \cos \theta = \frac{x}{r} \qquad \tan \theta = \frac{y}{x}$$

$$= \frac{-7}{\sqrt{170}} \qquad = \frac{11}{\sqrt{170}} \qquad = \frac{-7}{11}$$

c) To the nearest degree, what is θ ?

Use: $\cos \theta = \frac{11}{\sqrt{170}}$

The reference angle is:

$$\cos^{-1}\left(\frac{11}{\sqrt{170}}\right) = 32.4711\dots^\circ$$

In Quadrant 4, θ is approximately: $360^\circ - 32^\circ = 328^\circ$

5. To the nearest degree, which angles satisfy the equation $\cos \theta = -\frac{4}{7}$ for $0^\circ \leq \theta \leq 360^\circ$?

Use: $\cos \theta = -\frac{4}{7}$

The reference angle is:

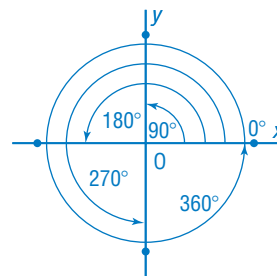
$$\cos^{-1}\left(\frac{4}{7}\right) \doteq 55^\circ$$

In Quadrant 2, θ is approximately: $180^\circ - 55^\circ = 125^\circ$

In Quadrant 3, θ is approximately: $180^\circ + 55^\circ = 235^\circ$

6. a) Sketch a diagram to show these angles in standard position:

0° ; 90° ; 180° ; 270° ; 360°



b) Without using technology, determine the sine, cosine, and tangent of each angle in part a.

For 0° , the terminal arm is on the positive x -axis, so $x = r$ and $y = 0$.

$$\begin{aligned} \text{Use: } \sin \theta &= \frac{y}{r} & \cos \theta &= \frac{x}{r} & \tan \theta &= \frac{y}{x} \\ \sin 0^\circ &= \frac{0}{r} & \cos 0^\circ &= \frac{r}{r} & \tan 0^\circ &= \frac{0}{r} \\ &= 0 & &= 1 & &= 0 \end{aligned}$$

For 90° , the terminal arm is on the positive y -axis, so $x = 0$ and $y = r$.

$$\begin{aligned} \sin 90^\circ &= \frac{r}{r} & \cos 90^\circ &= \frac{0}{r} & \tan 90^\circ &= \frac{r}{0}, \text{ which is undefined} \\ &= 1 & &= 0 & & \end{aligned}$$

For 180° , the terminal arm is on the negative x -axis, so $x = -r$ and $y = 0$.

$$\begin{aligned} \sin 180^\circ &= \frac{0}{r} & \cos 180^\circ &= \frac{-r}{r} & \tan 180^\circ &= \frac{0}{-r} \\ &= 0 & &= -1 & &= 0 \end{aligned}$$

For 270° , the terminal arm is on the negative y -axis, so $x = 0$ and $y = -r$.

$$\begin{aligned} \sin 270^\circ &= \frac{-r}{r} & \cos 270^\circ &= \frac{0}{r} & \tan 270^\circ &= \frac{-r}{0}, \text{ which is undefined} \\ &= -1 & &= 0 & & \end{aligned}$$

For 360° , the terminal arm is on the positive x -axis, so $x = r$ and $y = 0$.

$$\begin{aligned} \sin 360^\circ &= \frac{0}{r} & \cos 360^\circ &= \frac{r}{r} & \tan 360^\circ &= \frac{0}{r} \\ &= 0 & &= 1 & &= 0 \end{aligned}$$

7. Determine the exact primary trigonometric ratios for each angle in standard position.

a) 135°

The terminal arm of the angle lies in Quadrant 2, and its reference angle is: $180^\circ - 135^\circ = 45^\circ$

$$\begin{aligned} \sin 135^\circ &= \sin 45^\circ & \cos 135^\circ &= -\cos 45^\circ \\ &= \frac{1}{\sqrt{2}} & &= -\frac{1}{\sqrt{2}} \end{aligned}$$

$$\begin{aligned} \tan 135^\circ &= -\tan 45^\circ \\ &= -1 \end{aligned}$$

b) 300°

The terminal arm of the angle lies in Quadrant 4, and its reference angle is: $360^\circ - 300^\circ = 60^\circ$

$$\begin{aligned} \sin 300^\circ &= -\sin 60^\circ & \cos 300^\circ &= \cos 60^\circ \\ &= -\frac{\sqrt{3}}{2} & &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \tan 300^\circ &= -\tan 60^\circ \\ &= -\sqrt{3} \end{aligned}$$

c) 210°

The terminal arm of the angle lies in Quadrant 3, and its reference angle is: $210^\circ - 180^\circ = 30^\circ$

$$\begin{aligned}\sin 210^\circ &= -\sin 30^\circ & \cos 210^\circ &= -\cos 30^\circ \\ &= -\frac{1}{2} & &= -\frac{\sqrt{3}}{2} \\ \tan 210^\circ &= \tan 30^\circ \\ &= \frac{1}{\sqrt{3}}\end{aligned}$$

8. Angle θ is in standard position and its terminal arm lies in Quadrant 4. The cosine of its reference angle is $\frac{1}{8}$. Determine the exact values of $\sin \theta$, $\cos \theta$, and $\tan \theta$.

For the reference angle in Quadrant 1:

Use: $r^2 = x^2 + y^2$ Substitute: $x = 1, r = 8$

$$(8)^2 = (1)^2 + y^2$$

$$y^2 = 63$$

$$y = \sqrt{63}$$

In Quadrant 4, $x = 1, y = -\sqrt{63}, r = 8$

$$\begin{aligned}\text{Use: } \sin \theta &= \frac{y}{r} & \cos \theta &= \frac{x}{r} & \tan \theta &= \frac{y}{x} \\ &= \frac{-\sqrt{63}}{8} & &= \frac{1}{8} & &= \frac{-\sqrt{63}}{1}, \text{ or } -\sqrt{63}\end{aligned}$$

9. **Multiple Choice** Angle θ is in standard position, with $\tan \theta = -\frac{3}{2}$. Which statement could be correct?

A. $\sin \theta = \frac{3}{\sqrt{13}}$ B. $\cos \theta = -\frac{3}{\sqrt{13}}$

C. $\sin \theta = -\frac{2}{\sqrt{13}}$ D. $\cos \theta = \frac{3}{\sqrt{13}}$