REVIEW, pages 510–515

6.1

1. Point P(10, 4) is on the terminal arm of an angle θ in standard position.

a) Determine the distance of P from the origin.

The distance of P from the origin is r.

Use: $r = \sqrt{x^2 + y^2}$ Substitute: $x = 10$, $y = 4$

$r = \sqrt{10^2 + 4^2}$

$r = \sqrt{116}$, or $2\sqrt{29}$

b) Write the primary trigonometric ratios of θ.

$\sin\theta = \frac{y}{r}$

$= \frac{4}{2\sqrt{29}}$, or $\frac{2}{\sqrt{29}}$

$\cos\theta = \frac{x}{r}$

$= \frac{10}{2\sqrt{29}}$, or $\frac{5}{\sqrt{29}}$

$\tan\theta = \frac{y}{x}$

$= \frac{4}{10}$, or 0.4

c) What is the value of θ to the nearest degree?

Use: $\tan\theta = 0.4$

$\theta = \tan^{-1}(0.4)$

$\theta = 21.8014...^\circ$

θ is approximately 22°.

2. Angle θ is in standard position with its terminal arm in Quadrant 1 and $\sin\theta = \frac{2}{3}$.

a) Determine $\cos\theta$ and $\tan\theta$.

Use: $r^2 = x^2 + y^2$ Substitute: $r = 3$, $y = 2$

$3^2 = x^2 + 2^2$

$x^2 = 5$

$x = \sqrt{5}$

$\cos\theta = \frac{x}{r}$

$= \frac{\sqrt{5}}{3}$

$\tan\theta = \frac{y}{x}$

$= \frac{2}{\sqrt{5}}$

b) Determine the value of θ to the nearest degree.

Use: $\sin\theta = \frac{2}{3}$

$\theta = \sin^{-1}\left(\frac{2}{3}\right)$

$\theta = 41.8103...^\circ$

θ is approximately 42°.

6.2

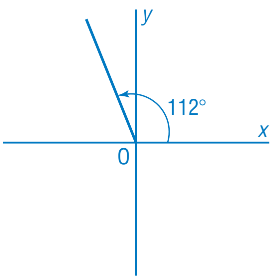
3. For each angle below:

i) Sketch it in standard position.

ii) Determine its reference angle.

a) 112°

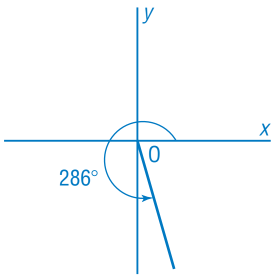
i) Since the angle is between 90° and 180°, the terminal arm is in Quadrant 2.



ii) The angle is in Quadrant 2, so its reference angle is: $180° - 112° = 68°$

b) 286°

i) Since the angle is between 270° and 360°, the terminal arm is in Quadrant 4.



ii) The angle is in Quadrant 4, so its reference angle is: $360° - 286° = 74°$

4. To the nearest degree, which values of θ satisfy the equation $\sin\theta = -\frac{3}{8}$ for $0° \leq \theta \leq 360°$?

The reference angle is:

$\sin^{-1}\left(\frac{3}{8}\right) \doteq 22°$

sin θ is negative in Quadrants 3 and 4 so:

$\theta \doteq 180° + 22°$ and $\theta \doteq 360° - 22°$

$\doteq 202°$ $\doteq 338°$

6.3

5. In ΔABC, BC = 20 cm, AB = 25 cm, and $\angle A = 45°$
Show that is it possible to draw ΔABC, then determine if these measurements illustrate an ambiguous case.

The ratio of the side opposite $\angle$A to the side adjacent to $\angle$A is:

$\frac{BC}{AB} = \frac{20}{25}$, which is 0.8

$\sin 45° = 0.7071...$

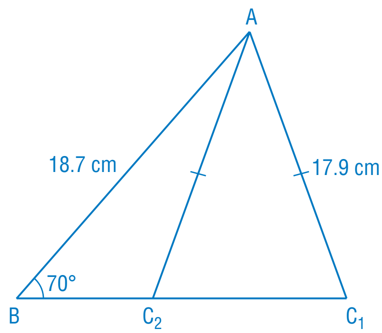
Since $\frac{BC}{AB} > \sin 45°$, it is possible to draw a triangle

Since $\sin 45° < 0.8 < 1$, then this is an ambiguous case

6.4

6. **a)** In ΔABC, AB = 18.7 cm, AC = 17.9 cm, and $\angle B = 70°$; determine the measure of BC to the nearest tenth of a centimetre.

Check for the ambiguous case.
The ratio of the side opposite $\angle B$ to the side adjacent to $\angle B$ is:
$\frac{AC}{AB} = \frac{17.9}{18.7}$, **which is 0.9572. . .**
$\sin 70° = 0.9396\ldots$
Since $\sin 70° < 0.9572\ldots < 1$,
then this is an ambiguous case, and 2 triangles can be constructed:
ΔABC_1 **is acute;** ΔABC_2 **is obtuse.**
Sketch a diagram.



This diagram is not drawn to scale.

In ΔABC_1
Determine $\angle C_1$.

Use: $\frac{\sin C_1}{c} = \frac{\sin B}{b}$

Substitute: $\angle B = 70°$, $c = 18.7$, $b = 17.9$

$$\frac{\sin C_1}{18.7} = \frac{\sin 70°}{17.9}$$

$$\sin C_1 = \frac{18.7 \sin 70°}{17.9}$$

$$\angle C_1 = \sin^{-1}\left(\frac{18.7 \sin 70°}{17.9}\right)$$

$$\angle C_1 = 79.0188\ldots°$$

So, $\angle A = 180° - (70° + 79.0188\ldots°)$
$= 30.9811\ldots°$

Use: $\frac{a}{\sin A} = \frac{b}{\sin B}$

Substitute: $\angle A = 30.9811\ldots°$
$\angle B = 70°$, $b = 17.9$

$$\frac{a}{\sin 30.9811\ldots°} = \frac{17.9}{\sin 70°}$$

$$a = \frac{17.9 \sin 30.9811\ldots°}{\sin 70°}$$

$$a = 9.8054\ldots$$

So, $BC \doteq 9.8$ **cm or 3.0 cm**

In ΔABC_2

$$\angle C_2 = 180° - \angle C_1$$
$$= 100.9811\ldots°$$

So, $\angle A = 180° - (70° + 100.9811\ldots°)$
$= 9.0188\ldots°$

Use: $\frac{a}{\sin A} = \frac{b}{\sin B}$

Substitute: $\angle A = 9.0188\ldots°$

$\angle B = 70°$, $b = 17.9$

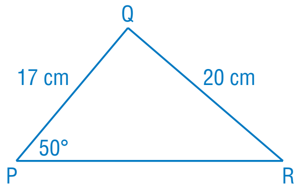
$$\frac{a}{\sin 9.0188\ldots°} = \frac{17.9}{\sin 70°}$$

$$a = \frac{17.9 \sin 9.0188\ldots°}{\sin 70°}$$

$$a = 2.9860\ldots$$

b) In ΔPQR, QR = 20 cm, PQ = 17 cm, and $\angle$P = 50°; determine the measure of $\angle$R to the nearest degree.

Check for the ambiguous case.
The ratio of the side opposite $\angle$P to the side adjacent to $\angle$P is:
$\frac{QR}{PQ} = \frac{20}{17}$, which is greater than 1, so only 1 triangle is possible
Sketch a diagram.



$\frac{\sin R}{r} = \frac{\sin P}{p}$

Substitute: $\angle P = 50°$, $r = 17$, $p = 20$

$\frac{\sin R}{17} = \frac{\sin 50°}{20}$

$\sin R = \frac{17 \sin 50°}{20}$

Since $\angle$R is acute:

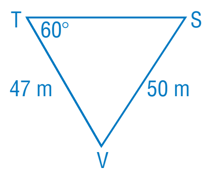
$\angle R = \sin^{-1}\left(\frac{17 \sin 50°}{20}\right)$

$\angle R = 40.6274...°$

$\angle R \doteq 41°$

7. Two tow trucks, on a straight road, are pulling a vehicle from a field. The cable from one truck is let out 47 m and it makes an angle of 60° with the road. The cable from the other truck is let out 50 m. To the nearest metre, how far apart are the trucks?

Sketch a diagram.
Check for the ambiguous case.
The ratio of the side opposite $\angle$T to the side adjacent to $\angle$T is:
$\frac{SV}{TV} = \frac{50}{47}$, which is >1, so only 1 triangle is possible



Determine $\angle$S first.

Use: $\frac{\sin S}{s} = \frac{\sin T}{t}$

Substitute: $\angle T = 60°$, $s = 47$, $t = 50$

$\frac{\sin S}{47} = \frac{\sin 60°}{50}$

$\sin S = \frac{47 \sin 60°}{50}$

$\angle S = \sin^{-1}\left(\frac{47 \sin 60°}{50}\right)$

Since $\angle$S is acute:
$\angle S = 54.4949...°$

So, $\angle V = 180° - (60° + 54.4949...°)$
$= 65.5050...°$

Then determine TS.

Use: $\frac{v}{\sin V} = \frac{t}{\sin T}$

Substitute: $\angle V = 65.5050...°$, $\angle T = 60°$, $t = 50$

$\frac{v}{\sin 65.5050...°} = \frac{50}{\sin 60°}$

$v = \frac{50 \sin 65.5050...°}{\sin 60°}$

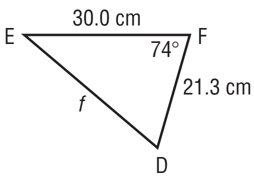
$v = 52.5387...$

The trucks are approximately 53 m apart.

6.5

8. Determine each indicated measure to the nearest tenth of a unit.

a)



Use: $f^2 = d^2 + e^2 - 2de \cos F$
Substitute: $d = 30$, $e = 21.3$, $\angle F = 74°$

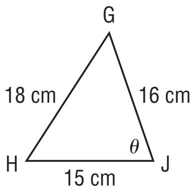
$f^2 = 30^2 + 21.3^2 - 2(30)(21.3) \cos 74°$

$f = \sqrt{30^2 + 21.3^2 - 2(30)(21.3) \cos 74°}$

$f = 31.6453...$

$f \doteq 31.6$ cm

b)



Use: $j^2 = g^2 + h^2 - 2gh \cos J$
Substitute: $j = 18$, $g = 15$, $h = 16$

$18^2 = 15^2 + 16^2 - 2(15)(16) \cos J$

$\cos J = \dfrac{15^2 + 16^2 - 18^2}{2(15)(16)}$

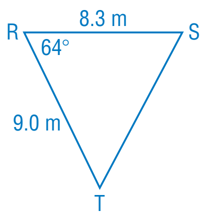
$\angle J = \cos^{-1}\left(\dfrac{15^2 + 16^2 - 18^2}{2(15)(16)}\right)$

$\angle J = 70.9081...°$

$\theta \doteq 70.9°$

9. In ΔRST, ∠R = 64°, RS = 8.3 m, and RT = 9.0 m
Solve this triangle. Give the side lengths to the nearest tenth of a metre and the angle measures to the nearest degree.

Since the given angle is between the two given sides, only 1 triangle is possible. Sketch a diagram. Determine ST first.



Use: $r^2 = s^2 + t^2 - 2st \cos R$
Substitute: $s = 9$, $t = 8.3$, $\angle R = 64°$

$r^2 = 9^2 + 8.3^2 - 2(9)(8.3) \cos 64°$

$r = \sqrt{9^2 + 8.3^2 - 2(9)(8.3) \cos 64°}$

$r = 9.1868...$

Determine ∠S.

Use: $\dfrac{\sin S}{s} = \dfrac{\sin R}{r}$ Substitute: $\angle R = 64°$, $s = 9$, $r = 9.1868...$

$\dfrac{\sin S}{9} = \dfrac{\sin 64°}{9.1868...}$

$\sin S = \dfrac{9 \sin 64°}{9.1868...}$

Since ∠S is acute:

$\angle S = \sin^{-1}\left(\dfrac{9 \sin 64°}{9.1868...}\right)$

$\angle S = 61.7049...°$

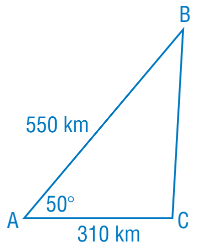
$\angle T = 180° - (64° + 61.7049...°)$

$= 54.2950...°$

So, TS $\doteq$ 9.2 m, ∠S $\doteq$ 62°, ∠T $\doteq$ 54°

10. Two airplanes leave an airport on flight paths that intersect at an angle of 50°. After one hour, one plane has travelled 550 km and the other has travelled 310 km.

a) To the nearest kilometre, how far apart are the planes?



Sketch a diagram.
Since the given angle is between the two given sides, only 1 triangle is possible.
In ΔABC, determine BC.
Use: $a^2 = b^2 + c^2 - 2bc \cos A$
Substitute: $b = 310$, $c = 550$, $\angle A = 50°$

$a^2 = 310^2 + 550^2 - 2(310)(550) \cos 50°$

$a = \sqrt{310^2 + 550^2 - 2(310)(550) \cos 50°}$

$a = 423.5674\ldots$

The planes are approximately 424 km apart.

b) To the nearest degree, what is the angle between the line joining the planes and the course of the faster plane?

Determine ∠B.
Use: $\frac{\sin B}{b} = \frac{\sin A}{a}$
Substitute: $\angle A = 50°$, $b = 310$, $a = 423.5674\ldots$

$\frac{\sin B}{310} = \frac{\sin 50°}{423.5674\ldots}$

$\sin B = \frac{310 \sin 50°}{423.5674\ldots}$

Since ∠B is acute:

$\angle B = \sin^{-1}\left(\frac{310 \sin 50°}{423.5674\ldots}\right)$

$\angle B = 34.1008\ldots°$

The angle between the line joining the planes and the course of the faster plane is approximately 34°.