Lesson 7.5 Exercises, pages 583–590

A

3. Identify the non-permissible values of the variable in each equation.

a) $5 = \frac{3x}{x}$

$x = 0$

b) $\frac{2}{x - 1} = \frac{6}{x}$

$x = 0$ and $x = 1$

c) $\frac{3}{6 - x} = \frac{2}{x + 5}$

$x = -5$ and $x = 6$

d) $\frac{-1}{3x + 6} = \frac{2x}{x - 2}$

$x = -2$ and $x = 2$

4. Solve each equation.

a) $\frac{4}{5} = \frac{8}{d}$

Non-permissible value: $d = 0$
Common denominator: $5d$

$5d\left(\frac{4}{5}\right) = 5d\left(\frac{8}{d}\right)$

$4d = 40$

$d = 10$

b) $\frac{e}{3} = \frac{12}{e}$

Non-permissible value: $e = 0$
Common denominator: $3e$

$3e\left(\frac{e}{3}\right) = 3e\left(\frac{12}{e}\right)$

$e^2 = 36$

$e = 6$ or $e = -6$

Solutions are $e = -6$ and $e = 6$.

5. Solve each equation.

a) $\frac{3}{2z} = \frac{4}{3z} - \frac{1}{2}$

Non-permissible value: $z = 0$
Common denominator: $6z$

$$\cancel{6z}^{3}\left(\frac{3}{\cancel{2z}}\right) = \cancel{6z}^{2}\left(\frac{4}{\cancel{3z}}\right) - \cancel{6z}^{3}\left(\frac{1}{\cancel{2}}\right)$$
$$9 = 8 - 3z$$
$$3z = -1$$
$$z = -\frac{1}{3}$$

b) $2 - \frac{4}{x} = \frac{6}{x^2}$

Non-permissible value: $x = 0$
Common denominator: x^2

$$x^2(2) - \cancel{x^2}^{1}\left(\frac{4}{\cancel{x}}\right) = \cancel{x^2}\left(\frac{6}{\cancel{x^2}}\right)$$
$$2x^2 - 4x = 6$$
$$2x^2 - 4x - 6 = 0$$
$$2(x^2 - 2x - 3) = 0$$
$$2(x - 3)(x + 1) = 0$$
$$x = 3 \text{ or } x = -1$$

Solutions are $x = -1$ and $x = 3$.

B

6. Solve each equation.

a) $\frac{3}{q - 2} = \frac{5}{q + 4}$

Non-permissible values: $q = 2$ and $q = -4$
Common denominator: $(q - 2)(q + 4)$

$$\cancel{(q - 2)}(q + 4)\left(\frac{3}{\cancel{q - 2}}\right) = (q - 2)\cancel{(q + 4)}\left(\frac{5}{\cancel{q + 4}}\right)$$
$$3q + 12 = 5q - 10$$
$$22 = 2q$$
$$q = 11$$

b) $\frac{3}{2s - 4} = \frac{3}{s - 2}$

Non-permissible value: $s = 2$
The numerators are the same, so equate the denominators.
$2s - 4 = s - 2$
$s = 2$
$s = 2$ is a non-permissible value, so the equation has no solution.

7. Solve each equation.

a) $\frac{a - 1}{a - 3} = \frac{a + 1}{a - 4}$

Non-permissible values: $a = 3$ and $a = 4$
Common denominator: $(a - 3)(a - 4)$

$$\cancel{(a - 3)}(a - 4)\left(\frac{a - 1}{\cancel{a - 3}}\right) = (a - 3)\cancel{(a - 4)}\left(\frac{a + 1}{\cancel{a - 4}}\right)$$
$$(a - 4)(a - 1) = (a - 3)(a + 1)$$
$$a^2 - 5a + 4 = a^2 - 2a - 3$$
$$-3a = -7$$
$$a = \frac{7}{3}$$

b) $\frac{2w + 1}{w - 4} = \frac{4w - 3}{2w + 1}$

Non-permissible values: $w = 4$ and $w = -\frac{1}{2}$

Common denominator: $(w - 4)(2w + 1)$

$$\cancel{(w - 4)}(2w + 1)\left(\frac{2w + 1}{\cancel{w - 4}}\right) = (w - 4)\cancel{(2w + 1)}\left(\frac{4w - 3}{\cancel{2w + 1}}\right)$$

$$(2w + 1)(2w + 1) = (w - 4)(4w - 3)$$

$$4w^2 + 4w + 1 = 4w^2 - 19w + 12$$

$$23w = 11$$

$$w = \frac{11}{23}$$

c) $\frac{6}{2x^2 + 2x} = \frac{x - 2}{x + 1}$

$\frac{6}{2x(x + 1)} = \frac{x - 2}{x + 1}$

Non-permissible values: $x = -1$ and $x = 0$

Common denominator: $2x(x + 1)$

$$\cancel{2x(x + 1)}\left(\frac{6}{\cancel{2x(x + 1)}}\right) = 2x\cancel{(x + 1)}\left(\frac{x - 2}{\cancel{x + 1}}\right)$$

$$6 = 2x^2 - 4x$$

$$2x^2 - 4x - 6 = 0$$

$$2(x^2 - 2x - 3) = 0$$

$$2(x - 3)(x + 1) = 0$$

$$x = 3 \text{ or } x = -1$$

$x = -1$ is a non-permissible value.

So, the only solution is $x = 3$.

8. Here is a student's solution for solving a rational equation. Identify the error in the solution. Write a correct solution.

$$\frac{1}{8} = 1 + \frac{2}{x}$$

$$\frac{1}{8} = \frac{3}{x}$$

$$\cancel{8}x\left(\frac{1}{\cancel{8}}\right) = 8\cancel{x}\left(\frac{3}{\cancel{x}}\right)$$

$$x = 24$$

There is an error in line 2: the student added $1 + \frac{2}{x}$ without using a common denominator. Correct solution:

Non-permissible value: $x = 0$

Common denominator: $8x$

$$\frac{1}{8} = 1 + \frac{2}{x}$$

$$\frac{1}{8} = \frac{x + 2}{x}$$

$$\cancel{8}x\left(\frac{1}{\cancel{8}}\right) = 8\cancel{x}\left(\frac{x + 2}{\cancel{x}}\right)$$

$$x = 8x + 16$$

$$7x = -16$$

$$x = -\frac{16}{7}$$

9. Solve each equation.

a) $\frac{1}{x} + \frac{1}{x-3} = \frac{x-2}{x-3}$

Non-permissible values: $x = 0$ and $x = 3$

$$\frac{1}{x} = \frac{x-2}{x-3} - \frac{1}{x-3}$$
$$\frac{1}{x} = \frac{x-3}{x-3}$$
$$\frac{1}{x} = 1$$
$$x = 1$$

b) $\frac{1}{u-2} + \frac{u-1}{u^2-4} = \frac{u+4}{u+2}$

$$\frac{1}{u-2} + \frac{u-1}{(u-2)(u+2)} = \frac{u+4}{u+2}$$

Non-permissible values: $u = -2$ and $u = 2$

Common denominator: $(u-2)(u+2)$

$$\cancel{(u-2)}(u+2)\left(\frac{1}{\cancel{u-2}}\right) + \cancel{(u-2)(u+2)}\left(\frac{u-1}{\cancel{(u-2)(u+2)}}\right) = (u-2)\cancel{(u+2)}\left(\frac{u+4}{\cancel{u+2}}\right)$$

$$u + 2 + u - 1 = (u-2)(u+4)$$
$$2u + 1 = u^2 + 2u - 8$$
$$u^2 - 9 = 0$$
$$u = 3 \text{ or } u = -3$$

Solutions are $u = -3$ and $u = 3$.

c) $\frac{6b^2}{b^2-25} + \frac{4b}{5-b} = \frac{b}{b+5}$

$$\frac{6b^2}{(b-5)(b+5)} - \frac{4b}{b-5} = \frac{b}{b+5}$$

Non-permissible values: $b = -5$ and $b = 5$

Common denominator: $(b-5)(b+5)$

$$\cancel{(b-5)(b+5)}\left(\frac{6b^2}{\cancel{(b-5)(b+5)}}\right) - \cancel{(b-5)}(b+5)\left(\frac{4b}{\cancel{b-5}}\right) = (b-5)\cancel{(b+5)}\left(\frac{b}{\cancel{b+5}}\right)$$

$$6b^2 - 4b(b+5) = b^2 - 5b$$
$$6b^2 - 4b^2 - 20b = b^2 - 5b$$
$$b^2 - 15b = 0$$
$$b(b-15) = 0$$
$$b = 0 \text{ or } b = 15$$

Solutions are $b = 0$ and $b = 15$.

d) $\dfrac{3z-1}{2z+1}+\dfrac{1}{6}=\dfrac{2z-1}{2z+1}+\dfrac{z+1}{z+3}$

$\dfrac{3z-1}{2z+1}-\dfrac{2z-1}{2z+1}+\dfrac{1}{6}=\dfrac{z+1}{z+3}$

$\dfrac{z}{2z+1}+\dfrac{1}{6}=\dfrac{z+1}{z+3}$

Non-permissible values: $z=-\dfrac{1}{2}$ and $z=-3$

Common denominator: $6(2z+1)(z+3)$

$6\cancel{(2z+1)}(z+3)\left(\dfrac{z}{\cancel{2z+1}}\right)+\cancel{6}(2z+1)(z+3)\left(\dfrac{1}{\cancel{6}}\right)=6(2z+1)\cancel{(z+3)}\left(\dfrac{z+1}{\cancel{z+3}}\right)$

$6z(z+3)+(2z+1)(z+3)=(12z+6)(z+1)$

$6z^2+18z+2z^2+7z+3=12z^2+18z+6$

$4z^2-7z+3=0$

$(4z-3)(z-1)=0$

$z=\dfrac{3}{4}$ or $z=1$

Solutions are $z=\dfrac{3}{4}$ and $z=1$.

10. Solve each equation.

a) $\dfrac{b}{b^2-4}=\dfrac{2}{b^2-b-6}$

$\dfrac{b}{(b-2)(b+2)}=\dfrac{2}{(b-3)(b+2)}$

Non-permissible values: $b=2$, $b=-2$, and $b=3$

Common denominator: $(b-2)(b+2)(b-3)$

$\cancel{(b-2)}\cancel{(b+2)}(b-3)\left(\dfrac{b}{\cancel{(b-2)}\cancel{(b+2)}}\right)=(b-2)\cancel{(b+2)}\cancel{(b-3)}\left(\dfrac{2}{\cancel{(b-3)}\cancel{(b+2)}}\right)$

$b^2-3b=2b-4$

$b^2-5b+4=0$

$(b-4)(b-1)=0$

$b=4$ or $b=1$

Solutions are $b=4$ and $b=1$.

b) $\dfrac{16}{2g^2+2g-12}=\dfrac{6}{g^2-9}$

$\dfrac{16}{2(g^2+g-6)}=\dfrac{6}{(g-3)(g+3)}$

$\dfrac{16}{2(g+3)(g-2)}=\dfrac{6}{(g-3)(g+3)}$

Non-permissible values: $g=-3$, $g=2$, and $g=3$

Common denominator: $2(g+3)(g-3)(g-2)$

$\cancel{2}\cancel{(g+3)}(g-3)\cancel{(g-2)}\left(\dfrac{16}{\cancel{2}\cancel{(g+3)}\cancel{(g-2)}}\right)=2\cancel{(g+3)}\cancel{(g-3)}(g-2)\left(\dfrac{6}{\cancel{(g-3)}\cancel{(g+3)}}\right)$

$16g-48=12g-24$

$4g=24$

$g=6$

c) $\frac{n}{n+1} + \frac{3n+5}{n^2+4n+3} = \frac{2}{n+3}$

$\frac{n}{n+1} + \frac{3n+5}{(n+1)(n+3)} = \frac{2}{n+3}$

Non-permissible values: $n = -1$ and $n = -3$

Common denominator: $(n+1)(n+3)$

$$\cancel{(n+1)}(n+3)\left(\frac{n}{\cancel{n+1}}\right) + \cancel{(n+1)(n+3)}\left(\frac{3n+5}{\cancel{(n+1)(n+3)}}\right) = (n+1)\cancel{(n+3)}\left(\frac{2}{\cancel{n+3}}\right)$$

$$n^2 + 3n + 3n + 5 = 2n + 2$$
$$n^2 + 4n + 3 = 0$$
$$(n+1)(n+3) = 0$$
$$n = -1 \text{ or } n = -3$$

$n = -1$ and $n = -3$ are non-permissible values.

So, the equation has no solution.

11. The solutions of the equation $4 = x + \frac{8}{x+m}$ are $x = 3$ and $x = -4$. Determine the value of m. Show how you can verify your answer.

Common denominator: $x + m$

$$(x+m)4 = (x+m)x + \cancel{(x+m)}\left(\frac{8}{\cancel{x+m}}\right)$$

$$4x + 4m = x^2 + mx + 8$$
$$x^2 + mx - 4x + 8 - 4m = 0$$
$$x^2 + x(m-4) + (8-4m) = 0$$

An equation with roots $x = 3$ and $x = -4$ is:

$$(x-3)(x+4) = 0$$
$$x^2 + x - 12 = 0$$

Compare the equations: $x^2 + x(m-4) + (8-4m) = 0$

$x^2 + x - 12 = 0$

So, $m - 4 = 1$ and $8 - 4m = -12$

$m = 5$ $\quad$ $m = 5$

The value of m is 5.

To verify the answer, substitute $m = 5$ in the original equation, then solve the equation.

12. The measure, d degrees, of each angle in a regular polygon with n sides is given by the equation $d = 180 - \frac{360}{n}$.

a) What is the measure of each angle in a regular polygon with 15 sides?

Substitute $n = 15$:

$d = 180 - \frac{360}{15}$

$d = 180 - 24$

$d = 156$

Each angle measures 156°.

b) When each angle in a regular polygon is 162°, how many sides does the polygon have?

Substitute $d = 162$:

$$162 = 180 - \frac{360}{n}$$

$$18 = \frac{360}{n}$$

$$n(18) = n\left(\frac{360}{n}\right)$$

$$18n = 360$$

$$n = 20$$

The polygon has 20 sides.

13. Without solving the equation, how do you know that the equation $\frac{12}{4x - 4} = \frac{4}{x - 1}$ has no solution?

Simplify the expression on the left.

$$\frac{\overset{3}{\cancel{12}}}{\cancel{4}(x - 1)} = \frac{4}{x - 1}$$

$$\frac{3}{x - 1} = \frac{4}{x - 1}$$

Since the expressions have the same denominator but different numerators, I know the equation has no solution.

C

14. **a)** Write a rational equation that has 4 as a solution.

Work backward.

$x = 4$	Write 4 as a difference of 2 numbers.
$x = 7 - 3$	Take 3 to the left side.
$x + 3 = 7$	Divide each side by $x + 3$.
$\frac{x + 3}{x + 3} = \frac{7}{x + 3}$	Simplify.
$1 = \frac{7}{x + 3}, x \neq -3$	

b) Write a rational equation that has -3 as a solution and has 3 as an extraneous root.

$\frac{1}{x - 3}$ has $x = 3$ as a non-permissible value.

Let both rational expressions in the equation have denominator $x - 3$.

$$\frac{\quad}{x - 3} = \frac{\quad}{x - 3}$$

Write an equation that has $x = 3$ and $x = -3$ as roots:

$x^2 = 9$

So, the rational equation becomes:

$$\frac{x^2}{x - 3} = \frac{9}{x - 3}$$

This equation has $x = 3$ and $x = -3$ as roots. Since $x = 3$ is a non-permissible value, $x = 3$ is an extraneous root. The only solution is $x = -3$.

15. Solve each equation.

a) $\frac{x+1}{x-3} = \frac{2x}{x+2}$

Non-permissible values: $x = 3$ and $x = -2$

Common denominator: $(x-3)(x+2)$

$$\cancel{(x-3)}(x+2)\left(\frac{x+1}{\cancel{x-3}}\right) = (x-3)\cancel{(x+2)}\left(\frac{2x}{\cancel{x+2}}\right)$$

$$x^2 + 3x + 2 = 2x^2 - 6x$$

$$x^2 - 9x - 2 = 0 \quad \text{Use the quadratic formula.}$$

$$x = \frac{9 \pm \sqrt{(-9)^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{9 \pm \sqrt{89}}{2}$$

Solutions are $x = \frac{9 + \sqrt{89}}{2}$ and $x = \frac{9 - \sqrt{89}}{2}$.

b) $\frac{w^2}{w+4} = \frac{5}{3}$

Non-permissible value: $w = -4$

Common denominator: $3(w+4)$

$$3\cancel{(w+4)}\left(\frac{w^2}{\cancel{w+4}}\right) = \cancel{3}(w+4)\left(\frac{5}{\cancel{3}}\right)$$

$$3w^2 = 5w + 20$$

$$3w^2 - 5w - 20 = 0 \quad \text{Use the quadratic formula.}$$

$$w = \frac{5 \pm \sqrt{(-5)^2 - 4(3)(-20)}}{2(3)}$$

$$w = \frac{5 \pm \sqrt{265}}{6}$$

Solutions are $w = \frac{5 + \sqrt{265}}{6}$ and $w = \frac{5 - \sqrt{265}}{6}$.

c) $\frac{1}{v} + \frac{1}{v-4} = 2$

Non-permissible values: $v = 0$ and $v = 4$

Common denominator: $v(v-4)$

$$\cancel{v}(v-4)\left(\frac{1}{\cancel{v}}\right) + v\cancel{(v-4)}\left(\frac{1}{\cancel{v-4}}\right) = v(v-4)(2)$$

$$v - 4 + v = 2v^2 - 8v$$

$$2v^2 - 10v + 4 = 0$$

$$v^2 - 5v + 2 = 0 \quad \text{Use the quadratic formula.}$$

$$v = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(2)}}{2(1)}$$

$$v = \frac{5 \pm \sqrt{17}}{2}$$

Solutions are $v = \frac{5 + \sqrt{17}}{2}$ and $v = \frac{5 - \sqrt{17}}{2}$.