Lesson 7.6 Exercises, pages 597–602

A Students should verify all solutions.

3. Lyle can fill the bathtub using the cold water tap in 8 min. When both the hot and cold water taps are fully open, he can fill the bathtub in 6 min. How long would it take Lyle to fill the bathtub using only the hot water tap?
An equation that represents this situation is: $\frac{6}{8} + \frac{6}{t} = 1$, where t is the time in minutes required to fill the bathtub using only the hot water tap. Solve the equation to solve the problem.

Non-permissible value: $t = 0$
Common denominator: $8t$

$$\frac{6}{8} + \frac{6}{t} = 1,\ t > 0$$

$$8t\left(\frac{6}{8}\right) + 8t\left(\frac{6}{t}\right) = 8t(1)$$

$$6t + 48 = 8t$$
$$48 = 2t$$
$$t = 24$$

It would take 24 min to fill the bathtub using only the hot water tap.

4. Bronwyn rides her electric bicycle 10 km/h faster than Aaron. Bronwyn can travel 60 km in the same time that it takes Aaron to travel 40 km. Determine Bronwyn's average speed and Aaron's average speed.
An equation that represents this situation is: $\frac{60}{s + 10} = \frac{40}{s}$, where s is Aaron's average speed in kilometres per hour.
Solve the equation to solve the problem.

Non-permissible values: $s = -10$ and $s = 0$
Common denominator: $s(s + 10)$

$$\frac{60}{s + 10} = \frac{40}{s},\ s > 0$$

$$s(s + 10)\left(\frac{60}{s + 10}\right) = s(s + 10)\left(\frac{40}{s}\right)$$

$$60s = 40s + 400$$
$$20s = 400$$
$$s = 20$$

Aaron's average speed is 20 km/h. Bronwyn's average speed is (20 + 10) km/h, or 30 km/h.

B

For questions 5 to 13, write an equation to model each situation. Then solve the equation to solve the problem.

5. It takes a painter 3 h to spray paint a fence. When two people paint the fence, one using a sprayer and the other using a brush, they can paint the fence in 2 h. How long would it take one person to paint the fence using only a brush?

Let t hours represent the time it takes one person to paint the fence using only a brush.
After 2 h, a painter has spray painted $\frac{2}{3}$ of the fence and the person using a brush has painted $\frac{2}{t}$ of the fence.
So, an equation is: $\frac{2}{3} + \frac{2}{t} = 1$
Non-permissible value: $t = 0$
Common denominator: $3t$

$$\frac{2}{3} + \frac{2}{t} = 1,\ t > 0$$

$$3t\left(\frac{2}{3}\right) + 3t\left(\frac{2}{t}\right) = 3t(1)$$

$$2t + 6 = 3t$$

$$t = 6$$

It would take one person 6 h to paint the fence using only a brush.

6. Jenny can clean out the garage in 5 h. When her son helps, they can clean out the garage in 3 h. How long would it take Jenny's son to clean out the garage on his own?

Let t hours represent the time it takes Jenny's son to clean out the garage on his own.
After 3 h, Jenny has cleaned out $\frac{3}{5}$ of the garage and Jenny's son has cleaned out $\frac{3}{t}$ of the garage.
So, an equation is: $\frac{3}{5} + \frac{3}{t} = 1,\ t > 0$
Non-permissible value: $t = 0$
Common denominator: $5t$

$$\frac{3}{5} + \frac{3}{t} = 1$$

$$5t\left(\frac{3}{5}\right) + 5t\left(\frac{3}{t}\right) = 5t(1)$$

$$3t + 15 = 5t$$

$$2t = 15$$

$$t = 7.5$$

It would take Jenny's son 7.5 h to clean out the garage on his own.

7. How much bleach should be added to 47 L of water to make a solution that is 6% bleach?

Let the volume of bleach added be v litres.
Then, total volume of the solution is $(v + 47)$ litres.

$$\frac{\text{volume of bleach}}{\text{total volume}} = \frac{6}{100}$$

$$\frac{v}{v + 47} = \frac{6}{100},\ v > 0$$

$v = -47$ is a non-permissible value.
A common denominator is: $100(v + 47)$

$$100\cancel{(v + 47)}\left(\frac{v}{\cancel{v + 47}}\right) = \cancel{100}(v + 47)\left(\frac{6}{\cancel{100}}\right)$$

$$100v = 6v + 282$$
$$94v = 282$$
$$v = 3$$

To create a solution that is 6% bleach, 3 L of bleach should be added to 47 L of water.

8. A boat travels 4 km upstream in the same time that it takes the boat to travel 10 km downstream. The average speed of the current is 3 km/h. What is the average speed of the boat in still water?

Let the average speed of the boat in still water be s kilometres per hour.
Average speed downstream: $(s + 3)$ km/h
Distance downstream: 10 km
Time downstream: $\frac{10}{s + 3}$ hours
Average speed upstream: $(s - 3)$ km/h
Distance upstream: 4 km
Time upstream: $\frac{4}{s - 3}$ hours
It takes the same time to travel upstream as it does to travel downstream.
So, an equation is: $\frac{10}{s + 3} = \frac{4}{s - 3},\ s > 3$
$s = 3$ and $s = -3$ are non-permissible values.
A common denominator is: $(s + 3)(s - 3)$

$$\cancel{(s + 3)}(s - 3)\left(\frac{10}{\cancel{s + 3}}\right) = (s + 3)\cancel{(s - 3)}\left(\frac{4}{\cancel{s - 3}}\right)$$

$$10s - 30 = 4s + 12$$
$$6s = 42$$
$$s = 7$$

The average speed of the boat in still water is 7 km/h.

9. A natural number is 4 more than another natural number. When the reciprocal of the greater number is subtracted from the reciprocal of the lesser number, the difference is $\frac{1}{15}$. What are the two numbers?

Let one natural number be x. Then the other natural number is $x + 4$.
The reciprocal of the lesser number is: $\frac{1}{x}$

The reciprocal of the greater number is: $\frac{1}{x + 4}$
Their difference is: $\frac{1}{15}$
So, an equation is: $\frac{1}{x} - \frac{1}{x + 4} = \frac{1}{15}, x \in \mathbb{N}$

$x = -4$ and $x = 0$ are non-permissible values.
A common denominator is: $15(x)(x + 4)$

$$15(\cancel{x})(x + 4)\left(\frac{1}{\cancel{x}}\right) - 15(x)\cancel{(x + 4)}\left(\frac{1}{\cancel{x + 4}}\right) = \cancel{15}(x)(x + 4)\left(\frac{1}{\cancel{15}}\right)$$

$$15x + 60 - 15x = x^2 + 4x$$
$$x^2 + 4x - 60 = 0$$
$$(x + 10)(x - 6) = 0$$

$x = -10$ or $x = 6$
Since $x \in \mathbb{N}$, $x = -10$ is not a solution.
So, the natural numbers are 6 and $6 + 4$, or 10.

10. It takes Marcy's apprentice 9 h longer to build a deck than it takes Marcy, an experienced carpenter. When they work together, they can build the deck in 20 h. How long would it take each person to build the deck working alone?

Let t hours represent the time it takes Marcy to build a deck.
Then, the time it takes Marcy's apprentice is $(t + 9)$ hours.
After 20 h, Marcy has built $\frac{20}{t}$ of the deck and Marcy's apprentice has built $\frac{20}{t + 9}$ of the deck.
So, an equation is: $\frac{20}{t} + \frac{20}{t + 9} = 1, t > 0$
Non-permissible values: $t = 0$ and $t = -9$
Common denominator: $t(t + 9)$

$$\frac{20}{t} + \frac{20}{t + 9} = 1$$

$$\cancel{t}(t + 9)\left(\frac{20}{\cancel{t}}\right) + t\cancel{(t + 9)}\left(\frac{20}{\cancel{t + 9}}\right) = t(t + 9)(1)$$

$$20t + 180 + 20t = t^2 + 9t$$
$$t^2 - 31t - 180 = 0$$
$$(t - 36)(t + 5) = 0$$

$t = 36$ or $t = -5$
Since time cannot be negative, $t = 36$
It would take Marcy 36 h to build the deck and it would take Marcy's apprentice 36 h + 9 h, or 45 h to build the deck.

11. The average speed of an airplane is 10 times that of a car. It takes the airplane 18 h less than the car to travel 1000 km. Determine the average speeds of the airplane and the car.

Let the average speed of the car be s kilometres per hour.
Then the average speed of the airplane is $10s$ kilometres per hour.
Distance: 1000 km
Time for car: $\frac{1000}{s}$ hours
Time for airplane: $\frac{1000}{10s}$ hours
It takes the airplane 18 h less than the car to travel this distance.
So, an equation is: $\frac{1000}{s} - \frac{1000}{10s} = 18, s > 0$
Non-permissible value: $s = 0$
Common denominator: $10s$

$$\frac{1000}{s} - \frac{1000}{10s} = 18$$

$$10\cancel{s}\left(\frac{1000}{\cancel{s}}\right) - \cancel{10s}\left(\frac{1000}{\cancel{10s}}\right) = 10s(18)$$

$$10\,000 - 1000 = 180s$$

$$9000 = 180s$$

$$s = 50$$

The average speed of the car is 50 km/h and the average speed of the airplane is 10(50 km/h), or 500 km/h.

12. Ann cycles 6 km to return a friend's bicycle. She then walks home. Her total time for the trip is 90 min. Ann cycles four times as fast as she walks. Determine Ann's average speeds for walking and for cycling.

Let Ann's average walking speed be s kilometres per hour.
Then her average cycling speed is $4s$ kilometres per hour.
Distance: 6 km
Time cycling: $\frac{6}{s}$ hours
Time walking: $\frac{6}{4s}$ hours
Total time taken is 90 min, or 1.5 h.
So, an equation is: $\frac{6}{s} + \frac{6}{4s} = 1.5, s > 0$
Non-permissible value: $s = 0$
Common denominator: $4s$

$$\frac{6}{s} + \frac{6}{4s} = 1.5$$

$$4\cancel{s}\left(\frac{6}{\cancel{s}}\right) + \cancel{4s}\left(\frac{6}{\cancel{4s}}\right) = 4s(1.5)$$

$$24 + 6 = 6s$$

$$30 = 6s$$

$$s = 5$$

Ann's average walking speed is 5 km/h and her average cycling speed is 4(5 km/h), or 20 km/h.

C

13. Henry's average running speed is 1 km/h greater than Brandon's. In a 10-km practice race for Footstock in Alberta, Brandon finished 2 min behind Henry. Determine the average running speed of each person.

Let Brandon's average running speed be s kilometres per hour.
Then Henry's average running speed is $(s + 1)$ kilometres per hour.
Distance: 10 km
Brandon's time: $\frac{10}{s}$ hours
Henry's time: $\frac{10}{s + 1}$ hours
It took Brandon 2 min, or $\frac{1}{30}$ h longer to finish the race.
So, an equation is: $\frac{10}{s} - \frac{1}{30} = \frac{10}{s + 1}$, $s > 0$
Non-permissible values: $s = 0$ and $s = -1$
Common denominator: $30s(s + 1)$

$$\frac{10}{s} - \frac{1}{30} = \frac{10}{s + 1}$$

$$30\cancel{s}(s + 1)\left(\frac{10}{\cancel{s}}\right) - \cancel{30}s(s + 1)\left(\frac{1}{\cancel{30}}\right) = 30s\cancel{(s + 1)}\left(\frac{10}{\cancel{s + 1}}\right)$$

$$300s + 300 - s^2 - s = 300s$$

$$s^2 + s - 300 = 0$$ Use the quadratic formula.

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Substitute: $a = 1, b = 1, c = -300$

$$s = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-300)}}{2(1)}$$

$$s = \frac{-1 \pm \sqrt{1201}}{2}$$

$s \doteq 16.8$ or $s \doteq -17.8$
Since speed cannot be negative, $s \doteq 16.8$.
Brandon's average running speed is approximately 16.8 km/h and Henry's average running speed is approximately $(16.8 + 1)$ km/h, or 17.8 km/h.