Lesson 8.2 Exercises, pages 638–646

A Students should verify the solutions to all equations.

4. Which values of x are not roots of each equation?

a) $|2x - 3| = 7$ $\quad x = 5$ or $x = -2$

Use mental math.
$x = 5$: L.S. $= 7$ R.S. $= 7$
$x = -2$: L.S. $= 7$ R.S. $= 7$
So, both $x = 5$ and $x = -2$ are roots.

b) $|-4x + 6| = 8$ $\quad x = -0.5$ or $x = 3$

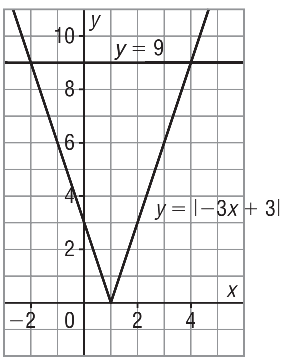
Use mental math.
$x = -0.5$: L.S. $= 8$ R.S. $= 8$
$x = 3$: L.S. $= 6$ R.S. $= 8$
So, $x = 3$ is not a root of the equation.

c) $|x^2 - 3x + 4| = 8$ $\quad x = 4$ or $x = -0.75$

Use a calculator.
$x = 4$: L.S. $= 8$ R.S. $= 8$
$x = -0.75$: L.S. $= 6.8125$ R.S. $= 8$
So, $x = -0.75$ is not a root of the equation.

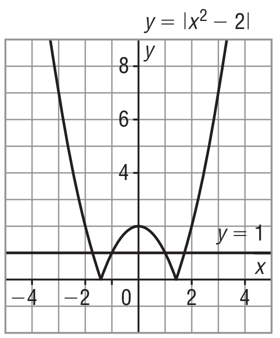
5. Use the graphs to determine the approximate solutions of each equation. Where necessary, give the solutions to the nearest tenth.

a) $|-3x + 3| = 9$



The line $y = 9$ intersects $y = |-3x + 3|$ at 2 points: $(-2, 9)$ and $(4, 9)$. So, the solutions are $x = -2$ and $x = 4$.

b) $|x^2 - 2| = 1$

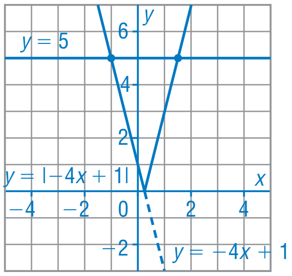


The line $y = 1$ intersects $y = |x^2 - 2|$ at 4 points, which appear to be: $(-1.7, 1)$, $(-1, 1)$, $(1, 1)$, and $(1.7, 1)$. So, the solutions are $x \doteq -1.7$, $x = -1$, $x = 1$, and $x \doteq 1.7$.

B

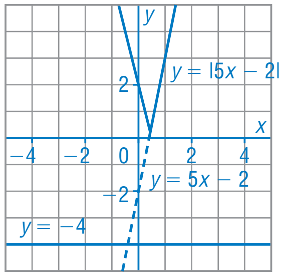
6. Solve by graphing.

a) $5 = |-4x + 1|$



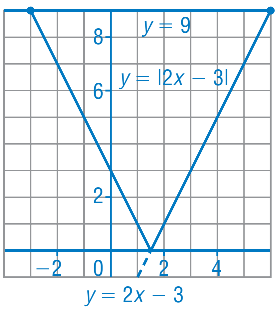
To graph $y = |-4x + 1|$, graph $y = -4x + 1$, then reflect, in the x-axis, the part of the graph that is below the x-axis. The line $y = 5$ intersects $y = |-4x + 1|$ at $(-1, 5)$ and $(1.5, 5)$. So, the solutions are $x = -1$ and $x = 1.5$.

b) $-4 = |5x - 2|$



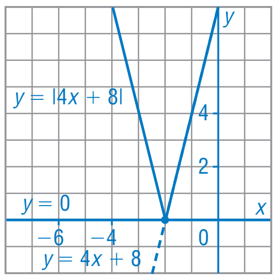
To graph $y = |5x - 2|$, graph $y = 5x - 2$, then reflect, in the x-axis, the part of the graph that is below the x-axis. The line $y = -4$ does not intersect $y = |5x - 2|$. So, the equation has no solution.

c) $|2x - 3| = 9$



To graph $y = |2x - 3|$, graph $y = 2x - 3$, then reflect, in the x-axis, the part of the graph that is below the x-axis. The line $y = 9$ intersects $y = |2x - 3|$ at $(-3, 9)$ and $(6, 9)$. So, the solutions are $x = -3$ and $x = 6$.

d) $|4x + 8| = 0$



To graph $y = |4x + 8|$, graph $y = 4x + 8$, then reflect, in the x-axis, the part of the graph that is below the x-axis. The line $y = 0$ intersects $y = |4x + 8|$ at $(-2, 0)$. So, the solution is $x = -2$.

7. Solve by graphing. Where necessary, give the solutions to the nearest tenth.

a) $|x^2 - 2x + 1| = 2$

Enter $y = |x^2 - 2x + 1|$ and $y = 2$ in the graphing calculator. The line $y = 2$ appears to intersect $y = |x^2 - 2x + 1|$ at 2 points: $(-0.4142\ldots, 2)$ and $(2.4142\ldots, 2)$.
So, the equation has 2 solutions: $x \doteq -0.4$ and $x \doteq 2.4$.

b) $4 = |2x^2 + 3x - 4|$

Enter $y = |2x^2 + 3x - 4|$ and $y = 4$ in the graphing calculator. The line $y = 4$ appears to intersect $y = |2x^2 + 3x - 4|$ at 4 points: $(-2.8860\ldots, 4)$, $(-1.5, 4)$, $(0, 4)$, and $(1.3860\ldots, 4)$. So, the equation has 4 solutions: $x \doteq -2.9$, $x = -1.5$, $x = 0$, and $x \doteq 1.4$.

c) $6 = |2x^2 + 7x + 3|$

Enter $y = |2x^2 + 7x + 3|$ and $y = 6$ in the graphing calculator. The line $y = 6$ appears to intersect $y = |2x^2 + 7x + 3|$ at 2 points: $(-3.8860\ldots, 6)$ and $(0.3860\ldots, 6)$.
So, the equation has 2 solutions: $x \doteq -3.9$ and $x \doteq 0.4$.

d) $|-x^2 + 2x - 4| = 2$

Enter $y = |-x^2 + 2x - 4|$ and $y = 2$ in the graphing calculator. The line $y = 2$ does not intersect $y = |-x^2 + 2x - 4|$. So, the equation has no solution.

8. Use algebra to solve each equation.

a) $|4x + 3| = 2$

$4x + 3 = 2$
if $4x + 3 \geq 0$
that is, if $x \geq -\frac{3}{4}$

When $x \geq -\frac{3}{4}$:

$$4x + 3 = 2$$
$$4x = -1$$
$$x = -\frac{1}{4}$$

$-\frac{1}{4} \geq -\frac{3}{4}$, so this root is a solution.

$-(4x + 3) = 2$
if $4x + 3 < 0$
that is, if $x < -\frac{3}{4}$

When $x < -\frac{3}{4}$:

$$-(4x + 3) = 2$$
$$4x + 3 = -2$$
$$4x = -5$$
$$x = -\frac{5}{4}$$

$-\frac{5}{4} < -\frac{3}{4}$, so this root is a solution.

The solutions are $x = -\frac{1}{4}$ and $x = -\frac{5}{4}$.

b) $3 = |2x + 5|$

$2x + 5 = 3$
if $2x + 5 \geq 0$
that is, if $x \geq -\frac{5}{2}$

When $x \geq -\frac{5}{2}$:

$$\begin{aligned} 2x + 5 &= 3 \\ 2x &= -2 \\ x &= -1 \end{aligned}$$

$-1 \geq -\frac{5}{2}$, so this root is a solution.

$-(2x + 5) = 2$
if $2x + 5 < 0$
that is, if $x < -\frac{5}{2}$

When $x < -\frac{5}{2}$:

$$\begin{aligned} -(2x + 5) &= 3 \\ 2x + 5 &= -3 \\ 2x &= -8 \\ x &= -4 \end{aligned}$$

$-4 < -\frac{5}{2}$, so this root is a solution.

The solutions are $x = -1$ and $x = -4$.

c) $2 = |x^2 + 4x + 5|$

When $x^2 + 4x + 5 \geq 0$:

$$\begin{aligned} x^2 + 4x + 5 &= 2 \\ x^2 + 4x + 3 &= 0 \\ (x + 1)(x + 3) &= 0 \end{aligned}$$

$x = -1$ or $x = -3$

When $x^2 + 4x + 5 < 0$:

$$\begin{aligned} -(x^2 + 4x + 5) &= 2 \\ x^2 + 4x + 5 &= -2 \\ x^2 + 4x + 7 &= 0 \end{aligned}$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(7)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{-12}}{2}$$

This is not a real number.

So, $x = -1$ and $x = -3$ are the solutions.

d) $|x^2 - 6x + 5| = 5$

When $x^2 - 6x + 5 \geq 0$:

$$\begin{aligned} x^2 - 6x + 5 &= 5 \\ x^2 - 6x &= 0 \\ x(x - 6) &= 0 \end{aligned}$$

$x = 0$ or $x = 6$

When $x^2 - 6x + 5 < 0$:

$$\begin{aligned} -(x^2 - 6x + 5) &= 5 \\ x^2 - 6x + 5 &= -5 \\ x^2 - 6x + 10 &= 0 \end{aligned}$$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(10)}}{2(1)}$$

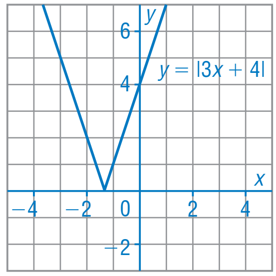
$$x = \frac{6 \pm \sqrt{-4}}{2}$$

This is not a real number.

So, $x = 0$ and $x = 6$ are the solutions.

9. For which values of c does the equation $|3x + 4| = c$ have:

a) 2 solutions?



This is the graph of $y = |3x + 4|$.
For the equation to have 2 solutions, the line $y = c$ must intersect the graph of $y = |3x + 4|$ at 2 points; that is, $c > 0$.

b) 1 solution?

For the equation to have 1 solution, the line $y = c$ must intersect the graph of $y = |3x + 4|$ at 1 point; that is, $c = 0$.

c) no solution?

For the equation to have no solution, the line $y = c$ must not intersect the graph of $y = |3x + 4|$; that is, $c < 0$.

10. A manufacturer rejects 275-g boxes of crackers when the actual mass of the box differs from the stated mass by more than 3.5 g.

a) Write an absolute value equation that can be used to determine the greatest and least masses that are acceptable.

Let m grams represent the mass of a box of crackers.
So, an equation is: $|m - 275| = 3.5$

b) Solve the equation. What is the least mass that is acceptable? What is the greatest mass?

When $m - 275 \geq 0$:
$$m - 275 = 3.5$$
$$m = 278.5$$

When $m - 275 < 0$:
$$-(m - 275) = 3.5$$
$$m - 275 = -3.5$$
$$m = 271.5$$

So, the least acceptable mass is 271.5 g and the greatest mass is 278.5 g.

11. Use algebra to solve each equation.

a) $2|2x + 1| = 7 + x$

$|2x + 1| = \frac{1}{2}(7 + x)$

$2x + 1 = \frac{1}{2}(7 + x)$ if $2x + 1 \geq 0$, that is, if $x \geq -\frac{1}{2}$

$-(2x + 1) = \frac{1}{2}(7 + x)$ if $2x + 1 < 0$, that is, if $x < -\frac{1}{2}$

When $x \geq -\frac{1}{2}$:

$$2x + 1 = \frac{1}{2}(7 + x)$$
$$4x + 2 = 7 + x$$
$$3x = 5$$
$$x = \frac{5}{3}$$

$\frac{5}{3} \geq -\frac{1}{2}$, so this root is a solution.

When $x < -\frac{1}{2}$:

$$-(2x + 1) = \frac{1}{2}(7 + x)$$
$$-4x - 2 = 7 + x$$
$$-9 = 5x$$
$$x = -\frac{9}{5}$$

$-\frac{9}{5} < -\frac{1}{2}$, so this root is a solution.

The solutions are $x = \frac{5}{3}$ and $x = -\frac{9}{5}$.

b) $-x + 11 = |x^2 - 11x|$

When $x^2 - 11x \geq 0$:

$$-x + 11 = x^2 - 11x$$
$$0 = x^2 - 10x - 11$$
$$0 = (x - 11)(x + 1)$$

$x = 11$ or $x = -1$

When $x^2 - 11x < 0$:

$$-x + 11 = -(x^2 - 11x)$$
$$-x + 11 = -x^2 + 11x$$
$$x^2 - 12x + 11 = 0$$
$$(x - 11)(x - 1) = 0$$

$x = 11$ or $x = 1$

So, $x = -1$, $x = 1$, and $x = 11$ are the solutions.

c) $\left|\frac{1}{2}x - \frac{3}{4}\right| = 4$

$0.5x - 0.75 = 4$ if $0.5x - 0.75 \geq 0$, that is, if $x \geq 1.5$

$-(0.5x - 0.75) = 4$ if $0.5x - 0.75 < 0$, that is, if $x < 1.5$

When $x \geq 1.5$:

$$0.5x - 0.75 = 4$$
$$0.5x = 4.75$$
$$x = 9.5$$

$9.5 \geq 1.5$, so this root is a solution.

When $x < 1.5$:

$$-(0.5x - 0.75) = 4$$
$$-0.5x + 0.75 = 4$$
$$-3.25 = 0.5x$$
$$x = -6.5$$

$-6.5 < 1.5$, so this root is a solution.

The solutions are $x = -6.5$ and $x = 9.5$.

d) $3x + 18 = 2|x^2 + 6x|$

When $x^2 + 6x \geq 0$:

$$3x + 18 = 2(x^2 + 6x)$$
$$3x + 18 = 2x^2 + 12x$$
$$2x^2 + 9x - 18 = 0$$
$$(x + 6)(2x - 3) = 0$$
$$x = -6 \text{ or } x = \frac{3}{2}$$

When $x^2 + 6x < 0$:

$$3x + 18 = -2(x^2 + 6x)$$
$$3x + 18 = -2x^2 - 12x$$
$$2x^2 + 15x + 18 = 0$$
$$(x + 6)(2x + 3) = 0$$
$$x = -6 \text{ or } x = -\frac{3}{2}$$

So, $x = -6$, $x = -\frac{3}{2}$, and $x = \frac{3}{2}$ are the solutions.

12. A student solved the equation $|2x^2 + 3x - 1| - 4 = -3$ and reasoned that since the absolute value of an expression cannot be negative, the equation has no solution.
Is the student correct? Explain. If the student is not correct, describe the error the student made and solve the equation.

The student is incorrect. The student should have simplified the equation first by adding 4 to both sides. Then the equation becomes $|2x^2 + 3x - 1| = 1$.

Correct solution:

$$2x^2 + 3x - 1 = 1$$
$$2x^2 + 3x - 2 = 0$$
$$(2x - 1)(x + 2) = 0$$
$$x = \frac{1}{2} \text{ or } x = -2$$

$$-(2x^2 + 3x - 1) = 1$$
$$2x^2 + 3x - 1 = -1$$
$$2x^2 + 3x = 0$$
$$x(2x + 3) = 0$$
$$x = 0 \text{ or } x = -\frac{3}{2}$$

So, $x = -2$, $x = -\frac{3}{2}$, $x = 0$, and $x = \frac{1}{2}$ are the solutions.

13. A car is travelling toward the British Columbia-Alberta border. The car is 150 km from the border and is travelling at an average speed of 100 km/h.

a) Write an absolute value equation to represent the distance, d kilometres, of the car from the border after t hours.

After 1 h, the car has travelled 100 km. After t hours, the car has travelled $100t$ kilometres. So, the distance from the border after t hours is represented by the equation: $d = |150 - 100t|$

b) Determine when the car is 25 km from the border. Explain your strategy.

Substitute: $d = 25$

$25 = |150 - 100t|$

When $150 - 100t \geq 0$:

$25 = 150 - 100t$

$100t = 125$

$t = 1\frac{1}{4}$

When $150 - 100t < 0$:

$25 = -(150 - 100t)$

$-25 = 150 - 100t$

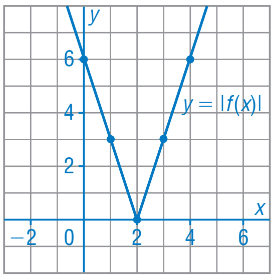
$100t = 175$

$t = 1\frac{3}{4}$

So, the car is 25 km from the border after $1\frac{1}{4}$ h and after $1\frac{3}{4}$ h. The car can be 25 km from the border on the Alberta side or 25 km from the border on the British Columbia side.

C

14. The function $y = f(x)$ is linear. The line $y = 6$ intersects $y = |f(x)|$ at $x = 4$ and $x = 0$.
The line $y = 3$ intersects $y = |f(x)|$ at $x = 1$ and $x = 3$.
What is an equation for the function $y = f(x)$?



$y = |f(x)|$ passes through the points (0, 6), (4, 6), (1, 3), and (3, 3).
Plot the points. The points are symmetrical about the line $x = 2$, so plot a point at (2, 0). Join the points from (2, 0) to (0, 6) and from (2, 0) to (4, 6) with straight lines. An equation for the right branch of the graph has the form $y = mx + b$.

$m = \frac{6 - 3}{4 - 3}$

$m = 3$

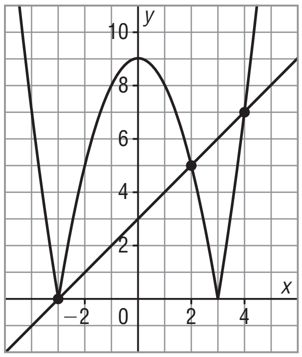
Use: $y = 3x + b$ Substitute: $x = 4$ and $y = 6$

$6 = 3(4) + b$

$b = -6$

So, an equation for the function $y = f(x)$ is $y = 3x - 6$.

15. A student used this graph to solve an absolute value equation. What might the equation have been? Explain your strategy.



The line has x-intercept -3 and y-intercept 3.
An equation of the line has the form $y = mx + 3$.
Use the point $(-3, 0)$. Substitute:
$x = -3$ and $y = 0$
$0 = m(-3) + 3$
$m = 1$
So, an equation of the line is: $y = x + 3$
Assume the middle piece of the graph was reflected in the x-axis.
So, the graph of the quadratic function opens up and has vertex $(0, -9)$.
So, the equation has the form $y = ax^2 - 9$.
An x-intercept of the graph is 3, so use the point $(3, 0)$.
Substitute $x = 3$ and $y = 0$.
$0 = a(3)^2 - 9$
$0 = 9a - 9$
$a = 1$
An equation for the quadratic function is $y = x^2 - 9$, and an equation for the absolute value function is $y = |x^2 - 9|$.
So, the student might have used the graph to solve the equation:
$x + 3 = |x^2 - 9|$