Lesson 8.4 Math Lab: Assess Your Understanding, pages 671–673

1. Without graphing, predict the number of vertical asymptotes of the graph of each reciprocal function. Identify the equation of each asymptote.

a) $y = \frac{1}{(x + 2)(x - 4)}$

The *x*-intercepts of the related quadratic function are -2 and 4.
There are 2 vertical asymptotes: $x = -2$ and $x = 4$

b) $y = \frac{1}{(-3x + 1)^2}$

The *x*-intercept of the related quadratic function is $\frac{1}{3}$.
There is 1 vertical asymptote: $x = \frac{1}{3}$

c) $y = \frac{1}{x^2}$

The *x*-intercept of the related quadratic function is 0.
There is 1 vertical asymptote: $x = 0$

d) $y = \frac{1}{4x^2 + 3}$

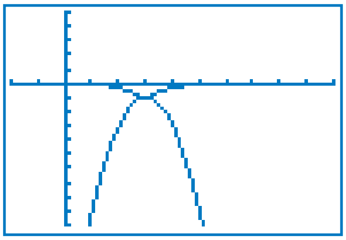
The related quadratic function has no *x*-intercepts.
There are no vertical asymptotes.

2. Look at your answers to question 1. When the equation of a reciprocal quadratic function is given in factored form, how can you tell how many vertical asymptotes its graph will have?

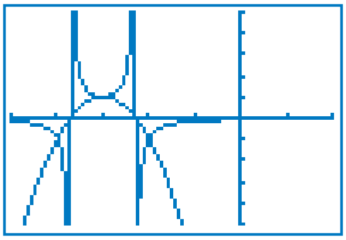
To tell how many vertical asymptotes the graph of a reciprocal quadratic function will have, I look at the expression in the denominator.
When the expression cannot be factored, there are no vertical asymptotes.
When the expression has two identical factors, there is 1 vertical asymptote.
When the expression has two different factors, there are 2 vertical asymptotes.

3. Graph $y = a(x - p)^2 + q$ and $y = \dfrac{1}{a(x - p)^2 + q}$ on the same screen for 6 different sets of values of a, p, and q. Sketch what you see on the screen. How can you use the signs of a, p, and q to determine the number of vertical asymptotes of the graph of the function $y = \dfrac{1}{a(x - p)^2 + q}$?

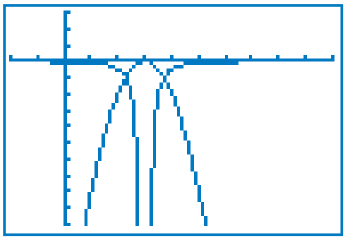
When a is negative, the graph opens down:
If q is also negative, the related quadratic function has no x-intercepts, so there are no vertical asymptotes.
For example, $y = -2(x - 3)^2 - 1$ and $y = \dfrac{1}{-2(x - 3)^2 - 1}$:



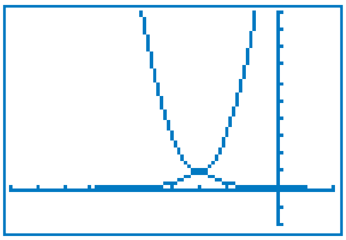
If q is positive, the related quadratic function has 2 x-intercepts, so there are 2 vertical asymptotes. For example, $y = -2(x + 3)^2 + 1$ and $y = \dfrac{1}{-2(x + 3)^2 + 1}$:



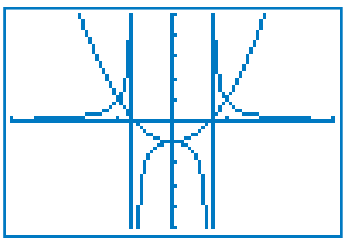
If $q = 0$, the related quadratic function has 1 x-intercept, so there is 1 vertical asymptote. For example, $y = -2(x - 3)^2$ and $y = \dfrac{1}{-2(x - 3)^2}$:



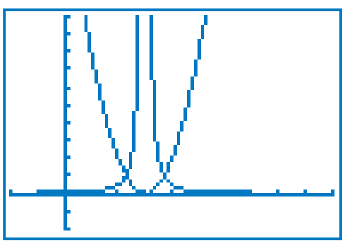
When a is positive, the graph opens up:
If q is also positive, the related quadratic function has no x-intercepts, so there are no vertical asymptotes. For example, $y = 2(x + 3)^2 + 1$ and $y = \dfrac{1}{2(x + 3)^2 + 1}$:



If q is negative, the related quadratic function has 2 x-intercepts, so there are 2 vertical asymptotes. For example, $y = 2x^2 - 1$ and $y = \dfrac{1}{2x^2 - 1}$:



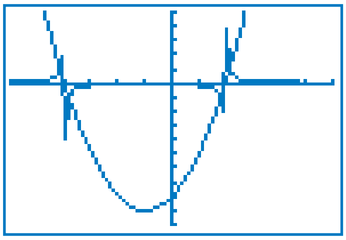
If $q = 0$, the related quadratic function has 1 x-intercept, so there is 1 vertical asymptote. For example, $y = 2(x - 3)^2$ and $y = \dfrac{1}{2(x - 3)^2}$:



4. Predict the vertical asymptotes of the graph of each reciprocal function. Graph to check your predictions.

a) $y = \dfrac{1}{(x + 1)^2 - 9}$

Since the value of a is positive and the value of q is negative, I predict the graph of the reciprocal function will have 2 vertical asymptotes.



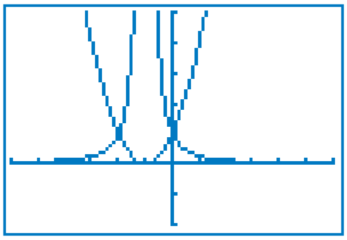
$y = \dfrac{1}{(x + 1)^2 - 9}$ is undefined when

$$(x + 1)^2 - 9 = 0$$
$$(x + 1)^2 = 9$$
$$x + 1 = 3 \text{ or } x + 1 = -3$$
$$x = 2 \qquad x = -4$$

So, the lines $x = 2$ and $x = -4$ are vertical asymptotes.
The graph shows my prediction is correct.

b) $y = \dfrac{1}{(x + 1)^2}$

Since the value of a is positive and the value of q is 0, I predict the graph of the reciprocal function will have 1 vertical asymptote.



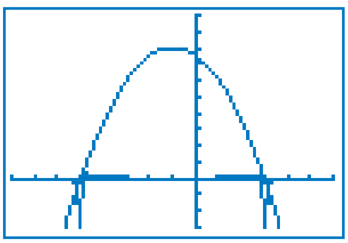
$y = \dfrac{1}{(x + 1)^2}$ is undefined when

$$(x + 1)^2 = 0$$
$$x + 1 = 0$$
$$x = -1$$

So, the line $x = -1$ is a vertical asymptote.
The graph shows my prediction is correct.

c) $y = \dfrac{1}{-(x + 1)^2 + 16}$

Since the value of a is negative and the value of q is positive, I predict the graph of the reciprocal function will have 2 vertical asymptotes.



$y = \dfrac{1}{-(x + 1)^2 + 16}$ is undefined when

$$-(x + 1)^2 + 16 = 0$$
$$-(x + 1)^2 = -16$$
$$(x + 1)^2 = 16$$
$$x + 1 = 4 \text{ or } x + 1 = -4$$
$$x = 3 \qquad x = -5$$

So, the lines $x = 3$ and $x = -5$ are vertical asymptotes.
The graph shows my prediction is correct.