Checkpoint: Assess Your Understanding, pages 648–651

8.1

1. **Multiple Choice** Which statement about $y = -4x + 8$ and $y = |-4x + 8|$ is false?

   A. Both functions have the same x-intercept.

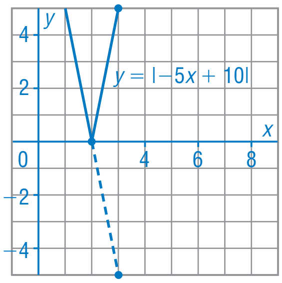
   B. Both functions have the same y-intercept.

   C. Both functions have the same domain.

   D. Both functions have the same range.

2. Sketch a graph of each absolute function.
Identify the intercepts, domain, and range.

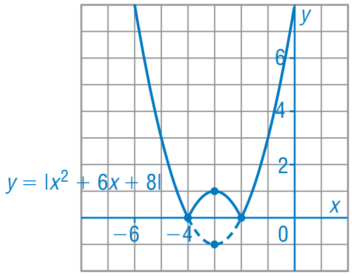
a) $y = |-5x + 10|$



Draw the graph of
$y = -5x + 10$.
It has x-intercept 2 and
y-intercept 10.
Reflect, in the x-axis,
the part of the graph that is
below the x-axis.

The x-intercept is 2 and the
y-intercept is 10. The domain
of $y = |-5x + 10|$ is $x \in \mathbb{R}$,
and the range is $y \geq 0$.

b) $y = |x^2 + 6x + 8|$



Draw the graph of: $y = x^2 + 6x + 8$
$y = (x + 2)(x + 4)$
The graph opens up and has x-intercepts
-4 and -2. The axis of symmetry is
$x = \frac{-4 - 2}{2}$, or -3 and the vertex
is at $(-3, -1)$. Reflect, in the x-axis,
the part of the graph that is below the
x-axis.

From the graph, the x-intercepts are
-4 and -2, and the y-intercept is 8.
The domain of $y = |x^2 + 6x + 8|$ is
$x \in \mathbb{R}$ and the range is $y \geq 0$.

3. Write each absolute value function in piecewise notation.

a) $y = |2x - 7|$

$y = 2x - 7$ when
$2x - 7 \geq 0$
$x \geq \frac{7}{2}$
$y = -(2x - 7)$,
or $y = -2x + 7$ when
$2x - 7 < 0$
$x < \frac{7}{2}$
So, using piecewise notation:

$$y = \begin{cases} 2x - 7, \text{ if } x \geq \frac{7}{2} \\ -2x + 7, \text{ if } x < \frac{7}{2} \end{cases}$$

b) $y = |(x + 4)^2 - 1|$

Determine the x-intercepts of
the graph of $y = (x + 4)^2 - 1$.
$0 = (x + 4)^2 - 1$
$(x + 4)^2 = 1$
$x = -3$ or $x = -5$
The graph opens up, so between
the x-intercepts, the graph is below
the x-axis.
For the graph of $y = (x + 4)^2 - 1$:
For $x \leq -5$ or $x \geq -3$, the value of
$(x + 4)^2 - 1 \geq 0$
For $-5 < x < -3$, the value of
$(x + 4)^2 - 1 < 0$;
that is, $y = -((x + 4)^2 - 1)$, or
$y = -(x + 4)^2 + 1$
So, using piecewise notation:

$$y = \begin{cases} (x + 4)^2 - 1, \text{ if } x \leq -5 \text{ or } x \geq -3 \\ -(x + 4)^2 + 1, \text{ if } -5 < x < -3 \end{cases}$$

8.2

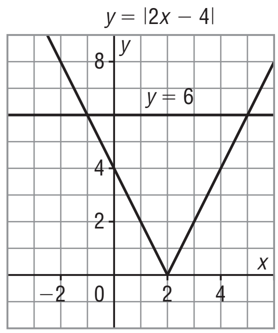
4. Multiple Choice How many solutions does the equation $|x^2 + x - 9| = 6$ have?

A. 1 solution **B.** 2 solutions **C.** 3 solutions **D.** 4 solutions

D

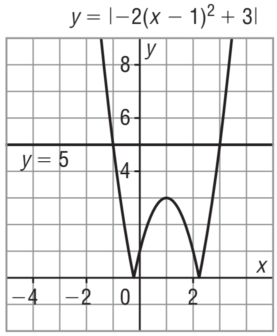
5. Use the graphs to determine the solutions of each equation.

a) $|2x - 4| = 6$



The line $y = 6$ intersects $y = |2x - 4|$ at 2 points: $(-1, 6)$ and $(5, 6)$. So, the solutions are $x = -1$ and $x = 5$.

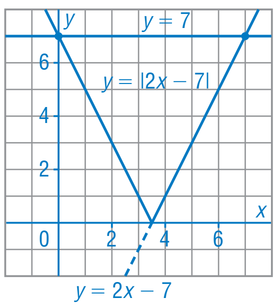
b) $5 = |-2(x - 1)^2 + 3|$



The line $y = 5$ intersects $y = |-2(x - 1)^2 + 3|$ at 2 points: $(-1, 5)$ and $(3, 5)$. So, the solutions are $x = -1$ and $x = 3$.

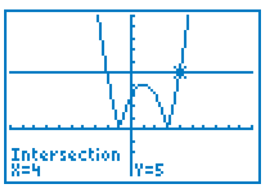
6. Solve by graphing.

a) $7 = |2x - 7|$



To graph $y = |2x - 7|$, graph $y = 2x - 7$, then reflect, in the x-axis, the part of the graph that is below the x-axis. The line $y = 7$ intersects $y = |2x - 7|$ at $(0, 7)$ and $(7, 7)$. So, the solutions are $x = 0$ and $x = 7$.

b) $|(x - 1)^2 - 4| = 5$



Enter $y = |(x - 1)^2 - 4|$ and $y = 5$ in the graphing calculator. The line $y = 5$ intersects $y = |(x - 1)^2 - 4|$ at 2 points: $(-2, 5)$ and $(4, 5)$. So, the equation has 2 solutions: $x = -2$ and $x = 4$.

7. Use algebra to solve each equation.

a) $9 = |-2x + 6|$

$-2x + 6 = 9$
if $-2x + 6 \geq 0$
that is, if $x \leq 3$
When $x \leq 3$:

$$-2x + 6 = 9$$
$$-2x = 3$$
$$x = -\frac{3}{2}, \text{ or } -1.5$$

$-1.5 \leq 3$, so this root is a solution.

$-(-2x + 6) = 9$
if $-2x + 6 < 0$
that is, if $x > 3$
When $x > 3$:

$$-(-2x + 6) = 9$$
$$-2x + 6 = -9$$
$$-2x = -15$$
$$x = \frac{15}{2}, \text{ or } 7.5$$

$7.5 > 3$, so this root is a solution.

The solutions are $x = -1.5$ and $x = 7.5$.

b) $|x^2 - 4x - 5| = 7$

When $x^2 - 4x - 5 \geq 0$:

$$x^2 - 4x - 5 = 7$$
$$x^2 - 4x - 12 = 0$$
$$(x - 6)(x + 2) = 0$$
$$x = 6 \text{ or } x = -2$$

When $x^2 - 4x - 5 < 0$:

$$-(x^2 - 4x - 5) = 7$$
$$-x^2 + 4x + 5 = 7$$
$$-x^2 + 4x - 2 = 0$$
$$x^2 - 4x + 2 = 0$$
$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$
$$x = \frac{4 \pm \sqrt{8}}{2}$$
$$x = \frac{4 \pm 2\sqrt{2}}{2}$$
$$x = 2 \pm \sqrt{2}$$

So, $x = 6$, $x = -2$, $x = 2 + \sqrt{2}$, and $x = 2 - \sqrt{2}$ are the solutions.