

CUMULATIVE REVIEW Chapters 1-8, pages 701–712

1

1. Identify this series as arithmetic or geometric, then determine its sum.

$$4 + 2.5 + 1 + \dots - 32$$

The series is arithmetic because each term is 1.5 less than the preceding term. Use $t_n = t_1 + d(n - 1)$ to determine n .

Substitute: $t_n = -32$, $t_1 = 4$, $d = -1.5$

$$-32 = 4 - 1.5(n - 1)$$

$$36 = 1.5(n - 1)$$

$$24 = n - 1$$

$$n = 25$$

Use: $S_n = \frac{n(t_1 + t_n)}{2}$ Substitute: $n = 25$, $t_1 = 4$, $t_n = -32$

$$S_{25} = \frac{25(4 - 32)}{2}$$

$$S_{25} = -350$$

2

2. Solve each equation. Verify the roots.

a) $\sqrt{5x + 1} = \sqrt{3x + 8}$

$$x \geq -\frac{1}{5}; x \geq -\frac{8}{3}$$

$$\text{so, } x \geq -\frac{1}{5}$$

$$5x + 1 = 3x + 8$$

$$2x = 7$$

$$x = 3.5$$

Verify. L.S. = R.S., so

the root is $x = 3.5$.

b) $4 - 5\sqrt{2x} = 8 - 3\sqrt{2x}$

$$x \geq 0$$

$$-4 = 2\sqrt{2x}$$

The square root cannot be negative, so there is no solution.

3

3. Solve each quadratic equation.

a) $(x - 2)(2x - 3) = 4$

$$2x^2 - 3x - 4x + 6 - 4 = 0$$

$$2x^2 - 7x + 2 = 0$$

Substitute:

$$a = 2, b = -7, c = 2$$

in:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(2)(2)}}{2(2)}$$

$$x = \frac{7 \pm \sqrt{33}}{4}$$

b) $x(4x + 1) = 4(5 - 2x^2)$

$$4x^2 + x = 20 - 8x^2$$

$$12x^2 + x - 20 = 0$$

Use decomposition.

$$12x^2 + 16x - 15x - 20 = 0$$

$$4x(3x + 4) - 5(3x + 4) = 0$$

$$(3x + 4)(4x - 5) = 0$$

Either $3x + 4 = 0$

$$x = -\frac{4}{3}$$

Or $4x - 5 = 0$

$$x = \frac{5}{4}$$

4

4. An outdoor theatre sells 800 tickets for a show at \$26 per ticket. A survey indicates that if the ticket price is decreased, the number of tickets sold will increase by 50 for each \$1 decrease.
What ticket price will maximize the revenue?
What is the maximum revenue?

Write a function.

For each \$1 decrease in price, 50 more tickets will be sold.

Let x represent the number of \$1 decreases in price.

Price of a Ticket (\$)	Number of Tickets Sold	Revenue (\$)
26	800	$26(800) = 20\,800$
$26 - (1) = 25$	$800 + 50(1) = 850$	$25(850) = 21\,250$
$26 - (2) = 24$	$800 + 50(2) = 900$	$24(900) = 21\,600$
$26 - x$	$800 + 50x$	$(26 - x)(800 + 50x)$

Let the revenue be R dollars.

A function is: $R = (26 - x)(800 + 50x)$

Use a graphing calculator to graph this function.

Use the CALC feature to determine the coordinates of the vertex: (5, 22 050)

The ticket price, in dollars, is: $26 - 5 = 21$

The maximum revenue is: \$22 050

5

5. Graph each inequality.

a) $4x^2 + 3x \leq 10$

Solve: $4x^2 + 3x = 10$

$$4x^2 + 3x - 10 = 0$$

$$(4x - 5)(x + 2) = 0$$

$$x = 1.25 \text{ or } x = -2$$

When $x \leq -2$, such as $x = -3$, L.S. = 27; R.S. = 10; so $x = -3$ does not satisfy the inequality.

When $-2 \leq x \leq 1.25$, such as $x = 0$, L.S. = 0; R.S. = 10; so $x = 0$ does satisfy the inequality.

The solution is: $-2 \leq x \leq 1.25$, $x \in \mathbb{R}$



b) $y > -x^2 + 4x - 1$

Graph the related quadratic function:

$$y = -x^2 + 4x - 1$$

$$y = -(x^2 - 4x + 4 - 4) - 1$$

$$y = -(x - 2)^2 + 4 - 1$$

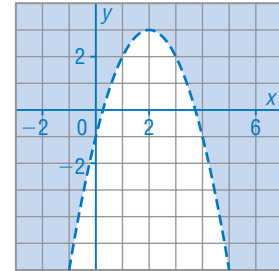
$$y = -(x - 2)^2 + 3$$

The graph opens down, is congruent

to $y = -x^2$, and has vertex (2, 3).

The curve is broken; shade the region above the curve.

Graph of $y > -x^2 + 4x - 1$



6

6. An aircraft is 120 km from a radar antenna, in a direction E35°N. To the nearest kilometre, how far is the aircraft due east and due north of the antenna?

Sketch a diagram.

The distance due east is the x-coordinate of A.

$$x = r \cos \theta \quad \text{Substitute: } r = 120, \theta = 35^\circ$$

$$x = 120 \cos 35^\circ$$

$$x = 98.2982 \dots$$

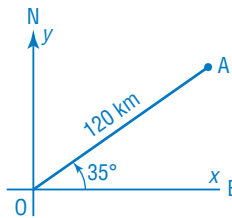
The distance due north is the y-coordinate of A.

$$y = r \sin \theta \quad \text{Substitute: } r = 120, \theta = 35^\circ$$

$$y = 120 \sin 35^\circ$$

$$y = 68.8291 \dots$$

The aircraft is approximately 98 km east and 69 km north.



7. To the nearest degree, what values of α satisfy each equation for $0^\circ \leq \alpha \leq 360^\circ$?

a) $\tan \alpha = \frac{9}{7}$

b) $\cos \alpha = -0.3$

In Quadrant 1, $\alpha = \tan^{-1}\left(\frac{9}{7}\right)$

$$\alpha = 52.1250 \dots^\circ$$

$$\alpha \doteq 52^\circ$$

$\tan \alpha$ is also positive in

Quadrant 3, where

α is approximately:

$$180^\circ + 52^\circ = 232^\circ$$

In Quadrant 1, $\alpha = \cos^{-1}(0.3)$

$$\alpha = 72.5243 \dots^\circ$$

$$\alpha \doteq 73^\circ$$

$\cos \alpha$ is negative in

Quadrants 2 and 3, so

α is approximately: $180^\circ - 73^\circ = 107^\circ$

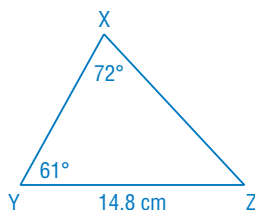
α is approximately: $180^\circ + 73^\circ = 253^\circ$

8. Solve each triangle. Give the angle measures to the nearest degree and the side lengths to the nearest tenth of a centimetre.

a) In $\triangle XYZ$, $\angle X = 72^\circ$, $YZ = 14.8$ cm, and $\angle Y = 61^\circ$

Sketch a diagram. Since 2 angles are given, only 1 triangle is possible.

$$\begin{aligned}\angle Z &= 180^\circ - (72^\circ + 61^\circ) \\ &= 47^\circ\end{aligned}$$



To determine XY

$$\text{Use: } \frac{z}{\sin Z} = \frac{x}{\sin X}$$

Substitute:

$$\angle Z = 47^\circ, \angle X = 72^\circ,$$

$$x = 14.8$$

$$\frac{z}{\sin 47^\circ} = \frac{14.8}{\sin 72^\circ}$$

$$z = \frac{14.8 \sin 47^\circ}{\sin 72^\circ}$$

$$z = 11.3810 \dots$$

XY is approximately

11.4 cm.

To determine XZ

$$\text{Use: } \frac{y}{\sin Y} = \frac{x}{\sin X}$$

Substitute:

$$\angle Y = 61^\circ, \angle X = 72^\circ,$$

$$x = 14.8$$

$$\frac{y}{\sin 61^\circ} = \frac{14.8}{\sin 72^\circ}$$

$$y = \frac{14.8 \sin 61^\circ}{\sin 72^\circ}$$

$$y = 13.6105 \dots$$

XZ is approximately

13.6 cm.

b) In $\triangle UVW$, $\angle W = 48^\circ$, $VW = 12.4$ cm, and $UW = 11.7$ cm

Sketch a diagram. Since 2 sides and the contained angle are given, only 1 triangle is possible.

To determine UV, use:

$$w^2 = u^2 + v^2 - 2uv \cos W$$

$$\text{Substitute: } u = 12.4, v = 11.7, \angle W = 48^\circ$$

$$w^2 = 12.4^2 + 11.7^2 - 2(12.4)(11.7) \cos 48^\circ$$

$$w = \sqrt{12.4^2 + 11.7^2 - 2(12.4)(11.7) \cos 48^\circ}$$

$$w = 9.8231 \dots$$

So, $UV \doteq 9.8$ cm

To determine $\angle V$, use:

$$\frac{\sin V}{v} = \frac{\sin W}{w}$$

$$\text{Substitute: } \angle W = 48^\circ, v = 11.7, w = 9.8231 \dots$$

$$\frac{\sin V}{11.7} = \frac{\sin 48^\circ}{9.8231 \dots}$$

$$\sin V = \frac{11.7 \sin 48^\circ}{9.8231 \dots}$$

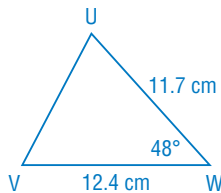
$$\angle V = \sin^{-1}\left(\frac{11.7 \sin 48^\circ}{9.8231 \dots}\right)$$

$$\angle V = 62.2674 \dots^\circ$$

$$\angle V \doteq 62^\circ$$

And, $\angle U \doteq 180^\circ - (62^\circ + 48^\circ)$

$$\doteq 70^\circ$$



9. From the top of a 50-m observation tower, a fire ranger observes smoke in two locations. One is on a bearing of 040° with an angle of depression of 8° , and the other is on a bearing of 205° with an angle of depression of 13° . To the nearest metre, how far apart are the sources of smoke?

Sketch a diagram.

The smoke is at points A and B.

The distance between the sources of smoke is AB.

In $\triangle TRA$, $\angle R$ is: $90^\circ - 13^\circ = 77^\circ$

$$\tan 77^\circ = \frac{AT}{50}$$

$$AT = 50 \tan 77^\circ$$

In $\triangle TRB$, $\angle R$ is: $90^\circ - 8^\circ = 82^\circ$

$$\tan 82^\circ = \frac{BT}{50}$$

$$BT = 50 \tan 82^\circ$$

In $\triangle TAB$, $\angle T$ is: $205^\circ - 40^\circ = 165^\circ$

Use: $t^2 = a^2 + b^2 - 2ab \cos T$

Substitute: $a = 50 \tan 82^\circ$, $b = 50 \tan 77^\circ$, $\angle T = 165^\circ$

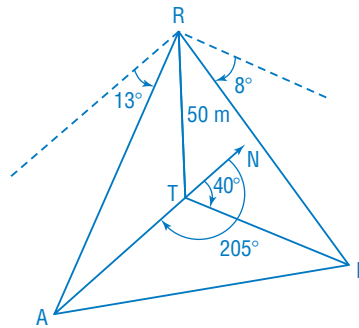
$$t^2 = (50 \tan 82^\circ)^2 + (50 \tan 77^\circ)^2 - 2(50 \tan 82^\circ)(50 \tan 77^\circ) \cos 165^\circ$$

$$t^2 = 50^2 [(\tan 82^\circ)^2 + (\tan 77^\circ)^2 - 2(\tan 82^\circ)(\tan 77^\circ) \cos 165^\circ]$$

$$t = 50 \sqrt{(\tan 82^\circ)^2 + (\tan 77^\circ)^2 - 2(\tan 82^\circ)(\tan 77^\circ) \cos 165^\circ}$$

$$t = 567.7365 \dots$$

The sources of smoke are approximately 568 m apart.



The diagram is not drawn to scale.

7

10. Determine the non-permissible values for each rational expression, then simplify it.

a) $\frac{16a^3b^4}{-8a^5b}$

$$= \frac{\overset{2}{\cancel{16}} a^{\overset{3}{\cancel{3}}-5} b^{\overset{4}{\cancel{4}}-1}}{-1 \overset{1}{\cancel{8}} a^{\overset{5}{\cancel{5}}} b}$$

$$= \frac{-2b^3}{a^2}, a, b \neq 0$$

b) $\frac{2x^2 - 7x - 15}{6x^2 + 13x + 6}$

$$= \frac{\overset{2}{\cancel{2x+3}}(x-5)}{\overset{2}{\cancel{2x+3}}(3x+2)}$$

$$= \frac{x-5}{3x+2}, x \neq -\frac{3}{2}, -\frac{2}{3}$$

11. Simplify each expression.

$$\begin{aligned} \text{a) } & \frac{6a^2 - 26a - 20}{a^2 - 12a + 35} \cdot \frac{2a^2 - 9a - 35}{3a^2 + 20a + 12} \\ &= \frac{2(\cancel{3a+2})(\cancel{a-5})}{(\cancel{a-5})(\cancel{a-7})} \cdot \frac{(2a+5)(\cancel{a-7})}{(\cancel{3a+2})(a+6)} \\ &= \frac{2(2a+5)}{a+6}, a \neq -6, -\frac{2}{3}, 5, 7 \end{aligned}$$

$$\begin{aligned} \text{b) } & \frac{8y^2 - 20y - 48}{6y^2 - 7y - 3} \div \frac{24y^2 - 88y - 32}{4y^2 - 12y + 9} \\ &= \frac{4(2y^2 - 5y - 12)}{(2y-3)(3y+1)} \div \frac{8(3y^2 - 11y - 4)}{(2y-3)(2y-3)} \\ &= \frac{4(2y+3)(y-4)}{(2y-3)(3y+1)} \div \frac{8(3y+1)(y-4)}{(2y-3)(2y-3)} \\ &= \frac{\cancel{4}(2y+3)(\cancel{y-4})}{(\cancel{2y-3})(3y+1)} \cdot \frac{(\cancel{2y-3})(2y-3)}{\cancel{2} \cdot 8(3y+1)(\cancel{y-4})} \\ &= \frac{(2y+3)(2y-3)}{2(3y+1)^2}, y \neq -\frac{1}{3}, \frac{3}{2}, 4 \end{aligned}$$

12. Simplify.

$$\text{a) } \frac{9a}{7b^2} + \frac{3a^2b}{4ab^2} - \frac{8b^4}{3ab^3}$$

Simplify each rational expression before adding or subtracting.

$$\begin{aligned} & \frac{9a}{7b^2} + \frac{3\cancel{a}^2\cancel{b}}{4\cancel{a}b^2} - \frac{8b^4}{3a\cancel{b}^3} \\ &= \frac{9a}{7b^2} \cdot \frac{12a}{12a} + \frac{3a}{4b} \cdot \frac{21ab}{21ab} - \frac{8b}{3a} \cdot \frac{28b^2}{28b^2} \\ &= \frac{108a^2 + 63a^2b - 224b^3}{84ab^2}, a, b \neq 0 \end{aligned}$$

$$\begin{aligned} \text{b) } & \frac{x^2 - 1}{x^2 + 7x + 6} + \frac{x^2 + 5x - 14}{x^2 - 2x - 3} \\ &= \frac{x^2 - 1}{(x+1)(x+6)} + \frac{x^2 + 5x - 14}{(x+1)(x-3)} \\ &= \frac{(x^2 - 1)}{(x+1)(x+6)} \cdot \frac{(x-3)}{(x-3)} + \frac{(x^2 + 5x - 14)}{(x+1)(x-3)} \cdot \frac{(x+6)}{(x+6)} \\ &= \frac{x^3 - 3x^2 - x + 3 + x^3 + 6x^2 + 5x^2 + 30x - 14x - 84}{(x+1)(x+6)(x-3)} \\ &= \frac{2x^3 + 8x^2 + 15x - 81}{(x+1)(x+6)(x-3)}, x \neq -6, -1, 3 \end{aligned}$$

13. Solve each equation.

a) $\frac{x+3}{x-1} = \frac{8}{2x^2-2x}$

$$\frac{x+3}{x-1} = \frac{8}{2x(x-1)}$$

$$2x(x-1)\left(\frac{x+3}{x-1}\right) = \left(\frac{8}{2x(x-1)}\right)2x(x-1), x \neq 0, 1$$

$$2x^2 + 6x - 8 = 0$$

$$x^2 + 3x - 4 = 0$$

$$(x-1)(x+4) = 0$$

So, $x = 1$ or $x = -4$

Since $x = 1$ is a non-permissible value, the solution is $x = -4$.

b) $\frac{6}{m^2-25} = \frac{18}{2m^2-2m-12}$ Divide by 6.

$$\frac{1}{(m-5)(m+5)} = \frac{3}{2(m-3)(m+2)}, m \neq -5, -2, 3, 5$$

Multiply each side by the common denominator:

$$2(m-5)(m+5)(m-3)(m+2)$$

$$2(m-3)(m+2) = 3(m-5)(m+5)$$

$$2m^2 - 2m - 12 = 3m^2 - 75$$

$$m^2 + 2m - 63 = 0$$

$$(m+9)(m-7) = 0$$

So, $m = -9$ or $m = 7$

14. Write an equation to model the following situation, then solve the equation.

Pump 1 can empty a swimming pool in 12 h. When Pump 2 is used as well, the pool is emptied in 7.5 h. How long would it take for Pump 2 alone to empty the pool?

Let the volume of the pool be V litres and the time it takes Pump 2 to empty the pool be t hours.

The rate at which the pool is emptied by:

Pump 1 is: $\frac{V}{12}$ litres per hour

Pump 2 is: $\frac{V}{t}$ litres per hour

Pump 1 and Pump 2 together: $\frac{V}{7.5}$

An equation is: $\frac{V}{12} + \frac{V}{t} = \frac{V}{7.5}$ Divide by V .

$$\frac{1}{12} + \frac{1}{t} = \frac{1}{7.5} \quad \text{Multiply by } 90t.$$

$$90t\left(\frac{1}{12}\right) + 90t\left(\frac{1}{t}\right) = 90t\left(\frac{1}{7.5}\right)$$

$$7.5t + 90 = 12t$$

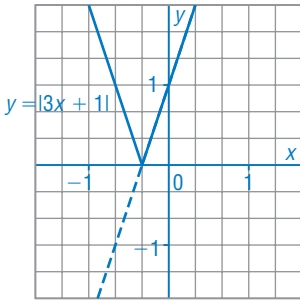
$$4.5t = 90$$

$$t = 20$$

Pump 2 alone can empty the pool in 20 h.

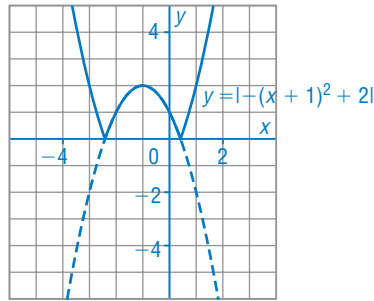
15. Sketch a graph of each absolute value function. Identify the domain and range of the function.

a) $y = |3x + 1|$



Draw the graph of $y = 3x + 1$. It has x -intercept $-\frac{1}{3}$. Reflect, in the x -axis, the part of the graph that is below the x -axis. From the graph, the domain of $y = |3x + 1|$ is $x \in \mathbb{R}$, and the range is $y \geq 0$.

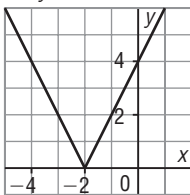
b) $y = |-(x + 1)^2 + 2|$



The graph of $y = -(x + 1)^2 + 2$ opens down with vertex $(-1, 2)$, and is congruent to the graph of $y = -x^2$. Reflect, in the x -axis, the part of the graph that is below the x -axis to get the graph of $y = |-(x + 1)^2 + 2|$. From the graph, the domain of $y = |-(x + 1)^2 + 2|$ is $x \in \mathbb{R}$ and the range is $y \geq 0$.

16. Write each absolute value function in piecewise notation.

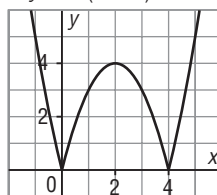
a) $y = |-2x - 4|$



The x -intercept is -2 . So, the graph of $y = |-2x - 4|$ is equal to the graph of $y = -2x - 4$ when $x \leq -2$. And, the graph of $y = |-2x - 4|$ is equal to the graph of $y = 2x + 4$ when $x > -2$. In piecewise notation:

$$y = \begin{cases} -2x - 4, & \text{if } x \leq -2 \\ 2x + 4, & \text{if } x > -2 \end{cases}$$

b) $y = |-(x - 2)^2 + 4|$



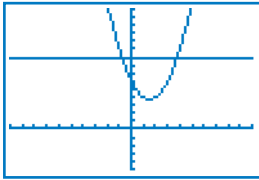
The x -intercepts are 0 and 4. So, the graph of $y = |-(x - 2)^2 + 4|$ is equal to the graph of $y = -(x - 2)^2 + 4$ for $0 \leq x \leq 4$. And, the graph of $y = |-(x - 2)^2 + 4|$ is equal to the graph of $y = (x - 2)^2 - 4$ for $x < 0$ or $x > 4$. In piecewise notation:

$$y = \begin{cases} -(x - 2)^2 + 4, & \text{if } 0 \leq x \leq 4 \\ (x - 2)^2 - 4, & \text{if } x < 0 \text{ or } x > 4 \end{cases}$$

17. Solve by graphing. Give the solutions to the nearest tenth.

$$|-x^2 + 3x - 6| = 9$$

Use a graphing calculator.



From the graph, the solutions are: $x \approx -0.8$ and $x \approx 3.8$
(The solutions can be verified using a calculator.)

18. Use algebra to solve each equation.

a) $2|2x - 1| = 9 - x$ Divide by 2.

$$|2x - 1| = 4.5 - 0.5x$$

Write 2 equations.

If $2x - 1 \geq 0$

then $x \geq 0.5$, and

$$2x - 1 = 4.5 - 0.5x$$

$$2.5x = 5.5$$

$$x = 2.2$$

If $2x - 1 < 0$

then $x < 0.5$, and

$$-(2x - 1) = 4.5 - 0.5x$$

$$1.5x = -3.5$$

$$x = -2.\bar{3}$$

The solutions are: $x = 2.2$ and $x = -2.\bar{3}$

(The solutions can be verified using mental math.)

b) $|x^2 - 7x + 6| = 4$

Write 2 equations.

If $x^2 - 7x + 6 \geq 0$

Then, $x^2 - 7x + 6 = 4$

$$x^2 - 7x + 2 = 0$$

Use: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Substitute:

$$a = 1, b = -7, c = 2$$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{7 \pm \sqrt{41}}{2}$$

(The solutions can be verified using a calculator.)

The solutions are: $x = \frac{7 \pm \sqrt{41}}{2}$, $x = 2$, and $x = 5$

If $x^2 - 7x + 6 < 0$

Then, $-(x^2 - 7x + 6) = 4$

$$x^2 - 7x + 10 = 0$$

$$(x - 2)(x - 5) = 0$$

$$x = 2 \text{ or } x = 5$$

Verify using mental math.

Both roots are solutions.

19. Graph each pair of functions on the same grid. State the equations of the asymptotes, the domain, and the range of each reciprocal function.

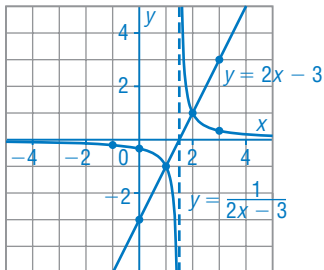
a) $y = 2x - 3$ and $y = \frac{1}{2x - 3}$ b) $y = -x + 2$ and $y = \frac{1}{-x + 2}$

The graph of $y = 2x - 3$ has slope 2 and y-intercept -3 .

The graph of $y = \frac{1}{2x - 3}$ has horizontal asymptote $y = 0$ and vertical asymptote $x = 1.5$.

Points $(1, -1)$ and $(2, 1)$ are common to both graphs. Some points on $y = 2x - 3$ are $(0, -3)$, $(-1, -5)$, and $(3, 3)$.

So, points on $y = \frac{1}{2x - 3}$ are $(0, -0.\bar{3})$, $(-1, -0.2)$, and $(3, 0.\bar{3})$. From the graph, $y = \frac{1}{2x - 3}$ has domain $x \in \mathbb{R}$, $x \neq 1.5$; and range $y \in \mathbb{R}$, $y \neq 0$

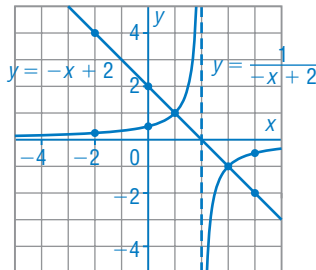


The graph of $y = -x + 2$ has slope -1 and y-intercept 2.

The graph of $y = \frac{1}{-x + 2}$ has horizontal asymptote $y = 0$ and vertical asymptote $x = 2$.

Points $(1, 1)$ and $(3, -1)$ are common to both graphs. Some points on $y = -x + 2$ are $(0, 2)$, $(-2, 4)$, and $(4, -2)$.

So, points on $y = \frac{1}{-x + 2}$ are $(0, 0.5)$, $(-2, 0.25)$, and $(4, -0.5)$. From the graph, $y = \frac{1}{-x + 2}$ has domain $x \in \mathbb{R}$, $x \neq 2$; and range $y \in \mathbb{R}$, $y \neq 0$



20. Determine the equations of the vertical asymptotes of the graph of the reciprocal function $y = \frac{1}{(x + 3)^2 - 4}$.

Vertical asymptotes occur for values of x where y is undefined; that is, when

$$(x + 3)^2 - 4 = 0$$

$$(x + 3)^2 = 4$$

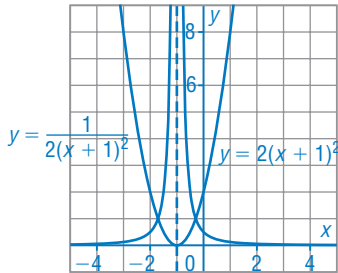
$$x + 3 = \pm 2$$

$$x = -1 \text{ or } x = -5$$

The equations of the vertical asymptotes are $x = -1$ and $x = -5$.

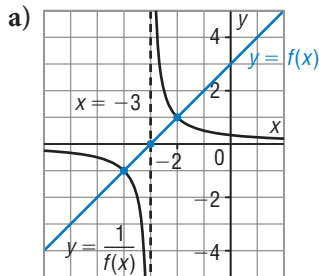
21. Graph these functions on the same grid:

$$y = 2(x + 1)^2 \text{ and } y = \frac{1}{2(x + 1)^2}$$

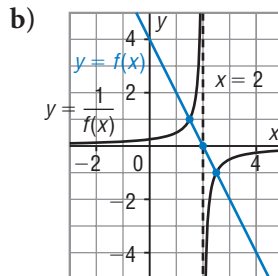


The graph of $y = 2(x + 1)^2$ opens up, has vertex $(-1, 0)$, and is congruent to $y = 2x^2$. The graph of $y = \frac{1}{2(x + 1)^2}$ has vertical asymptote $x = -1$ and horizontal asymptote $y = 0$. Plot points where the line $y = 1$ intersects the graph of $y = 2(x + 1)^2$. These points are common to both graphs. The graph of the reciprocal function has Shape 2.

22. Use each graph of the reciprocal function $y = \frac{1}{f(x)}$ to graph the linear function $y = f(x)$.

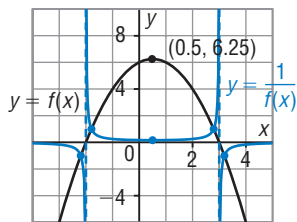


Vertical asymptote is $x = -3$, so the graph of $y = f(x)$ has x -intercept -3 . Mark points at $y = 1$ and $y = -1$ on the graph of $y = \frac{1}{f(x)}$, then draw a line through these points for the graph of $y = f(x)$.



Vertical asymptote is $x = 2$, so the graph of $y = f(x)$ has x -intercept 2 . Mark points at $y = 1$ and $y = -1$ on the graph of $y = \frac{1}{f(x)}$, then draw a line through these points for the graph of $y = f(x)$.

23. Use the graph of the quadratic function $y = f(x)$ to sketch a graph of the reciprocal function $y = \frac{1}{f(x)}$. Identify any vertical asymptotes.



The graph of $y = f(x)$ has two x -intercepts, so the graph of

$y = \frac{1}{f(x)}$ has 2 vertical asymptotes, $x = -2$ and $x = 3$,

and has Shape 3. The horizontal asymptote is $y = 0$.

Plot points where the lines $y = 1$ and $y = -1$ intersect the graph of $y = f(x)$.

These points are common to both graphs. The graph of $y = f(x)$ has vertex

$(0.5, 6.25)$, so point $(0.5, \frac{1}{6.25})$, or $(0.5, 0.16)$ lies on $y = \frac{1}{f(x)}$.