Lesson 1.2 Exercises, pages 20–25

A

3. Write each binomial in the form $x - a$. What is the value of a?

a) $x + 4$

$x + 4 = x - (-4)$
$a = -4$

b) $x - 1$

$x - 1$ is in the form $x - a$.
$a = 1$

c) $11 + x$

$11 + x = x - (-11)$
$a = -11$

d) $-7 + x$

$-7 + x = x - 7$
$a = 7$

4. a) Determine the remainder when $x^3 - 4x^2 - 7x + 10$ is divided by each binomial.

i) $x - 1$

Let $P(x) = x^3 - 4x^2 - 7x + 10$
$P(1) = (1)^3 - 4(1)^2 - 7(1) + 10$
$= 1 - 4 - 7 + 10$
$= 0$
The remainder is 0.

ii) $x + 3$

$P(-3) = (-3)^3 - 4(-3)^2 - 7(-3) + 10$
$= -27 - 36 + 21 + 10$
$= -32$
The remainder is -32.

iii) $x + 2$

$P(-2)$
$= (-2)^3 - 4(-2)^2 - 7(-2) + 10$
$= -8 - 16 + 14 + 10$
$= 0$
The remainder is 0.

iv) $x - 2$

$P(2)$
$= (2)^3 - 4(2)^2 - 7(2) + 10$
$= 8 - 16 - 14 + 10$
$= -12$
The remainder is -12.

b) Which binomials in part a are factors of $x^3 - 4x^2 - 7x + 10$? How do you know?

$x - 1$ and $x + 2$ are factors of $x^3 - 4x^2 - 7x + 10$ because the value of the polynomial when $x = 1$ and when $x = -2$ is 0.

5. Which values of a, $a \in \mathbb{Z}$, should be chosen to test for binomial factors of the form $x - a$ of the polynomial $x^4 + 3x^3 - 8x^2 - 12x + 16$? How did you choose the values?

I chose values of a that are factors of the constant term in the polynomial, 16. Factors of 16 are: 1, −1, 2, −2, 4, −4, 8, −8, 16, −16

B

6. a) Determine the remainder when each polynomial is divided by $x - 2$.

i) $x^2 - 7x + 11$

Let $P(x) = x^2 - 7x + 11$
$P(2) = (2)^2 - 7(2) + 11$
$= 4 - 14 + 11$
$= 1$
The remainder is 1.

ii) $2x^3 - 3x^2 - 6x + 8$

Let $P(x) = 2x^3 - 3x^2 - 6x + 8$
$P(2) = 2(2)^3 - 3(2)^2 - 6(2) + 8$
$= 16 - 12 - 12 + 8$
$= 0$
The remainder is 0.

iii) $3x^3 - 2x^2 - 10x + 6$

Let $P(x) = 3x^3 - 2x^2 - 10x + 6$
$P(2) = 3(2)^3 - 2(2)^2 - 10(2) + 6$
$= 24 - 8 - 20 + 6$
$= 2$
The remainder is 2.

iv) $x^4 - 2x^3 + 3x^2 - 8$

Let $P(x) = x^4 - 2x^3 + 3x^2 - 8$
$P(2) = (2)^4 - 2(2)^3 + 3(2)^2 - 8$
$= 16 - 16 + 12 - 8$
$= 4$
The remainder is 4.

b) Explain the relationship between the remainder when a polynomial $P(x)$ is divided by $x - a$, $a \in \mathbb{Z}$, and $P(a)$.

When a polynomial $P(x)$ is divided by $x - a$, the remainder is $P(a)$.
This result comes from the division statement: $P(x) = (x - a)Q(x) + R$
When $x = a$, $x - a = 0$, so $(x - a)Q(x) = 0$
Then, $P(a) = R$

7. Determine the remainder.

a) $(2x^3 - x^2 + 3x - 2) \div (x - 3)$

Let $P(x) = 2x^3 - x^2 + 3x - 2$
$P(3) = 2(3)^3 - (3)^2 + 3(3) - 2$
$= 54 - 9 + 9 - 2$
$= 52$
The remainder is 52.

b) $(3x^3 - 2x^2 - 4x + 6) \div (x - 2)$

Let $P(x) = 3x^3 - 2x^2 - 4x + 6$
$P(2) = 3(2)^3 - 2(2)^2 - 4(2) + 6$
$= 24 - 8 - 8 + 6$
$= 14$
The remainder is 14.

8. When $2x^3 + kx^2 - 3x + 2$ is divided by $x - 2$, the remainder is 4. Determine the value of k.

Let $P(x) = 2x^3 + kx^2 - 3x + 2$
$P(2) = 2(2)^3 + k(2)^2 - 3(2) + 2$
$= 16 + 4k - 6 + 2$
$= 12 + 4k$
The remainder is 4.
So, $12 + 4k = 4$ Solve for k.
$4k = -8$
$k = -2$
The value of k is -2.

9. Determine one binomial factor of each polynomial.

a) $x^4 + 6x^3 + 5x^2 - 24x - 36$

Sample response:
Let $P(x) = x^4 + 6x^3 + 5x^2 - 24x - 36$
The factors of -36 are: 1, -1, 2, -2, 3, -3, 4, -4, 6, -6, 9, -9, 12, -12, 18, -18, 36, -36
Use mental math to substitute $x = 1$, then $x = -1$ to determine that neither $x - 1$ nor $x + 1$ is a factor.
Try $x = 2$: $P(2) = (2)^4 + 6(2)^3 + 5(2)^2 - 24(2) - 36$
$= 0$
So, $x - 2$ is a factor of $x^4 + 6x^3 + 5x^2 - 24x - 36$.

b) $x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 12$

Sample response:
Let $P(x) = x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 12$
The factors of 12 are: 1, -1, 2, -2, 3, -3, 4, -4, 6, -6, 12, -12
Use mental math to substitute $x = 1$:
$P(1) = 0$
So, $x - 1$ is a factor of $x^5 + 3x^4 - 5x^3 - 15x^2 + 4x + 12$.

10. a) Show that $x + 5$ is a factor of $x^3 + 4x^2 - 11x - 30$.

Let $P(x) = x^3 + 4x^2 - 11x - 30$

$P(-5) = (-5)^3 + 4(-5)^2 - 11(-5) - 30$

$= 0$

The remainder is 0, so $x + 5$ is a factor of $x^3 + 4x^2 - 11x - 30$.

b) Determine the other binomial factors of the polynomial. Verify that the factors are correct.

Divide by $x + 5$ to determine the other factor.

−5	1	4	−11	−30
		−5	5	30
	1	−1	−6	0

So, $x^3 + 4x^2 - 11x - 30 = (x + 5)(x^2 - x - 6)$

Factor the trinomial.

$x^2 - x - 6 = (x + 2)(x - 3)$

So, $x^3 + 4x^2 - 11x - 30 = (x + 2)(x - 3)(x + 5)$

To verify, expand:

$(x + 2)(x - 3)(x + 5) = (x^2 - x - 6)(x + 5)$

$= x^3 + 5x^2 - x^2 - 5x - 6x - 30$

$= x^3 + 4x^2 - 11x - 30$

Since this is the original polynomial, the factors are correct.

11. Fully factor each polynomial.

a) $x^3 + 6x^2 + 3x - 10$

Let $P(x) = x^3 + 6x^2 + 3x - 10$

The factors of −10 are: 1, −1, 2, −2, 5, −5, 10, −10

Use mental math to substitute $x = 1$:

$P(1) = 0$

So, $x - 1$ is a factor.

Divide to determine the other factor.

1	1	6	3	−10
		1	7	10
	1	7	10	0

So, $x^3 + 6x^2 + 3x - 10 = (x - 1)(x^2 + 7x + 10)$

Factor the trinomial: $x^2 + 7x + 10 = (x + 2)(x + 5)$

So, $x^3 + 6x^2 + 3x - 10 = (x - 1)(x + 2)(x + 5)$

b) $x^4 - 5x^2 + 4$

Let $P(x) = x^4 - 5x^2 + 4$
The factors of 4 are: 1, −1, 2, −2, 4, −4
Use mental math to substitute $x = 1$:
$P(1) = 0$; so, $x - 1$ is a factor.
Use mental math to substitute $x = -1$:
$P(-1) = 0$; so, $x + 1$ is a factor.
Try $x = 2$: $P(2) = (2)^4 - 5(2)^2 + 4$
$= 0$
So, $x - 2$ is a factor.
Try $x = -2$: $P(-2) = (-2)^4 - 5(-2)^2 + 4$
$= 0$
So, $x + 2$ is a factor.
Since the original polynomial has degree 4, it can have at most 4 binomial factors.
So, $x^4 - 5x^2 + 4 = (x - 1)(x + 1)(x - 2)(x + 2)$

12. a) What value of b will ensure $x + 3$ is a factor of $bx^3 - 2x^2 + x - 6$?

Let $P(x) = bx^3 - 2x^2 + x - 6$
If $x + 3$ is a factor, $P(-3) = 0$
$P(-3) = b(-3)^3 - 2(-3)^2 + (-3) - 6$
$= -27b - 27$
Let $P(-3) = 0$
$-27b - 27 = 0$
$b = -1$
So, the value of b is −1.

b) What value of d will ensure $x + 2$ is a factor of $3x^5 - dx^4 + 4x^3 - 2dx^2 + x + 10$?

Let $P(x) = 3x^5 - dx^4 + 4x^3 - 2dx^2 + x + 10$
If $x + 2$ is a factor, $P(-2) = 0$
$P(-2) = 3(-2)^5 - d(-2)^4 + 4(-2)^3 - 2d(-2)^2 + (-2) + 10$
$= -120 - 24d$
Let $P(-2) = 0$
$-120 - 24d = 0$
$d = \frac{120}{-24}$, or -5
So, the value of d is −5.

13. Determine whether $x + b$ is a factor of $(x + b)^5 + (x + p)^5 + (b - p)^5$, $b, p \in \mathbb{R}$.

Let $P(x) = (x + b)^5 + (x + p)^5 + (b - p)^5$
If $x + b$ is a factor, $P(-b) = 0$
$P(-b) = (-b + b)^5 + (-b + p)^5 + (b - p)^5$
$= 0 + (-(b - p))^5 + (b - p)^5$
$= 0 - (b - p)^5 + (b - p)^5$
$= 0$
Since the remainder is 0, $(x + b)$ is a factor of $(x + b)^5 + (x + p)^5 + (b - p)^5$, $b, p \in \mathbb{R}$.

C

14. When $mx^3 - 2x^2 + nx - 4$ is divided by $x + 2$, the remainder is 4. When $mx^3 - 2x^2 + nx - 4$ is divided by $x - 1$, the remainder is -11. Determine the values of m and n.

Let $P(x) = mx^3 - 2x^2 + nx - 4$
$P(-2) = 4$
$P(-2) = m(-2)^3 - 2(-2)^2 + n(-2) - 4$
$4 = -8m - 2n - 12$
$0 = -8m - 2n - 16$ ①
$P(1) = -11$
$P(1) = m(1)^3 - 2(1)^2 + n(1) - 4$
$-11 = m + n - 6$
$0 = m + n + 5$ ②
Solve the system of equations:
$0 = -8m - 2n - 16$ ①
$0 = m + n + 5$ ②
Solve equation ② for m: $m = -n - 5$
Substitute for m in equation ①.
$0 = -8m - 2n - 16$ ①
$0 = -8(-n - 5) - 2n - 16$
$0 = 8n + 40 - 2n - 16$
$0 = 6n + 24$
$n = -4$
Substitute $n = -4$ in equation ②.
$0 = m - 4 + 5$
$m = -1$
So, $m = -1$ and $n = -4$

15. Determine each remainder.

a) $(8x^2 - 6x + 3) \div (4x + 1)$

$$\begin{array}{r} 2x - 2 \\ 4x + 1 \overline{)\, 8x^2 - 6x + 3} \\ \underline{8x^2 + 2x} \quad\;\; \\ -8x + 3 \\ \underline{-8x - 2} \\ 5 \end{array}$$

The remainder is 5.

b) $(3x^3 + 2x^2 - 6x - 1) \div (3x + 2)$

$$\begin{array}{r} x^2 \qquad\quad - 2 \\ 3x + 2 \overline{)\, 3x^3 + 2x^2 - 6x - 1} \\ \underline{3x^3 + 2x^2} \qquad\qquad \\ 0 - 6x - 1 \\ \underline{- 6x - 4} \\ 3 \end{array}$$

The remainder is 3.