Lesson 1.5 Exercises, pages 61–67

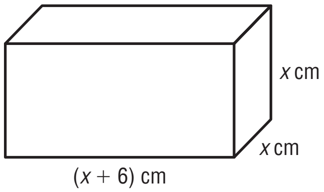
Use technology to graph the functions.

A

3. A box has length $(x + 2)$ units, width $(x - 5)$ units, and height $(2x + 3)$ units. Write a polynomial function to represent its volume, V, in terms of x.

The formula for the volume of a rectangular prism is: $V = lwh$
Substitute: $l = x + 2$, $w = x - 5$, $h = 2x + 3$
So, a polynomial function that represents the volume, V, of the box is:
$V(x) = (x + 2)(x - 5)(2x + 3)$

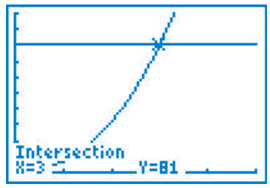
4. a) Write a polynomial function to represent the volume, V, of this square prism in terms of x.



The prism has height x centimetres, width x centimetres, and length $(x + 6)$ centimetres. So, a polynomial function that represents the volume, V, of the prism is:
$V(x) = x(x)(x + 6)$
or, $V(x) = x^2(x + 6)$

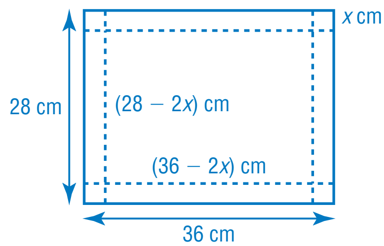
b) Graph the function. Sketch the graph. Use the graph to determine the dimensions of the prism if its volume is 81 cm^3.

Enter the equations $y = x^2(x + 6)$ and $y = 81$ into a graphing calculator. Determine the coordinates of the point of intersection of the graphs: (3, 81). So, the dimensions of the prism are: height 3 cm, width 3 cm, and length $(3 + 6)$ cm, or 9 cm



B

5. A piece of cardboard 36 cm long and 28 cm wide is used to make an open box. Equal squares of side length x centimetres are cut from the corners and the sides are folded up.



a) Write expressions to represent the length, width, and height of the box in terms of x.

Sketch a diagram. The box has height x centimetres, width $(28 - 2x)$ centimetres, and length $(36 - 2x)$ centimetres.

b) Write a polynomial function to represent the volume of the box in terms of x.

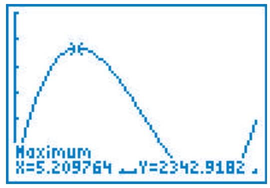
The formula for the volume of the box is: $V = lwh$
So, a polynomial function that represents the volume, V, of the box is:
$V(x) = x(36 - 2x)(28 - 2x)$

c) Graph the function. Sketch the graph. What is the domain?

Enter the equation $y = x(36 - 2x)(28 - 2x)$ into a graphing calculator. The dimensions of the box are positive. The cardboard has width 28 cm. So, the side length of a square cut from each corner must be less than $\frac{28 \text{ cm}}{2}$, or 14 cm. So, the domain is: $0 < x < 14$

d) What is the maximum volume of the box? What is the side length of the square that should be cut out to create a box with this volume? Give your answers to the nearest tenth.

Determine the coordinates of the local maximum point: (5.2097. . . , 2342.9182. . .).
The maximum volume of the box is approximately 2342.9 cm^3. This occurs when each square that is cut out has a side length of approximately 5.2 cm.



6. The volume, in cubic centimetres, of an expandable gift box can be represented by the polynomial function $V(x) = -x^3 + 35x^2 + 200x$. The height of the box in centimetres is $40 - x$. Assume the length is greater than the width.

a) Determine binomial expressions for the length and width of the box in terms of x.

Divide the volume by the height: $(x^3 - 35x^2 - 200x) \div (x - 40)$

$$\begin{array}{r}
x^2 + 5x \\
x - 40 \overline{) x^3 - 35x^2 - 200x} \\
\underline{x^3 - 40x^2} \\
5x^2 - 200x \\
\underline{5x^2 - 200x} \\
0
\end{array}$$

Factor.

$$\begin{aligned}
-x^3 + 35x^2 + 200x &= (-x + 40)(x^2 + 5x) \\
&= x(-x + 40)(x + 5)
\end{aligned}$$

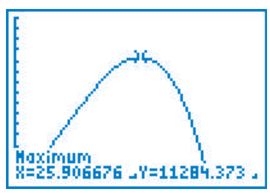
So, the length of the box is $(x + 5)$ centimetres and the width is x centimetres.

b) Graph the function. Sketch the graph. What do the x-intercepts represent?

Enter the equation $y = x(-x + 40)(x + 5)$ into a graphing calculator.
The x-intercepts represent the values of x for which the volume of the box is 0 cm^3 (a box does not exist for these values).

c) To the nearest cubic centimetre, what is the maximum volume of the gift box?

Determine the coordinates of the local maximum point: (25.9066. . . , 11 284.373. . .)
The maximum volume of the box is approximately 11 284 cm^3.



7. Fred and Ted are twins. They were born 3 years after their older sister, Bethany. This year, the product of their three ages is 5726 greater than the sum of their ages. How old are the twins?

Let Bethany's age in years be x.
Then, in years, Fred's age is $x - 3$ and Ted's age is $x - 3$.
The sum of their ages is: $x + (x - 3) + (x - 3) = 3x - 6$
Product of ages $-$ 5726 $=$ sum of ages
So, $x(x - 3)(x - 3) - 5726 = 3x - 6$
$x(x - 3)^2 - 5726 - 3x + 6 = 0$
$x(x - 3)^2 - 5720 - 3x = 0$
Enter the equation $y = x(x - 3)^2 - 5720 - 3x$ into a graphing calculator.
The x-intercept is 20, which is Bethany's age.
So, the twins are 20 $-$ 3, or 17 years old.

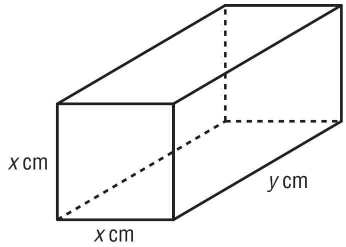
8. Ann, Stan, and Fran are triplets. They were born 4 years before their sister, Kim. This year, the product of their four ages is 49 092 greater than the sum of their ages. How old is Kim?

Let Kim's age in years be x.
Then, in years, the age of each triplet is $x + 4$.
The sum of their ages is: $x + (x + 4) + (x + 4) + (x + 4) = 4x + 12$
Product of ages $-$ 49 092 $=$ sum of ages
So, $x(x + 4)(x + 4)(x + 4) - 49\,092 = 4x + 12$
$x(x + 4)^3 - 49\,092 - 4x - 12 = 0$
$x(x + 4)^3 - 49\,104 - 4x = 0$
Enter the equation $y = x(x + 4)^3 - 49\,104 - 4x$ into a graphing calculator.
Age cannot be negative, so the positive x-intercept, which is 12, represents Kim's age. So, Kim is 12 years old.

9. A carton of juice has dimensions 6.4 cm by 3.8 cm by 10.9 cm. The manufacturer wants to design a box with double the capacity by increasing each dimension by x centimetres. To the nearest tenth of a centimetre, what are the dimensions of the larger carton?

The capacity of the larger carton is: $2(6.4)(3.8)(10.9)$ cm^3 $=$ 530.176 cm^3
The dimensions of the larger carton, in centimetres, are:
$(6.4 + x)$, $(3.8 + x)$, and $(10.9 + x)$
The formula for the volume of a rectangular prism is: $V = lwh$
So, the volume of the larger carton is: $V = (6.4 + x)(3.8 + x)(10.9 + x)$
Substitute: $V = 530.176$
$530.176 = (6.4 + x)(3.8 + x)(10.9 + x)$
Enter $y = (6.4 + x)(3.8 + x)(10.9 + x) - 530.176$ into a graphing calculator.
Determine the x-intercept, which represents the length, in centimetres, by which each dimension is increased.
The x-intercept is approximately 1.6.
So, each dimension is increased by approximately 1.6 cm.
The approximate dimensions of the larger carton are:
(6.4 cm + 1.6 cm) by (3.8 cm + 1.6 cm) by (10.9 cm + 1.6 cm), or 8.0 cm by 5.4 cm by 12.5 cm

10. A package sent by a courier has the shape of a square prism. The sum of the length of the prism and the perimeter of its base is 100 cm.



a) Write a polynomial function to represent the volume V of the package in terms of x.

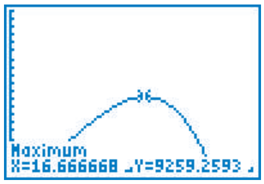
The perimeter of the base, in centimetres, is $4x$.
The length of the prism, in centimetres, is y.
So, $4x + y = 100$ Solve for y.
$y = 100 - 4x$
The formula for the volume of a rectangular prism is: $V = lwh$
So, the volume of the prism is:
$V = x^2y$ Substitute: $y = 100 - 4x$
$V = x^2(100 - 4x)$
$V = 4x^2(25 - x)$
So, a polynomial function that represents the volume of the package is:
$V(x) = 4x^2(25 - x)$

b) Graph the function. Sketch the graph.

Enter the equation $y = 4x^2(25 - x)$ into a graphing calculator.

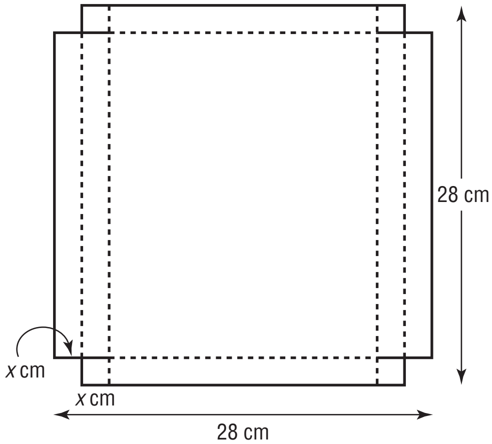
c) To the nearest tenth of a centimetre, what are the dimensions of the package for which its volume is maximized?

Determine the coordinates of the local maximum point:
(16.6666. . . , 9259.2593. . .)
The maximum volume of the package is approximately 9259.3 cm^3.
This occurs when the value of x is 16.6666. . . cm.
So, the side length of the base is approximately 16.7 cm and the length of the package is: 100 cm − 4(16.6666. . . cm) ≐ 33.3 cm



C

11. An open box with locking tabs is to be made from a square piece of cardboard with side length 28 cm. This is done by cutting equal squares of side length x centimetres from the corners and folding along the dotted lines as shown.



a) Write a polynomial function to represent the volume, V, of the box in terms of x.

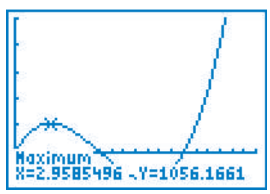
The width of the box, in centimetres, is $(28 - 4x)$.
The length of the box, in centimetres, is $(28 - 2x)$.
The height of the box, in centimetres, is x.
The formula for the volume of a rectangular prism is: $V = lwh$
So, the volume of the box is: $V = x(28 - 4x)(28 - 2x)$
$= 4x(7 - x)(2)(14 - x)$
$= 8x(7 - x)(14 - x)$
A polynomial function that represents the volume of the box is:
$V(x) = 8x(7 - x)(14 - x)$

b) Graph the function. Sketch the graph. State the domain.

Enter the equation $y = 8x(7 - x)(14 - x)$ into a graphing calculator.
The dimensions of the box are positive.
The cardboard has side length 28 cm.
Since each of two sides has 4 small squares removed, the side length of a square cut from each corner must be less than $\frac{28 \text{ cm}}{4}$, or 7 cm.
So, the domain is: $0 < x < 7$

c) To the nearest centimetre, what is the value of x for the box with maximum volume?

Determine the coordinates of the local maximum point:
(2.9585. . ., 1056.1661. . .)
The maximum volume of the box is approximately 1056.2 cm^3.
This occurs when the value of x is approximately 3 cm.



12. A manufacturer designs a cylindrical can with no top. The surface area of the can is 300 cm^2. The can has base radius r centimetres.

a) Write a polynomial function to model the capacity, C cubic centimetres, of the can as a function of r.

The formula for the surface area of a cylinder with no top is:

$SA = \pi r^2 + 2\pi rh$ Substitute: $SA = 300$

$300 = \pi r^2 + 2\pi rh$ Solve for h.

$300 - \pi r^2 = 2\pi rh$

$\dfrac{300 - \pi r^2}{2\pi r} = h$

The formula for the capacity of a cylinder is:

$C = \pi r^2 h$ Substitute: $h = \dfrac{300 - \pi r^2}{2\pi r}$

$C = \cancel{\pi} r^{\cancel{2}}\left(\dfrac{300 - \pi r^2}{2\cancel{\pi}\cancel{r}}\right)$

$C = r\left(\dfrac{300 - \pi r^2}{2}\right)$

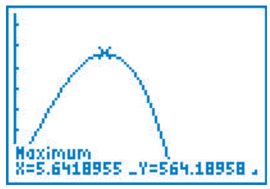
$C = \dfrac{300r - \pi r^3}{2}$

So, a polynomial function that models the capacity of the can is:

$C(r) = \dfrac{300r - \pi r^3}{2}$

b) Graph the function. Sketch the graph. To the nearest tenth of a centimetre, what are the radius and height of the can when it has a maximum capacity?

Enter $y = \dfrac{300x - \pi x^3}{2}$ into a graphing calculator.



To determine the maximum capacity, determine the coordinates of the local maximum point: (5.6418. . ., 564.1895. . .)

The maximum capacity of the can is approximately 564.2 cm^3.

This occurs when the radius of the can is approximately 5.6418. . . cm.

To determine the height of the can, use:

$h = \dfrac{300 - \pi r^2}{2\pi r}$ Substitute: $r = 5.6418\ldots$

$h = \dfrac{300 - \pi(5.6418\ldots)^2}{2\pi(5.6418\ldots)}$

$h = 5.6418\ldots$

So, the approximate dimensions of the can with maximum capacity are: radius 5.6 cm and height 5.6 cm