Checkpoint: Assess Your Understanding, pages 28–31

1.1

1. Multiple Choice When synthetic division is used to divide $x^4 - 7x^2 + 2x + 3$ by $x + 1$, the result is: 1 −1 −6 8 −5
Which is the correct division statement?

A. $x^4 - 7x^2 + 2x + 3 = (x + 1)(x^4 - x^3 - 6x^2 + 8x - 5) + 0$

[✓ **B.**] $x^4 - 7x^2 + 2x + 3 = (x + 1)(x^3 - x^2 - 6x + 8) - 5$

C. $x^4 - 7x^2 + 2x + 3 = (x + 1)(x^3 - x^2 - 6x + 8) + 5$

D. $x^4 - 7x^2 + 2x - 2 = (x + 1)(x^3 - x^2 - 6x + 8)$

2. a) Use long division to divide:

i) $2x^2 - 11x - 19$ by $x - 7$ **ii)** $3x^3 + 4x^2 - 15x + 30$ by $x + 4$

$$\begin{array}{r}
2x + 3 \\
x - 7\overline{)2x^2 - 11x - 19} \\
\underline{2x^2 - 14x} \\
3x - 19 \\
\underline{3x - 21} \\
2
\end{array}$$

The quotient is $2x + 3$ and the remainder is 2.

$$\begin{array}{r}
3x^2 - 8x + 17 \\
x + 4\overline{)3x^3 + 4x^2 - 15x + 30} \\
\underline{3x^3 + 12x^2} \\
-8x^2 - 15x \\
\underline{-8x^2 - 32x} \\
17x + 30 \\
\underline{17x + 68} \\
-38
\end{array}$$

The quotient is $3x^2 - 8x + 17$ and the remainder is -38.

b) Verify the answer to one division in part a.

Sample response:

In part ii, multiply the quotient by the divisor, then add the remainder.

$(x + 4)(3x^2 - 8x + 17) + (-38)$

$= 3x^3 - 8x^2 + 17x + 12x^2 - 32x + 68 - 38$

$= 3x^3 + 4x^2 - 15x + 30$

Since this is the original polynomial, the answer is correct.

3. Use synthetic division to divide $2x^4 - 7x^3 - 29x^2 - 8x + 12$ by each binomial.

a) $x - 1$

$$\begin{array}{r|rrrrr} 1 & 2 & -7 & -29 & -8 & 12 \\ & & 2 & -5 & -34 & -42 \\ \hline & 2 & -5 & -34 & -42 & -30 \end{array}$$

The quotient is $2x^3 - 5x^2 - 34x - 42$ and the remainder is -30.

b) $x - 6$

$$\begin{array}{r|rrrrr} 6 & 2 & -7 & -29 & -8 & 12 \\ & & 12 & 30 & 6 & -12 \\ \hline & 2 & 5 & 1 & -2 & 0 \end{array}$$

The quotient is $2x^3 + 5x^2 + x - 2$ and the remainder is 0.

c) $x + 2$

$$\begin{array}{r|rrrrr} -2 & 2 & -7 & -29 & -8 & 12 \\ & & -4 & 22 & 14 & -12 \\ \hline & 2 & -11 & -7 & 6 & 0 \end{array}$$

The quotient is $2x^3 - 11x^2 - 7x + 6$ and the remainder is 0.

d) $x + 3$

$$\begin{array}{r|rrrrr} -3 & 2 & -7 & -29 & -8 & 12 \\ & & -6 & 39 & -30 & 114 \\ \hline & 2 & -13 & 10 & -38 & 126 \end{array}$$

The quotient is $2x^3 - 13x^2 + 10x - 38$ and the remainder is 126.

1.2

4. **Multiple Choice** When a polynomial $P(x)$ is divided by $x + 3$, the remainder is -4. Which statement is true?

A. $P(-4) = -3$ **B.** $P(3) = -4$

[✓ **C.**] $P(-3) = -4$ **D.** $P(-3) = 0$

5. **a)** Determine the remainder when $3x^4 + 8x^3 - 15x^2 - 32x + 12$ is divided by $x + 1$.

Let $P(x) = 3x^4 + 8x^3 - 15x^2 - 32x + 12$

$$\begin{aligned} P(-1) &= 3(-1)^4 + 8(-1)^3 - 15(-1)^2 - 32(-1) + 12 \\ &= 3 - 8 - 15 + 32 + 12 \\ &= 24 \end{aligned}$$

The remainder is 24.

b) Is $x + 1$ a factor of the polynomial in part a? If your answer is yes, explain how you know. If your answer is no, determine a binomial of the form $x - a$, $a \in \mathbb{Z}$, that is a factor.

$x + 1$ is not a factor of the polynomial because the remainder after dividing by $x + 1$ is not 0.

The factors of the constant term, 12, are: 1, −1, 2, −2, 3, −3, 4, −4, 6, −6, 12, −12. Use mental math to substitute $x = 1$ to determine that $x - 1$ is not a factor.

Try $x = 2$: $$\begin{aligned} P(2) &= 3(2)^4 + 8(2)^3 - 15(2)^2 - 32(2) + 12 \\ &= 48 + 64 - 60 - 64 + 12 \\ &= 0 \end{aligned}$$

So, $x - 2$ is a factor of $3x^4 + 8x^3 - 15x^2 - 32x + 12$.

6. For each polynomial, determine one factor of the form $x - a$, $a \in \mathbb{Z}$.

a) $x^3 - 5x^2 - 17x + 21$

Sample response:

Let $P(x) = x^3 - 5x^2 - 17x + 21$

The factors of 21 are: 1, −1, 3, −3, 7, −7, 21, −21

Try $x = 1$: $$\begin{aligned} P(1) &= (1)^3 - 5(1)^2 - 17(1) + 21 \\ &= 0 \end{aligned}$$

So, $x - 1$ is a factor of $x^3 - 5x^2 - 17x + 21$.

b) $4x^4 - 15x^3 - 32x^2 + 33x + 10$

Sample response:

Let $P(x) = 4x^4 - 15x^3 - 32x^2 + 33x + 10$

The factors of 10 are: 1, −1, 2, −2, 5, −5, 10, −10

Try $x = 1$: $$\begin{aligned} P(1) &= 4(1)^4 - 15(1)^3 - 32(1)^2 + 33(1) + 10 \\ &= 0 \end{aligned}$$

So, $x - 1$ is a factor of $4x^4 - 15x^3 - 32x^2 + 33x + 10$.

7. Factor this polynomial.

$4x^4 - 12x^3 + 3x^2 + 13x - 6$

Let $P(x) = 4x^4 - 12x^3 + 3x^2 + 13x - 6$

The factors of −6 are: 1, −1, 2, −2, 3, −3, 6, −6

Try $x = 1$: $P(1) = 4(1)^4 - 12(1)^3 + 3(1)^2 + 13(1) - 6$

$= 2$

So, $x - 1$ is not a factor.

Try $x = -1$: $P(-1) = 4(-1)^4 - 12(-1)^3 + 3(-1)^2 + 13(-1) - 6$

$= 0$

So, $x + 1$ is a factor.

Divide to determine the other factor.

$$\begin{array}{r|rrrrr} -1 & 4 & -12 & 3 & 13 & -6 \\ & & -4 & 16 & -19 & 6 \\ \hline & 4 & -16 & 19 & -6 & 0 \end{array}$$

So, $4x^4 - 12x^3 + 3x^2 + 13x - 6 = (x + 1)(4x^3 - 16x^2 + 19x - 6)$

Let $P_1(x) = 4x^3 - 16x^2 + 19x - 6$

Try $x = 2$: $P_1(2) = 4(2)^3 - 16(2)^2 + 19(2) - 6$

$= 0$

So, $x - 2$ is a factor.

Divide to determine the other factor.

$$\begin{array}{r|rrrr} 2 & 4 & -16 & 19 & -6 \\ & & 8 & -16 & 6 \\ \hline & 4 & -8 & 3 & 0 \end{array}$$

So, $4x^4 - 12x^3 + 3x^2 + 13x - 6 = (x + 1)(x - 2)(4x^2 - 8x + 3)$

Factor the trinomial: $4x^2 - 8x + 3 = (2x - 1)(2x - 3)$

So, $4x^4 - 12x^3 + 3x^2 + 13x - 6 = (x + 1)(x - 2)(2x - 1)(2x - 3)$