PRACTICE TEST, pages 78–80

1. **Multiple Choice** Which statement is true?

A. When $2x^3 + 4x^2 - 2x - 1$ is divided by $x - 2$, the remainder is 3.

B. The binomial $x + 1$ is a factor of $4x^4 - x^3 - 3x + 2$.

C. When $2x^4 - 7x^3 + 6x^2 - 14x + 20$ is divided by $x - 3$, the remainder is -5.

[✓ **D.**] The binomial $x + 2$ is a factor of $5x^3 + 7x^2 + 12$.

2. **Multiple Choice** Which statement about the graph of a quartic function is false?

A. The graph may open up.

B. The graph may have a zero of multiplicity 3.

[✓ **C.**] The graph may fall to the left and rise to the right.

D. The graph may have a zero of multiplicity 2.

3. Divide $2x^4 + 11x^3 - 10 - 5x + 14x^2$ by $x + 2$.
Write the division statement.

Write the polynomial in descending order: $2x^4 + 11x^3 + 14x^2 - 5x - 10$

$$\begin{array}{r|rrrrr} -2 & 2 & 11 & 14 & -5 & -10 \\ & & -4 & -14 & 0 & 10 \\ \hline & 2 & 7 & 0 & -5 & 0 \end{array}$$

$2x^4 + 11x^3 + 14x^2 - 5x - 10 = (x + 2)(2x^3 + 7x^2 - 5)$

4. Does the polynomial $x^4 - x^3 - 14x^2 + x + 16$ have a factor of $x + 3$? How do you know?

Let $P(x) = x^4 - x^3 - 14x^2 + x + 16$

$P(-3) = (-3)^4 - (-3)^3 - 14(-3)^2 - 3 + 16$

$= 81 + 27 - 126 - 3 + 16$

$= -5$

$P(-3) \neq 0$, so $x + 3$ is not a factor of the polynomial.

5. Factor: $4x^4 - 20x^3 + 17x^2 + 26x - 15$

Let $P(x) = 4x^4 - 20x^3 + 17x^2 + 26x - 15$

The factors of -15 are: 1, -1, 3, -3, 5, -5, 15, -15

Use mental math to determine that $x - 1$ is not a factor and $x + 1$ is a factor. Divide to determine the other factor.

$$\begin{array}{r|rrrrr} -1 & 4 & -20 & 17 & 26 & -15 \\ & & -4 & 24 & -41 & 15 \\ \hline & 4 & -24 & 41 & -15 & 0 \end{array}$$

So, $4x^4 - 20x^3 + 17x^2 + 26x - 15 = (x + 1)(4x^3 - 24x^2 + 41x - 15)$

Let $P_1(x) = 4x^3 - 24x^2 + 41x - 15$

Try $x = 3$: $P_1(3) = 4(3)^3 - 24(3)^2 + 41(3) - 15$

$= 0$

So, $x - 3$ is a factor. Divide to determine the other factor.

$$\begin{array}{r|rrrr} 3 & 4 & -24 & 41 & -15 \\ & & 12 & -36 & 15 \\ \hline & 4 & -12 & 5 & 0 \end{array}$$

Factor the trinomial: $4x^2 - 12x + 5 = (2x - 5)(2x - 1)$

So, $4x^4 - 20x^3 + 17x^2 + 26x - 15 = (x + 1)(x - 3)(2x - 5)(2x - 1)$

6. Sketch the graph of this polynomial function.

$g(x) = 4x^4 + 11x^3 - 7x^2 - 11x + 3$

Factor the polynomial.

The factors of the constant term, 3, are: 1, −1, 3, −3

Use mental math to substitute $x = 1$, then $x = -1$ in $g(x)$ to determine that both $x - 1$ and $x + 1$ are factors.

Divide by $x - 1$.

$$\begin{array}{r|rrrrr} 1 & 4 & 11 & -7 & -11 & 3 \\ & & 4 & 15 & 8 & -3 \\ \hline & 4 & 15 & 8 & -3 & 0 \end{array}$$

So, $4x^4 + 11x^3 - 7x^2 - 11x + 3 = (x - 1)(4x^3 + 15x^2 + 8x - 3)$

Divide $4x^3 + 15x^2 + 8x - 3$ by $x + 1$.

$$\begin{array}{r|rrrr} -1 & 4 & 15 & 8 & -3 \\ & & -4 & -11 & 3 \\ \hline & 4 & 11 & -3 & 0 \end{array}$$

So, $4x^4 + 11x^3 - 7x^2 - 11x + 3$

$= (x - 1)(x + 1)(4x^2 + 11x - 3)$

Factor the trinomial:

$4x^2 + 11x - 3 = (x + 3)(4x - 1)$

So, $4x^4 + 11x^3 - 7x^2 - 11x + 3$

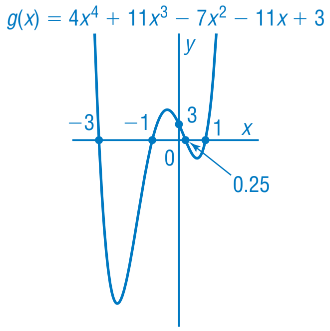
$= (x - 1)(x + 1)(x + 3)(4x - 1)$

The zeros of $g(x)$ are:

1, −1, −3, 0.25

So, the x-intercepts of the graph are:

1, −1, −3, 0.25



Each zero has multiplicity 1, so the graph crosses the x-axis at each x-intercept. The equation has degree 4, so it is an even-degree polynomial function. The leading coefficient is positive, so the graph opens up.

The constant term is 3, so the y-intercept is 3.

7. Canada Post defines a small packet as one for which the sum of its length, width, and height is less than or equal to 90 cm. A company produces several different small packets, each with length 15 cm longer than its height.

a) Write a polynomial function to represent possible volumes of one of these packets in terms of its height x. Assume the sum of the dimensions is maximized.

The sum of the length, width, and height of the packet is 90 cm.
The formula for the volume, V, of the packet is: $V = lwh$
Let x represent the height of the packet, in centimetres.
Then the length, in centimetres, is $x + 15$, and the width, in centimetres, is $90 - (x + x + 15)$, or $75 - 2x$.
So, $V(x) = x(x + 15)(75 - 2x)$

b) Graph the function.

Enter the equation: $y = x(x + 15)(75 - 2x)$ into a graphing calculator.

c) To the nearest cubic centimetre, what is the maximum possible volume of one of these packets? What are its dimensions to the nearest tenth of a centimetre?

Determine the coordinates of the local maximum point:
(23.1124. . . , 25 347.18. . .)
The maximum volume of the packet is approximately 25 347 cm^3.
This occurs when the height of the packet is approximately 23.1 cm.
So, the length is approximately 23.1 cm + 15 cm = 38.1 cm and the width is: 75 cm − 2(23.1124. . . cm) ≑ 28.8 cm.
The maximum volume of one of these packets is approximately 25 347 cm^3. Its dimensions are approximately 38.1 cm by 28.8 cm by 23.1 cm.