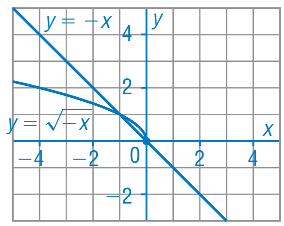
Lesson 2.1 Exercises, pages 90–96

A

4. **a)** Complete the table of values.

x	-4	-2	-1	0	1	2	4
$y = -x$	4	2	1	0	-1	-2	-4
$y = \sqrt{-x}$	2	$\doteq 1.4$	1	0	–	–	–

b) For each function in part a, sketch its graph then state its domain and range.



For $y = -x$: the domain is $x \in \mathbb{R}$; and the range is $y \in \mathbb{R}$.
For $y = \sqrt{-x}$: the domain is $x \leq 0$; and the range is $y \geq 0$.

c) How are the graphs in part b related?

Both graphs pass through $(-1, 1)$ and $(0, 0)$. The graph of $y = \sqrt{-x}$ is above the graph of $y = -x$ between these points. Every value of y for $y = \sqrt{-x}$ is the square root (if it exists) of the corresponding y-value on the graph of $y = -x$.

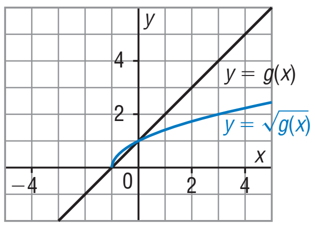
5. For each graph of $y = g(x)$ below:

i) Mark points where $y = 0$ or $y = 1$.

ii) Sketch the graph of $y = \sqrt{g(x)}$.

iii) Identify the domain and range of $y = \sqrt{g(x)}$.

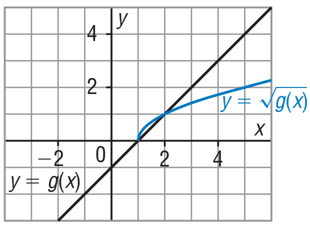
a)



ii) Choose the point $(3, 4)$ on $y = g(x)$. The corresponding point on $y = \sqrt{g(x)}$ is $(3, 2)$. Draw a curve through this point and the marked points.

iii) Domain is: $x \geq -1$
Range is: $y \geq 0$

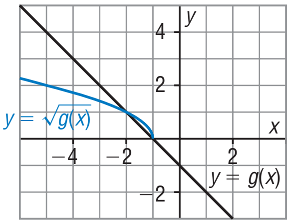
b)



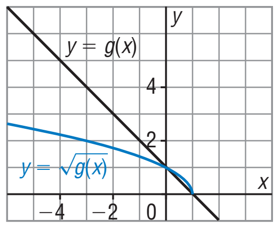
ii) Choose the point $(5, 4)$ on $y = g(x)$. The corresponding point on $y = \sqrt{g(x)}$ is $(5, 2)$. Draw a curve through this point and the marked points.

iii) Domain is: $x \geq 1$
Range is: $y \geq 0$

c)



d)



ii) Choose the point (−5, 4) on $y = g(x)$. The corresponding point on $y = \sqrt{g(x)}$ is (−5, 2). Draw a curve through this point and the marked points.

iii) Domain is: $x \leq -1$
Range is: $y \geq 0$

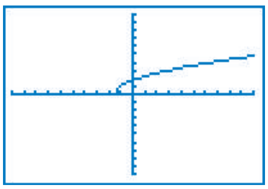
ii) Choose the point (−3, 4) on $y = g(x)$. The corresponding point on $y = \sqrt{g(x)}$ is (−3, 2). Draw a curve through this point and the marked points.

iii) Domain is: $x \leq 1$
Range is: $y \geq 0$

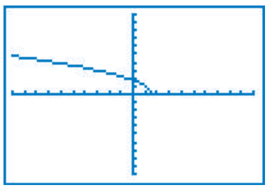
B

6. a) Use technology to graph each function. Sketch each graph.

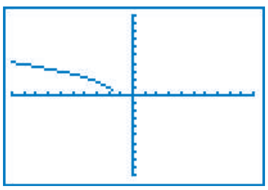
i) $y = \sqrt{2x + 3}$



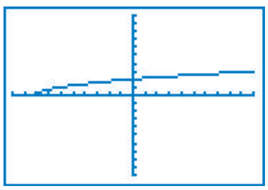
ii) $y = \sqrt{-2x + 3}$



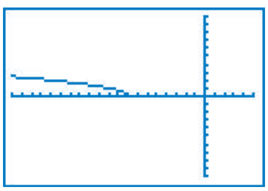
iii) $y = \sqrt{-2x - 3}$



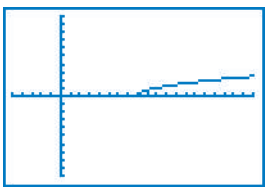
iv) $y = \sqrt{0.5x + 4}$



v) $y = \sqrt{-0.5x - 4}$



vi) $y = \sqrt{0.5x - 4}$



b) Given a linear function of the form $f(x) = ax + b$, $a, b \neq 0$

i) For which values of a does the graph of $y = \sqrt{f(x)}$ open to the right? Use examples to support your answer.

The graph opens to the right when $a > 0$, as in part a, i, iv, and vi.

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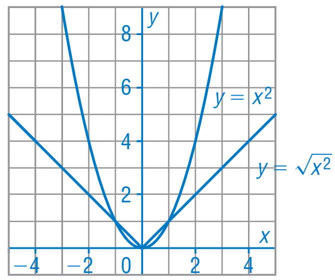
ii) For which values of a does the graph of $y = \sqrt{f(x)}$ open to the left? Use examples to support your answer.

The graph opens to the left when $a < 0$, as in part a, ii, iii, and v.

7. a) Complete this table of values for $y = x^2$ and $y = \sqrt{x^2}$, then graph the functions on the same grid.

x	-3	-2	-1	0	1	2	3
$y = x^2$	9	4	1	0	1	4	9
$y = \sqrt{x^2}$	3	2	1	0	1	2	3



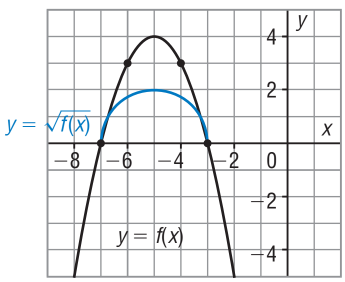
b) What other function describes the graph of $y = \sqrt{x^2}$? Explain why.

The graph of $y = \sqrt{x^2}$ is the same as the graph of $y = |x|$. For any number x, $x^2 = (-x)^2$; both $\sqrt{x^2}$ and $\sqrt{(-x)^2}$ are equal to a positive root, which is $|x|$.

8. a) For the graph of each quadratic function $y = f(x)$ below:

- Sketch the graph of $y = \sqrt{f(x)}$.
- State the domain and range of $y = \sqrt{f(x)}$.

i)

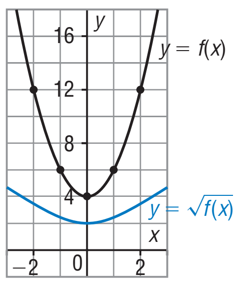


Mark points where $y = 0$ or $y = 1$. Choose, then mark another point on the graph of $y = \sqrt{f(x)}$.

x	$y = f(x)$	$y = \sqrt{f(x)}$
-5	4	2

Join all points with a smooth curve.
Domain is: $-7 \leq x \leq -3$
Range is: $0 \leq y \leq 2$

ii)



There are no points where $y = 0$ or $y = 1$. Choose, then mark other points on the graph of $y = \sqrt{f(x)}$.

x	$y = f(x)$	$y = \sqrt{f(x)}$
-2	12	$\doteq 3.5$
0	4	2
2	12	$\doteq 3.5$

Join all points with a smooth curve.
Domain is: $x \in \mathbb{R}$
Range is: $y \geq 2$

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b) Choose one pair of functions $y = f(x)$ and $y = \sqrt{f(x)}$ from part a. If the domains are different and the ranges are different, explain why.

Sample response: For part a, i, the domains are different because the radical function only exists for those values of x where $y \geq 0$; while the quadratic function exists for all real values of x. The ranges are different because the value of y for the radical function can only be 0 or a positive number; while the range of the quadratic function is all real numbers less than or equal to 4, which includes all negative real numbers.

9. Solve each radical equation by graphing. Give the solution to the nearest tenth where necessary.

a) $x - 5 = 2\sqrt{x + 3}$

Write the equation as:
$x - 5 - 2\sqrt{x + 3} = 0$
Graph the related function:
$f(x) = x - 5 - 2\sqrt{x + 3}$
The zero is: 13
So, the root is: $x = 13$

b) $x = \sqrt{4 - x} + 2$

Write the equation as:
$x - \sqrt{4 - x} - 2 = 0$
Graph the related function:
$f(x) = x - \sqrt{4 - x} - 2$
The zero is: 3
So, the root is: $x = 3$

c) $3\sqrt{2x + 1} = x + 4$

Write the equation as:
$3\sqrt{2x + 1} - x - 4 = 0$
Graph the related function:
$f(x) = 3\sqrt{2x + 1} - x - 4$
The approximate zeros are:
0.75735931 and 9.2426407
So, the roots are: $x \doteq 0.8$ and $x \doteq 9.2$

d) $1 + \sqrt{x - 3} = \sqrt{2x - 6}$

Write the equation as:
$1 + \sqrt{x - 3} - \sqrt{2x - 6} = 0$
Graph the related function:
$f(x) = 1 + \sqrt{x - 3} - \sqrt{2x - 6}$
The approximate zero is:
8.8284271
So, the root is: $x \doteq 8.8$

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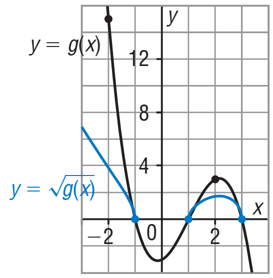
10. For the graph of each cubic function $y = g(x)$ below:

- Sketch the graph of $y = \sqrt{g(x)}$.
- State the domain and range of $y = \sqrt{g(x)}$.

a)

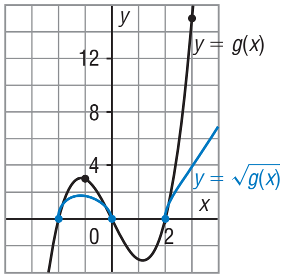


Mark points where $y = 0$ or $y = 1$. Identify and mark the coordinates of other points on the graph of $y = \sqrt{g(x)}$.

x	$y = g(x)$	$y = \sqrt{g(x)}$
-2	15	$\doteq 3.9$
2	3	$\doteq 1.7$

Join the points with 2 smooth curves.
Domain is: $x \leq -1$ or $1 \leq x \leq 3$
Range is: $y \geq 0$

b)



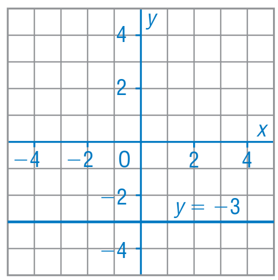
Mark points where $y = 0$ or $y = 1$. Identify and mark the coordinates of other points on the graph of $y = \sqrt{g(x)}$.

x	$y = g(x)$	$y = \sqrt{g(x)}$
-1	3	$\doteq 1.7$
3	15	$\doteq 3.9$

Join the points with 2 smooth curves.
Domain is: $-2 \leq x \leq 0$ or $x \geq 2$
Range is: $y \geq 0$

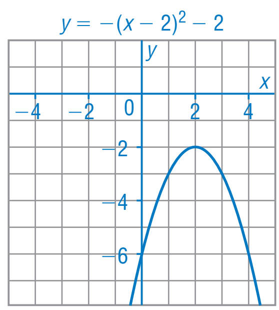
11. **a)** Sketch the graph of a linear function $y = g(x)$ for which $y = \sqrt{g(x)}$ is not defined. Explain how you know that $y = \sqrt{g(x)}$ is not defined.

Sample response: For $y = \sqrt{g(x)}$ to be undefined, the value of $g(x)$ must always be negative; for example, a function is $y = -3$.



b) Sketch the graph of a quadratic function $y = f(x)$ for which $y = \sqrt{f(x)}$ is not defined. Explain how you know that $y = \sqrt{f(x)}$ is not defined.

Sample response: For $y = \sqrt{f(x)}$ to be undefined, the value of $f(x)$ must always be negative; so the graph of $y = f(x)$ must always lie beneath the x-axis; for example, a function is $y = -(x - 2)^2 - 2$.

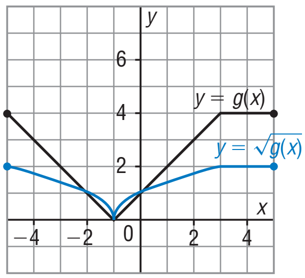


c) For every cubic function $y = g(x)$, the function $y = \sqrt{g(x)}$ exists. Explain why.

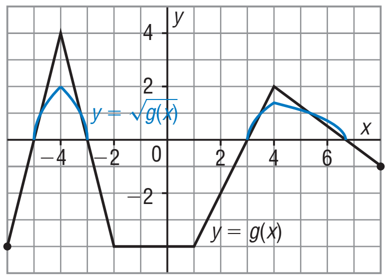
Since the graph of every cubic function either begins in Quadrant 2 and ends in Quadrant 4, or begins in Quadrant 3 and ends in Quadrant 1, there is always part of the graph above the x-axis; that is, the function has positive values, so its square root exists.

12. For each graph of $y = g(x)$, sketch the graph of $y = \sqrt{g(x)}$.

a)



b)



Mark points where $y = 0$ or $y = 1$. Identify and mark the coordinates of other points on the graph of $y = \sqrt{g(x)}$.

x	$y = g(x)$	$y = \sqrt{g(x)}$
−5	4	2
−3	2	≐ 1.4
1	2	≐ 1.4
3	4	2

Join the points with 2 smooth curves, and a line segment.

Mark points where $y = 0$ and $y = 1$. Identify and mark the coordinates of other points on the graph of $y = \sqrt{g(x)}$.

x	$y = g(x)$	$y = \sqrt{g(x)}$
−4	4	2
4	2	≐ 1.4

Join the points with 2 smooth curves.

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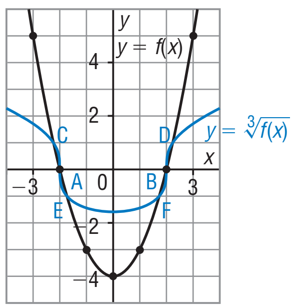
13. When a satellite is h kilometres above Earth, the time for one complete orbit, t minutes, can be calculated using this formula:
$t = 1.66 \times 10^{-4}\sqrt{(h + 6370)^3}$
A communications satellite is to be positioned so that it is always above the same point on Earth's surface. It takes 24 h for this satellite to complete one orbit. What should the height of the satellite be?

Substitute $t = (24)(60)$, or 1440.
$1440 = 1.66 \times 10^{-4}\sqrt{(h + 6370)^3}$
Write the equation with all the terms on one side.
$1440 - 1.66 \times 10^{-4}\sqrt{(h + 6370)^3} = 0$
Write a related function.
$f(h) = 1440 - 1.66 \times 10^{-4}\sqrt{(h + 6370)^3}$
Graph the function, then determine the approximate zero, which is 35 848.513.
So, to the nearest kilometre, the satellite should be 35 849 km high.

C

14. Given the graph of $y = f(x)$, sketch the graph of $y = \sqrt[3]{f(x)}$ without using graphing technology. What are the invariant points on the graph of $y = \sqrt[3]{f(x)}$?



Since $\sqrt[3]{-1} = -1$, $\sqrt[3]{0} = 0$, and $\sqrt[3]{1} = 1$, the invariant points occur where: $y = 0$, label these points A and B; $y = 1$, label these points C and D; and $y = -1$, label these points E and F. Since the cube root of a number between 0 and 1 is greater than the number, the graph of $y = \sqrt[3]{f(x)}$ lies above the graph of $y = f(x)$ between A and C, and between B and D. Since the cube root of a number between 0 and −1 is less than the number, the graph of $y = \sqrt[3]{f(x)}$ lies below the graph of $y = f(x)$ between A and E, and between B and F.
Identify and mark the coordinates of other points.

x	$y = f(x)$	$y = \sqrt[3]{f(x)}$
−3	5	≐ 1.7
0	−4	≐ −1.6
3	5	≐ 1.7

Join the points with a smooth curve.

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