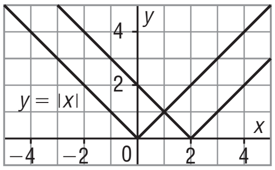
Lesson 3.1 Exercises, pages 169–175

A

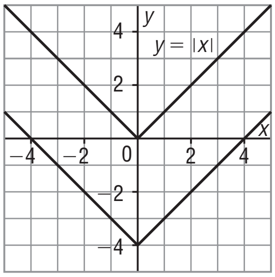
4. On each grid, the graph of $y = |x|$ and its image after a single translation are shown. What was the translation? What is the equation of the image graph?

a)



The graph of $y = |x|$ was translated 2 units right. The image graph has equation $y = |x - 2|$.

b)



The graph of $y = |x|$ was translated 4 units down. The image graph has equation $y + 4 = |x|$.

5. For each equation of a translation image, describe how the graph of $y = f(x)$ was translated.

a) $y = f(x - 7)$

Compare the equation to $y = f(x - h)$: $h = 7$
So, the graph of $y = f(x)$ was translated 7 units right.

b) $y + 5 = f(x)$

Write $y + 5 = f(x)$ as $y - (-5) = f(x)$. Compare the equation to $y - k = f(x)$: $k = -5$
So, the graph of $y = f(x)$ was translated 5 units down.

c) $y = f(x + 6)$

Write $y = f(x + 6)$ as
$y = f(x - (-6))$.
Compare the equation to
$y = f(x - h)$: $h = -6$
So, the graph of $y = f(x)$ was translated 6 units left.

d) $y - 4 = f(x)$

Compare the equation to
$y - k = f(x)$: $k = 4$
So, the graph of $y = f(x)$ was translated 4 units up.

6. The graph of $y = g(x)$ is translated as described below. Write the equation of each translation image in terms of the function g.

a) a translation of 3 units right

The translation is horizontal, so the equation of the image graph has the form $y = g(x - h)$. The translation is 3 units right, so $h = 3$. The equation of the translation image is: $y = g(x - 3)$

b) a translation of 8 units down

The translation is vertical, so the equation of the image graph has the form $y - k = g(x)$. The translation is 8 units down, so $k = -8$. The equation of the translation image is: $y - (-8) = g(x)$, or $y + 8 = g(x)$

c) a translation of 9 units up

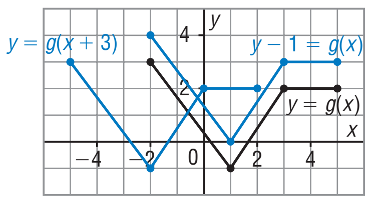
The translation is vertical, so the equation of the image graph has the form $y - k = g(x)$. The translation is 9 units up, so $k = 9$. The equation of the translation image is: $y - 9 = g(x)$

d) a translation of 7 units left

The translation is horizontal, so the equation of the image graph has the form $y = g(x - h)$. The translation is 7 units left, so $h = -7$. The equation of the translation image is: $y = g(x - (-7))$, or $y = g(x + 7)$

B

7. Here is the graph of $y = g(x)$. On the same grid, sketch the graph of each function below. State the domain and range of each function.



a) $y - 1 = g(x)$

Compare the equation to
$y - k = g(x)$: $k = 1$
So, translate each point on the graph of $y = g(x)$ 1 unit up.
Both functions have the same domain: $-2 \leq x \leq 5$
The range of $y = g(x)$ is:
$-1 \leq y \leq 3$
The range of $y - 1 = g(x)$ is:
$0 \leq y \leq 4$

b) $y = g(x + 3)$

Write $y = g(x + 3)$ as $y = g(x - (-3))$.
Compare the equation to
$y = g(x - h)$: $h = -3$
So, translate each point on the graph of $y = g(x)$ 3 units left.
The domain of $y = g(x)$ is: $-2 \leq x \leq 5$
The domain of $y = g(x + 3)$ is:
$-5 \leq x \leq 2$
Both functions have the same range: $-1 \leq y \leq 3$

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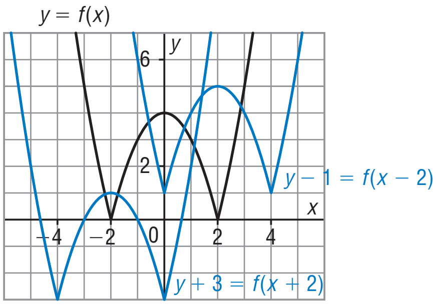
8. Here is the graph of $y = f(x)$. On the same grid, sketch the graph of each function below. State the domain and range of each function.



a) $y + 3 = f(x + 2)$

Write $y + 3 = f(x + 2)$ as $y - (-3) = f(x - (-2))$.
Compare the equation to $y - k = f(x - h)$:
$h = -2$ and $k = -3$
Mark some lattice points on $y = f(x)$.
Translate each point 2 units left and 3 units down, then join the points with a smooth curve.
Both functions have domain: $x \in \mathbb{R}$
The range of $y = f(x)$ is: $y \geq 0$
The range of $y + 3 = f(x + 2)$ is: $y \geq -3$

b) $y - 1 = f(x - 2)$

Compare the equation to $y - k = f(x - h)$:
$h = 2$ and $k = 1$
Mark some lattice points on $y = f(x)$.
Translate each point 2 units right and 1 unit up, then join the points with a smooth curve.
Both functions have domain: $x \in \mathbb{R}$
The range of $y = f(x)$ is: $y \geq 0$
The range of $y - 1 = f(x - 2)$ is: $y \geq 1$

9. The point A(3, 7) lies on the graph of $y = f(x)$. What are the coordinates of its image A′ on the graph of $y - 2 = f(x - 8)$? How do you know?

Compare the equation $y - 2 = f(x - 8)$ to $y - k = f(x - h)$: $h = 8$ and $k = 2$
So, each point on the graph of $y = f(x)$ is translated 8 units right and 2 units up to create the graph of $y - 2 = f(x - 8)$.
So, the image of point A is A′(3 + 8, 7 + 2), or A′(11, 9).

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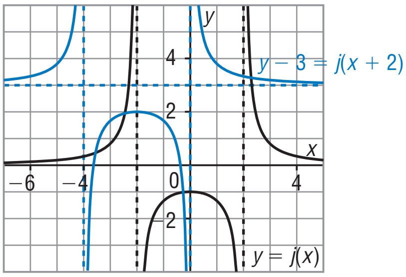
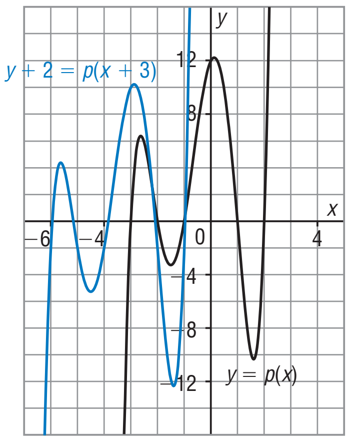
10. Here is the graph of $y = j(x)$. On the same grid, sketch the graph of $y - 3 = j(x + 2)$. Describe how the vertical and horizontal asymptotes are affected by the translations. What are the equations of the asymptotes of the image graph?



Compare $y - 3 = j(x + 2)$ to $y - k = j(x - h)$:
$h = -2$ and $k = 3$
So, translate the vertical asymptotes 2 units left and the horizontal asymptote 3 units up.
Then sketch the graph of $y - 3 = j(x + 2)$ so that it is congruent to the graph of $y = j(x)$.
The equations of the vertical asymptotes are $x = -4$ and $x = 0$.
The equation of the horizontal asymptote is $y = 3$.

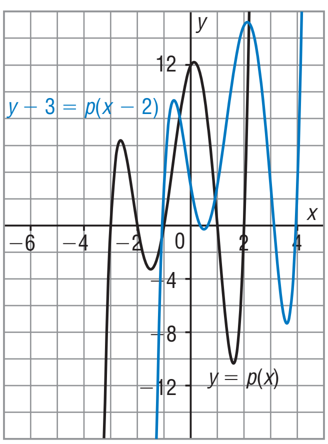
11. Use the graph of $y = p(x)$ to sketch the graph of each function below.

a) $y + 2 = p(x + 3)$



Compare $y + 2 = p(x + 3)$ to $y - k = p(x - h)$:
$h = -3$ and $k = -2$
Mark some lattice points on $y = p(x)$.
Translate each point 3 units left and 2 units down. Draw a smooth curve through the points so that the image graph is congruent to the graph of $y = p(x)$.

b) $y - 3 = p(x - 2)$



Compare $y - 3 = p(x - 2)$ to $y - k = p(x - h)$: $h = 2$ and $k = 3$
Mark some lattice points on $y = p(x)$.
Translate each point 2 units right and 3 units up.
Draw a smooth curve through the points so that the image graph is congruent to the graph of $y = p(x)$.

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12. The function $y = f(x)$ has domain $-7 \leq x \leq 12$ and range $-1 \leq y \leq 10$. What are the domain and range of $y + 8 = f(x - 3)$?

Write $y + 8 = f(x - 3)$ as $y - (-8) = f(x - 3)$.
Compare the equation to $y - k = f(x - h)$: $h = 3$ and $k = -8$
So, each point on the graph of $y = f(x)$ is translated 3 units right and 8 units down to create the graph of $y + 8 = f(x - 3)$.
So, the image graph has domain: $-7 + 3 \leq x \leq 12 + 3$, or $-4 \leq x \leq 15$
and range: $-1 - 8 \leq y \leq 10 - 8$, or $-9 \leq y \leq 2$

13. Describe how the graph of $y = \sqrt{x}$ has been translated to create the graph of each function below. Use graphing technology to check.

a) $y = \sqrt{x - 1}$

Compare $y = \sqrt{x - 1}$ to $y - k = \sqrt{x - h}$: $h = 1$ and $k = 0$
So, the graph of $y = \sqrt{x - 1}$ is the graph of $y = \sqrt{x}$ after a translation of 1 unit right.

b) $y = \sqrt{x + 4} - 2$

Write $y = \sqrt{x + 4} - 2$ as $y - (-2) = \sqrt{x - (-4)}$, then compare to $y - k = \sqrt{x - h}$: $h = -4$ and $k = -2$
So, the graph of $y = \sqrt{x + 4} - 2$ is the graph of $y = \sqrt{x}$ after a translation of 4 units left and 2 units down.

14. The graph of $y = x^3$ is translated 5 units left and 3 units up. What is the equation of the image graph?

The equation of the image graph has the form $y - k = (x - h)^3$.
$h = -5$ and $k = 3$
So, the equation of the image graph is: $y - 3 = (x - (-5))^3$, or
$y - 3 = (x + 5)^3$

15. The graph of $y = |x^2 - 2|$ is translated 2 units right and 7 units down. What is the equation of the image graph?

The equation of the image graph has the form $y - k = |(x - h)^2 - 2|$.
$h = 2$ and $k = -7$
So, the equation of the image graph is: $y - (-7) = |(x - 2)^2 - 2|$, or
$y + 7 = |(x - 2)^2 - 2|$

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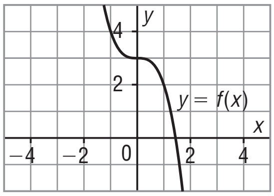
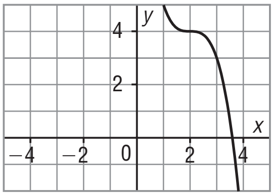
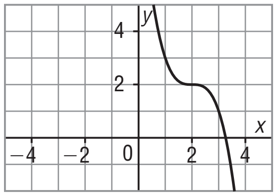
16. The graph of $y = f(x)$ has been translated to create the graphs below. Match each graph to its equation.



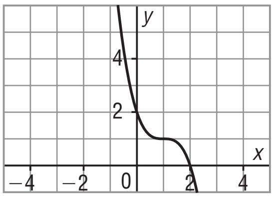
i) Graph A



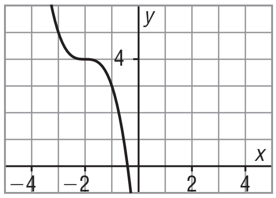
ii) Graph B



iii) Graph C



iv) Graph D



a) $y - 1 = f(x + 2)$

Compare $y - 1 = f(x - (-2))$ to $y - k = f(x - h)$: $h = -2$ and $k = 1$
So, the graph of $y = f(x)$ is translated 2 units left and 1 unit up.
This corresponds to Graph D.

b) $y - 1 = f(x - 2)$

Compare $y - 1 = f(x - 2)$ to $y - k = f(x - h)$: $h = 2$ and $k = 1$
So, the graph of $y = f(x)$ is translated 2 units right and 1 unit up.
This corresponds to Graph A.

c) $y + 1 = f(x - 2)$

Compare $y - (-1) = f(x - 2)$ to $y - k = f(x - h)$: $h = 2$ and $k = -1$
So, the graph of $y = f(x)$ is translated 2 units right and 1 unit down.
This corresponds to Graph B.

d) $y + 2 = f(x - 1)$

Compare $y + 2 = f(x - 1)$ to $y - k = f(x - h)$: $h = 1$ and $k = -2$
So, the graph of $y = f(x)$ is translated 1 unit right and 2 units down.
This corresponds to Graph C.

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17. The graph of $f(x) = x^2 - 5x - 6$ has zeros at -1 and 6.

a) The graph of $y = f(x)$ is translated horizontally. Neither zero of the image graph is negative. What is the shortest possible translation?

A translation of 1 unit right would move the zero -1 to 0. So, the shortest possible translation is 1 unit right.

b) The graph of $y = f(x)$ is translated horizontally. Neither zero of the image graph is positive. What is the shortest possible translation?

A translation of 6 units left would move the zero 6 to 0. So, the shortest possible translation is 6 units left.

C

18. The graph of $y = 2x^2 - 12x + 23$ is translated vertically so that its vertex lies on the x-axis. How could you use the discriminant to determine the translation? What is the translation?

The equation of the translation image has the form $y - k = 2x^2 - 12x + 23$, or $y = 2x^2 - 12x + 23 + k$.
The vertex lies on the x-axis when the value of the discriminant is 0.
In $b^2 - 4ac$, substitute: $a = 2$, $b = -12$, $c = 23 + k$

$$(-12)^2 - 4(2)(23 + k) = 0$$
$$144 - 8(23 + k) = 0$$
$$-8(23 + k) = -144$$
$$23 + k = 18$$
$$k = -5$$

So, the translation is 5 units down.

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