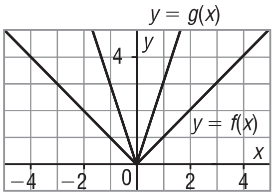
Lesson 3.4 Exercises, pages 226–232

A

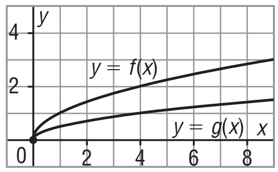
3. The graph of $y = g(x)$ is the image of the graph of $y = f(x)$ after a single transformation. Identify the transformation.

a)



A horizontal compression by a factor of $\frac{1}{3}$, or a vertical stretch by a factor of 3

b)



A vertical compression by a factor of $\frac{1}{2}$

4. Describe how the graph of each function below is related to the graph of $y = f(x)$.

a) $y + 5 = -2f(x)$

Compare $y - k = af(b(x - h))$ to $y + 5 = -2f(x)$: $k = -5$, $a = -2$
So, the graph of $y = f(x)$ is vertically stretched by a factor of 2, reflected in the x-axis, then translated 5 units down.

b) $y = f(3(x - 4))$

Compare $y - k = af(b(x - h))$ to $y = f(3(x - 4))$: $b = 3$, $h = 4$
So, the graph of $y = f(x)$ is horizontally compressed by a factor of $\frac{1}{3}$, then translated 4 units right.

c) $y = \frac{1}{2}f(x + 7)$

Compare $y - k = af(b(x - h))$ to $y = \frac{1}{2}f(x + 7)$: $a = \frac{1}{2}$, $h = -7$
So, the graph of $y = f(x)$ is vertically compressed by a factor of $\frac{1}{2}$, then translated 7 units left.

d) $y - 2 = f\left(\frac{1}{3}x\right)$

Compare $y - k = af(b(x - h))$ to $y - 2 = f\left(\frac{1}{3}x\right)$: $k = 2$, $b = \frac{1}{3}$

So, the graph of $y = f(x)$ is horizontally stretched by a factor of 3, then translated 2 units up.

5. The graph of $y = f(x)$ is transformed as described below. Write the equation of the image graph in terms of the function f.

a) a horizontal compression by a factor of $\frac{1}{4}$, a reflection in the y-axis, and a translation of 3 units left

The equation of the image graph has the form: $y - k = af(b(x - h))$
Since $b = -4$ and $h = -3$, the equation is: $y = f(-4(x + 3))$

b) a vertical compression by a factor of $\frac{1}{2}$, a reflection in the y-axis, and a translation of 7 units up

The equation of the image graph has the form: $y - k = af(b(x - h))$

Since $a = \frac{1}{2}$, $b = -1$, and $k = 7$, the equation is: $y - 7 = \frac{1}{2}f(-x)$

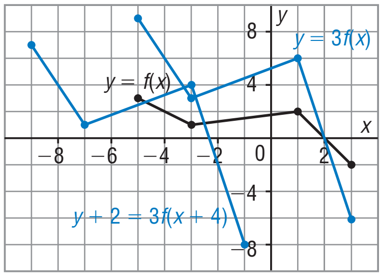
c) a horizontal stretch by a factor of 5, a vertical compression by a factor of $\frac{1}{3}$, and a translation of 6 units left and 3 units up

In $y - k = af(b(x - h))$, substitute $b = \frac{1}{5}$, $a = \frac{1}{3}$, $h = -6$, and $k = 3$.

The equation is: $y - 3 = \frac{1}{3}f\left(\frac{1}{5}(x + 6)\right)$

B

6. Here is the graph of $y = f(x)$. On the same grid, sketch and label its image after a vertical stretch by a factor of 3, and a translation of 4 units left and 2 units down.



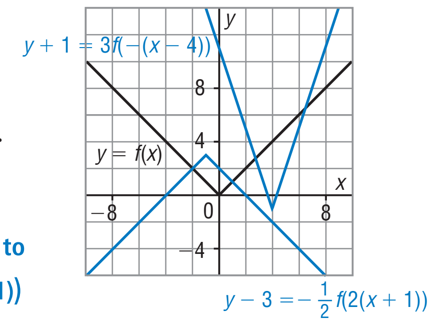
Perform the vertical stretch by a factor of 3 first. Point (x, y) on $y = f(x)$ corresponds to point $(x, 3y)$ on the image graph $y = 3f(x)$.

Point on $y = f(x)$	Point on $y = 3f(x)$
$(-5, 3)$	$(-5, 9)$
$(-3, 1)$	$(-3, 3)$
$(1, 2)$	$(1, 6)$
$(3, -2)$	$(3, -6)$

Plot the points, then join them in order with line segments to form the graph of $y = 3f(x)$. Then translate this graph 4 units left and 2 units down to form the graph of $y + 2 = 3f(x + 4)$.

7. Here is the graph of $y = f(x)$. On the same grid, sketch the graph of each function below then state its domain and range.

a) $y - 3 = -\frac{1}{2}f(2(x + 1))$



Compare: $y - k = af(b(x - h))$ to

$y - 3 = -\frac{1}{2}f(2(x + 1))$

$k = 3$, $a = -\frac{1}{2}$, $b = 2$, and $h = -1$

(x, y) corresponds to $\left(\frac{x}{2} - 1, -\frac{1}{2}y + 3\right)$

Point on $y = f(x)$	Point on $y - 3 = -\frac{1}{2}f(2(x + 1))$
$(-4, 4)$	$(-3, 1)$
$(0, 0)$	$(-1, 3)$
$(4, 4)$	$(1, 1)$

The domain is: $x \in \mathbb{R}$
The range is: $y \leq 3$

b) $y + 1 = 3f(-(x - 4))$

Compare: $y - k = af(b(x - h))$ to $y + 1 = 3f(-(x - 4))$
$k = -1$, $a = 3$, $b = -1$, and $h = 4$
(x, y) corresponds to $(-x + 4, 3y - 1)$

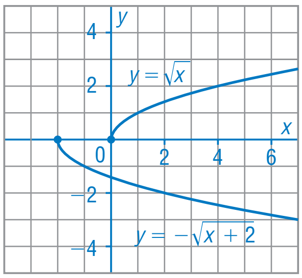
Point on $y = f(x)$	Point on $y + 1 = 3f(-(x - 4))$
$(-4, 4)$	$(8, 11)$
$(0, 0)$	$(4, -1)$
$(4, 4)$	$(0, 11)$

The domain is: $x \in \mathbb{R}$
The range is: $y \geq -1$

8. On each grid, graph $y = \sqrt{x}$, apply transformations to sketch the given function, then state its domain and range.

a) $y = -\sqrt{x + 2}$

$y = -\sqrt{x - (-2)}$
$a = -1$ and $h = -2$
(x, y) corresponds to $(x - 2, -y)$

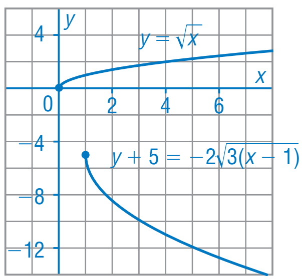


Point on $y = \sqrt{x}$	Point on $y = -\sqrt{x + 2}$
$(0, 0)$	$(-2, 0)$
$(1, 1)$	$(-1, -1)$
$(9, 3)$	$(7, -3)$

Domain is: $x \geq -2$
Range is: $y \leq 0$

b) $y + 5 = -2\sqrt{3(x - 1)}$

$k = -5$, $a = -2$, $b = 3$, and $h = 1$
(x, y) corresponds to $\left(\frac{x}{3} + 1, -2y - 5\right)$

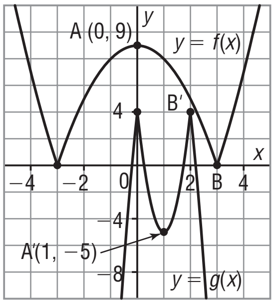


Point on $y = \sqrt{x}$	Point on $y + 5 = -2\sqrt{3(x - 1)}$
$(0, 0)$	$(1, -5)$
$(1, 1)$	$\left(\frac{4}{3}, -7\right)$
$(9, 3)$	$(4, -11)$

Domain is: $x \geq 1$
Range is: $y \leq -5$

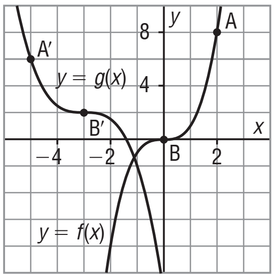
9. The graph of $y = g(x)$ is the image of the graph of $y = f(x)$ after a combination of transformations. Corresponding points are labelled. Write an equation of each image graph in terms of the function f.

a)



Write the equation for the image graph in the form $y - k = af(b(x - h))$.
Use the points A(0, 9) and B(3, 0) on the graph of $y = f(x)$.
Horizontal distance between A and B is: 3
Vertical distance between A and B is: 9
Use corresponding points A′(1, −5) and B′(2, 4) on the graph of $y = g(x)$.
Horizontal distance between A′ and B′ is: 1
Vertical distance between A′ and B′ is: 9
The horizontal distance is one-third of the original distance, so the graph of $y = f(x)$ is compressed horizontally by a factor of $\frac{1}{3}$: $b = 3$.
The vertical distance does not change, so the graph of $y = f(x)$ is not compressed or stretched vertically. From the graph, there is a reflection in the x-axis, so $a = -1$. To determine the coordinates of B(3, 0) after this compression and reflection, substitute: $x = 3$, $y = 0$, $a = -1$, and $b = 3$ in $\left(\frac{x}{b}, ay\right)$ to get $\left(\frac{3}{3}, 0\right)$, or (1, 0). Determine the translation that would move (1, 0) to B′(2, 4).
A translation of 1 unit right and 4 units up is required, so $h = 1$ and $k = 4$.
An equation for the image graph is: $y - 4 = -f(3(x - 1))$

b)



Write the equation for the image graph in the form $y - k = af(b(x - h))$.
Use the points A(2, 8) and B(0, 0) on the graph of $y = f(x)$.
Horizontal distance between A and B is: 2
Vertical distance between A and B is: 8
Use corresponding points A′(−5, 6) and B′(−3, 2) on the graph of $y = g(x)$.
Horizontal distance between A′ and B′ is: 2
Vertical distance between A′ and B′ is: 4
The horizontal distance does not change, so the graph of $y = f(x)$ is not compressed or stretched horizontally. From the graph, there is a reflection in the y-axis, so $b = -1$.
The vertical distance is halved, so the graph of $y = f(x)$ is compressed vertically by a factor of $\frac{1}{2}$: $a = \frac{1}{2}$. To determine the coordinates of A(2, 8) after this compression and reflection, substitute: $x = 2$, $y = 8$, $b = -1$, and $a = \frac{1}{2}$ in $\left(\frac{x}{b}, ay\right)$ to get $\left(\frac{2}{-1}, \frac{1}{2}(8)\right)$, or (−2, 4).
Determine the translation that would move (−2, 4) to A′(−5, 6).
A translation of 3 units left and 2 units up is required, so $h = -3$ and $k = 2$. An equation for the image graph is: $y - 2 = \frac{1}{2}f(-(x + 3))$

10. For each pair of functions below, describe the graph of the second function as a transformation image of the graph of the first function.

a) $y = |x|$ $\quad y + 6 = -2|3(x - 4)|$

Let $f(x) = |x|$, then compare $y + 6 = -2|3(x - 4)|$ to $y - k = af(b(x - h))$: $k = -6$, $a = -2$, $b = 3$, and $h = 4$.
The graph of $y + 6 = -2|3(x - 4)|$ is the image of the graph of $y = |x|$ after a vertical stretch by a factor of 2, a horizontal compression by a factor of $\frac{1}{3}$, a reflection in the x-axis, followed by a translation of 4 units right and 6 units down.

b) $y = \frac{1}{x}$ $\quad y - 3 = 2\left(\frac{5}{x + 1}\right)$

Let $f(x) = \frac{1}{x}$, then compare $y - 3 = 2\left(\frac{5}{x + 1}\right)$ to $y - k = af(b(x - h))$: $k = 3$, $a = 2$, $b = 5$, and $h = -1$.
The graph of $y - 3 = 2\left(\frac{5}{x + 1}\right)$ is the image of the graph of $y = \frac{1}{x}$ after a vertical stretch by a factor of 2, a horizontal compression by a factor of $\frac{1}{5}$, followed by a translation of 1 unit left and 3 units up.

c) $y = x^4$ $\quad y + 1 = \frac{1}{4}[-2(x + 3)]^4$

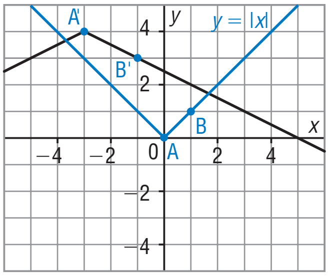
Let $f(x) = x^4$, then compare $y + 1 = \frac{1}{4}[-2(x + 3)]^4$ to $y - k = af(b(x - h))$: $k = -1$, $a = \frac{1}{4}$, $b = -2$, and $h = -3$.
The graph of $y + 1 = \frac{1}{4}[-2(x + 3)]^4$ is the image of the graph of $y = x^4$ after a vertical compression by a factor of $\frac{1}{4}$, a horizontal compression by a factor of $\frac{1}{2}$, a reflection in the y-axis, followed by a translation of 3 units left and 1 unit down.

11. A transformation image of the graph of $y = f(x)$ is represented by the equation $y - 1 = -2f\left(\frac{x + 5}{3}\right)$. The point (7, 5) lies on the image graph. What are the coordinates of the corresponding point on the graph of $y = f(x)$?

Compare $y - 1 = -2f\left(\frac{x + 5}{3}\right)$ to $y - k = af(b(x - h))$:
$k = 1$, $a = -2$, $b = \frac{1}{3}$, and $h = -5$
A point (x, y) on $y = f(x)$ corresponds to the point $(3x - 5, -2y + 1)$ on $y - 1 = -2f\left(\frac{x + 5}{3}\right)$. The image of a point (x, y) is (7, 5).
So, $3x - 5 = 7$, or $x = 4$; and $-2y + 1 = 5$, or $y = -2$
So, the corresponding point on $y = f(x)$ is $(4, -2)$.

C

12. This graph is the image of the graph of $y = |x|$ after a combination of transformations. Write an equation of the image.



Write the equation for the image graph in the form $y - k = a|b(x - h)|$.
Sketch the graph of $y = |x|$.
Use the points A(0, 0) and B(1, 1) on the graph of $y = |x|$.
Horizontal distance between A and B is: 1
Vertical distance between A and B is: 1
Use corresponding points A′(−3, 4) and B′(−1, 3) on the image graph.
Horizontal distance between A′ and B′ is: 2
Vertical distance between A′ and B′ is: 1
The horizontal distance is doubled, so the graph of $y = |x|$ is stretched horizontally by a factor of 2 and $b = \frac{1}{2}$.
The vertical distance does not change, so the graph of $y = |x|$ is not compressed or stretched vertically. From the graph, there is a reflection in the x-axis, so $a = -1$.
To determine the coordinates of B(1, 1) after this stretch and reflection, substitute: $x = 1$, $y = 1$, $b = \frac{1}{2}$, and $a = -1$ in $\left(\frac{x}{b}, ay\right)$ to get (2, −1).
Determine the translation that would move (2, −1) to B′(−1, 3).
A translation of 3 units left and 4 units up is required, so $h = -3$ and $k = 4$.
An equation for the image graph is: $y - 4 = -\left|\frac{1}{2}(x + 3)\right|$, or
$y - 4 = -\frac{1}{2}|x + 3|$
Use mental math to check this equation, by verifying that the point (1, 2) lies on the graph.