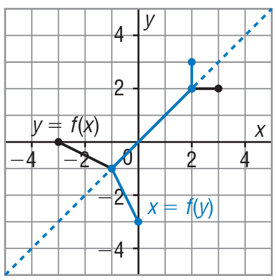
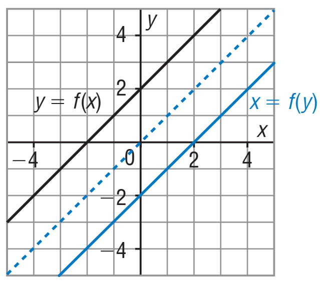
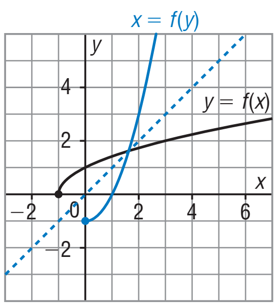
Lesson 3.5 Exercises, pages 243–249

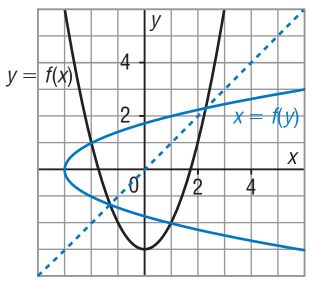
A

4. For each graph below, sketch its image after a reflection in the line $y = x$.

a)


b)


c)


d)


5. Determine an equation of the inverse of each function.

a) $y = 2x + 9$

$$x = 2y + 9$$
$$x - 9 = 2y$$
$$y = \frac{x - 9}{2}$$

An equation of the inverse is: $y = \frac{x - 9}{2}$

b) $y = x^2 + 5$

$$x = y^2 + 5$$
$$x - 5 = y^2$$
$$y = \pm\sqrt{x - 5}$$

An equation of the inverse is: $y = \pm\sqrt{x - 5}$

c) $y = \frac{5x - 3}{4}$

$$x = \frac{5y - 3}{4}$$
$$4x = 5y - 3$$
$$4x + 3 = 5y$$
$$y = \frac{4x + 3}{5}$$

An equation of the inverse is: $y = \frac{4x + 3}{5}$

d) $y = 2x^2 - 3$

$$x = 2y^2 - 3$$
$$x + 3 = 2y^2$$
$$y^2 = \frac{x + 3}{2}$$
$$y = \pm\sqrt{\frac{x + 3}{2}}$$

An equation of the inverse is: $y = \pm\sqrt{\frac{x + 3}{2}}$

B

6. For each function below:

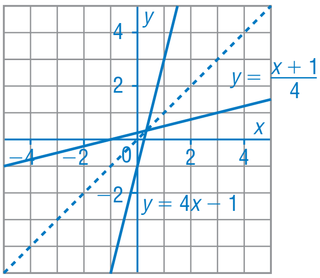
i) Determine an equation of its inverse.

ii) Sketch the graphs of the function and its inverse.

iii) Is the inverse a function? Explain.

a) $y = 4x - 1$

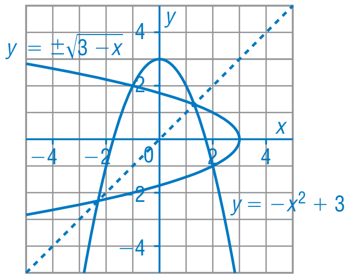
i) $x = 4y - 1$
$4y = x + 1$
$y = \frac{x + 1}{4}$

ii)


iii) The inverse is a function because its graph passes the vertical line test.

b) $y = -x^2 + 3$

i) $x = -y^2 + 3$
$y^2 = 3 - x$
$y = \pm\sqrt{3 - x}$

ii)


iii) The inverse is not a function because its graph does not pass the vertical line test.

7. Determine whether the functions in each pair are inverses of each other. Justify your answer.

a) $y = x^2 + 7, x \geq 0$ and $y = \sqrt{x - 7}$

An equation of the inverse of $y = x^2 + 7, x \geq 0$ is:

$$x = y^2 + 7$$
$$x - 7 = y^2$$
$$y = \pm\sqrt{x - 7}$$

Since the domain of $y = x^2 + 7$ is $x \geq 0$, then the range of the inverse is $y \geq 0$.
So, an equation of the inverse is $y = \sqrt{x - 7}$.
Since this equation matches the given equation, the functions are inverses of each other.

b) $y = 5x - 3$ and $y = \frac{3 - x}{5}$

An equation of the inverse of $y = 5x - 3$ is:

$$x = 5y - 3$$
$$x + 3 = 5y$$
$$y = \frac{x + 3}{5}$$

Since this is not equivalent to the given equation, the functions are not inverses of each other.

c) $y = (x + 4)^2 - 2, x \geq -4$ and $y = -\sqrt{x + 2} + 4$

An equation of the inverse of $y = (x + 4)^2 - 2, x \geq -4$ is:

$$x = (y + 4)^2 - 2$$
$$x + 2 = (y + 4)^2$$
$$y + 4 = \pm\sqrt{x + 2}$$
$$y = \pm\sqrt{x + 2} - 4$$

Since the domain of $y = (x + 4)^2 - 2$ is $x \geq -4$, then the range of the inverse is $y \geq -4$.
So, an equation of the inverse is $y = \sqrt{x + 2} - 4$.
Since this equation is not equivalent to the given equation, the functions are not inverses of each other.

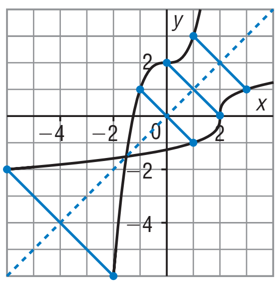
d) $y = \frac{1}{2}x - 6$ and $y = 2x + 12$

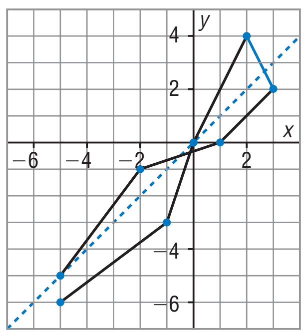
An equation of the inverse of $y = \frac{1}{2}x - 6$ is:

$$x = \frac{1}{2}y - 6$$
$$x + 6 = \frac{1}{2}y$$
$$y = 2x + 12$$

Since this matches the given equation, the functions are inverses of each other.

8. Determine whether the functions in each pair are inverses of each other. Justify your answer.

a) 

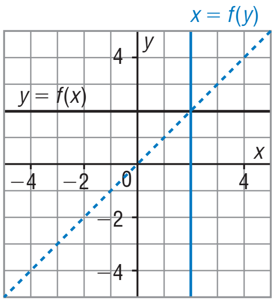
b) 

Corresponding pairs of points are equidistant from the line $y = x$, so the functions are inverses of each other.

Corresponding pairs of points are not equidistant from the line $y = x$, so the functions are not inverses of each other.

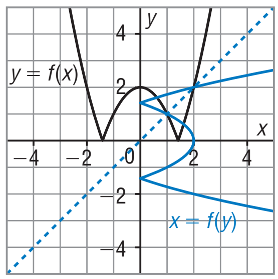
9. Sketch the graph of the inverse of each relation. Is the inverse a function? Justify your answer.

a)



The inverse is not a function because its graph does not pass the vertical line test.

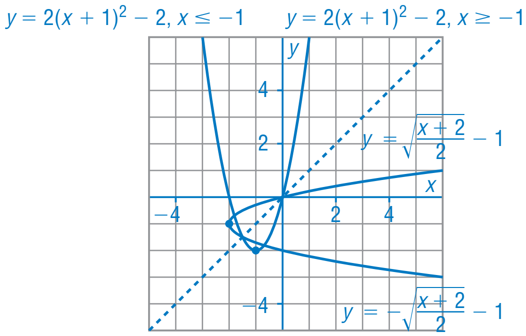
b)



The inverse is not a function because its graph does not pass the vertical line test.

10. Determine two ways to restrict the domain of each function below so its inverse is a function. Write the equation of the inverse each time. Use a graph to illustrate each way.

a) $y = 2(x + 1)^2 - 2$



Sketch the graphs of $y = 2(x + 1)^2 - 2$ and its inverse.
For an equation of the inverse, write $x = 2(y + 1)^2 - 2$, then solve for y.

$x + 2 = 2(y + 1)^2$

$\frac{x + 2}{2} = (y + 1)^2$

$y + 1 = \pm\sqrt{\frac{x + 2}{2}}$

$y = \pm\sqrt{\frac{x + 2}{2}} - 1$

The inverse is a function if the domain of $y = 2(x + 1)^2 - 2$ is restricted to either $x \leq -1$ or $x \geq -1$.

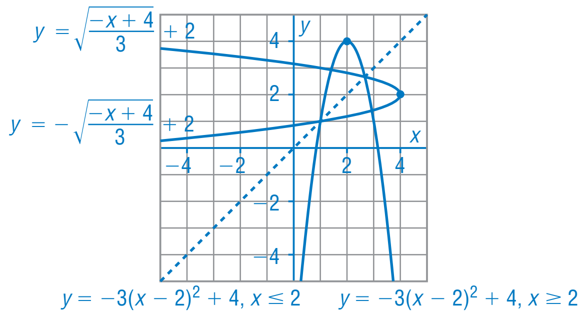
For $y = 2(x + 1)^2 - 2, x \leq -1$, the inverse is:

$y = -\sqrt{\frac{x + 2}{2}} - 1$

For $y = 2(x + 1)^2 - 2, x \geq -1$, the inverse is:

$y = \sqrt{\frac{x + 2}{2}} - 1$

b) $y = -3(x - 2)^2 + 4$



Sketch the graphs of $y = -3(x - 2)^2 + 4$ and its inverse.
For an equation of the inverse, write $x = -3(y - 2)^2 + 4$, then solve for y.

$$-x + 4 = 3(y - 2)^2$$
$$\frac{-x+4}{3} = (y - 2)^2$$
$$y - 2 = \pm\sqrt{\frac{-x+4}{3}}$$
$$y = \pm\sqrt{\frac{-x+4}{3}} + 2$$

The inverse is a function if the domain of $y = -3(x - 2)^2 + 4$ is restricted to either $x \leq 2$ or $x \geq 2$.

For $y = -3(x - 2)^2 + 4, x \leq 2$, the inverse is:

$$y = -\sqrt{\frac{-x+4}{3}} + 2$$

For $y = -3(x - 2)^2 + 4, x \geq 2$, the inverse is:

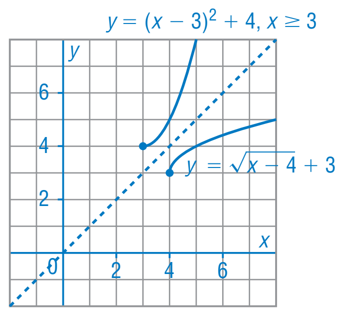
$$y = \sqrt{\frac{-x+4}{3}} + 2$$

11. The point A(5, −6) lies on the graph of $y = f(x)$. What are the coordinates of its image A′ on the graph of $y = f^{-1}(x - 3)$?

On the graph of $y = f^{-1}(x)$, the coordinates of the image of A are: (−6, 5)
On the graph of $y = f^{-1}(x - 3)$, this point is translated 3 units right, so the coordinates of A′ are: (−3, 5)

12. a) Graph the function $y = (x - 3)^2 + 4, x \geq 3$.

The graph is the part of a parabola, congruent to $y = x^2$, with vertex (3, 4), for which $x \geq 3$.



b) What is the range of the function?

From the graph, the range is: $y \geq 4$

c) Determine the equation of its inverse. Sketch its graph.

For an equation of the inverse, write $x = (y - 3)^2 + 4$, then solve for y.

$$x - 4 = (y - 3)^2$$
$$y - 3 = \pm\sqrt{x - 4}$$
$$y = \pm\sqrt{x - 4} + 3$$

Since the domain of $y = (x - 3)^2 + 4$ is restricted to $x \geq 3$, then the equation of the inverse is: $y = \sqrt{x - 4} + 3$

d) What are the domain and range of the inverse?

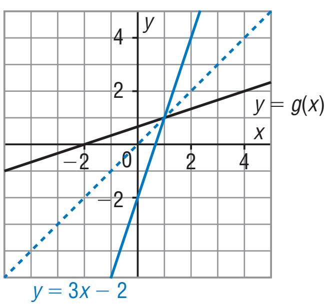
The domain of the inverse is: $x \geq 4$
The range of the inverse is: $y \geq 3$

13. Determine the coordinates of two points that lie on the graphs of both $y = x^2 + 2x - 6$ and its inverse. Explain the strategy you used.

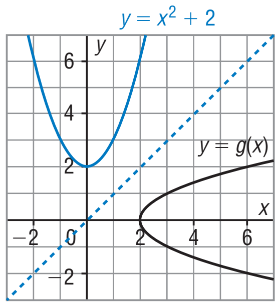
For the points to lie on both graphs, the points must lie on the line $y = x$. I input the equation in a graphing calculator, then looked at the table of values to determine two points with matching coordinates.
These points are: $(2, 2)$ and $(-3, -3)$

14. A graph was reflected in the line $y = x$. Its reflection image is shown. Determine an equation of the original graph in terms of x and y.

a)



b)



Use the line $y = x$ to sketch the graph of the inverse. This line has y-intercept -2, and slope 3, so its equation is: $y = 3x - 2$

Use the line $y = x$ to sketch the graph of the inverse. This curve is a parabola that has vertex $(0, 2)$, and is congruent to $y = x^2$. So, its equation is: $y = x^2 + 2$

C

15. Determine the equations of two functions such that the graphs of each function and its inverse coincide.

The graphs of these functions must be symmetrical about the line $y = x$. For example, any linear function with slope –1 has a graph that coincides with its inverse, such as $y = -x + 4$.

16. The graphs of $y = \dfrac{x + 5}{x + k}$ and its inverse coincide. Determine the value of k.

Given $y = \dfrac{x + 5}{x + k}$, interchange x and y.

$x = \dfrac{y + 5}{y + k}$ **Simplify.**

$xy + xk = y + 5$ **Solve for y.**

$y(x - 1) = 5 - xk$

$y = \dfrac{5 - xk}{x - 1}$

This equation must be the same as $y = \dfrac{x + 5}{x + k}$.

Compare the denominators: $x - 1 = x + k$

So, $k = -1$