Checkpoint: Assess Your Understanding, pages 213–218

3.1

1. **Multiple Choice** The graph of $y = -3x^3 + 4$ is translated 4 units right and 5 units down. What is an equation of the translation image?

A. $y = -3(x + 4)^3 + 9$

B. $y = -3(x - 4)^3 + 9$

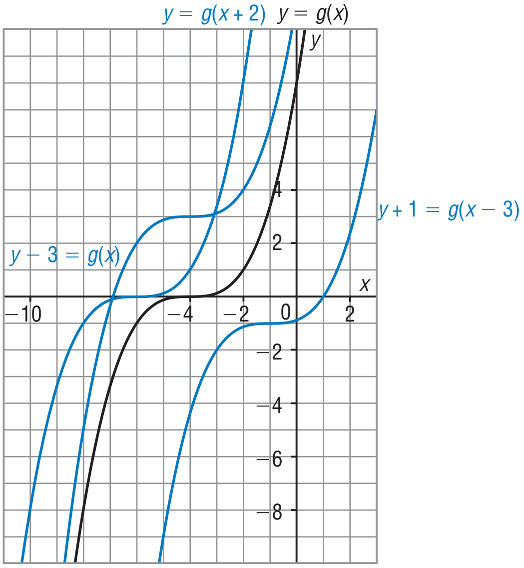
C. $y = -3(x + 4)^3 - 1$

[✓ **D.**] $y = -3(x - 4)^3 - 1$

2. Here is the graph of $y = g(x)$. On the same grid, sketch the graph of each function below. State the domain and range of each function.

a) $y - 3 = g(x)$

Compare the equation to $y - k = g(x)$: $k = 3$
So, mark some lattice points on $y = g(x)$ and translate each point 3 units up.
Both functions have domain: $x \in \mathbb{R}$
Both functions have range: $y \in \mathbb{R}$



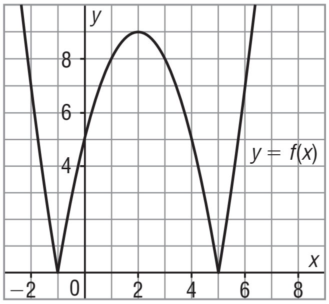
b) $y = g(x + 2)$

Write $y = g(x + 2)$ as $y = g(x - (-2))$.
Compare the equation to $y = g(x - h)$: $h = -2$
Translate each point on the graph of $y = g(x)$ 2 units left.
The domain is: $x \in \mathbb{R}$
The range is: $y \in \mathbb{R}$

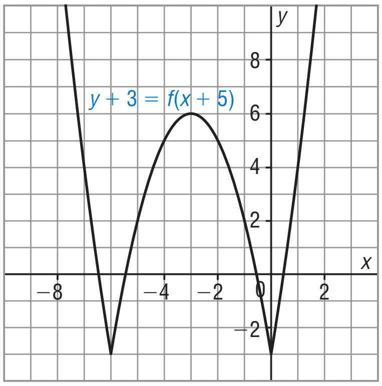
c) $y + 1 = g(x - 3)$

Write $y + 1 = g(x - 3)$ as $y - (-1) = g(x - 3)$.
Compare the equation to $y - k = g(x - h)$: $h = 3$ and $k = -1$
Translate each point on the graph of $y = g(x)$ 3 units right and 1 unit down.
The domain is: $x \in \mathbb{R}$
The range is: $y \in \mathbb{R}$

3. The graph of $y = f(x)$ was translated to create each graph below. Write an equation of each graph in terms of the function f.

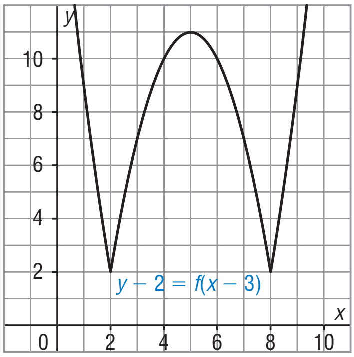


a)



The graph of $y = f(x)$ has a local maximum at (2, 9).
This graph has a local maximum at (−3, 6).
So, the graph of $y = f(x)$ was translated 5 units left and 3 units down.
The equation of the image graph has the form:
$y - k = f(x - h)$, where
$h = -5$ and $k = -3$
So, an equation of the image graph is: $y + 3 = f(x + 5)$

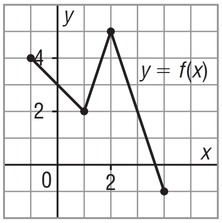
b)



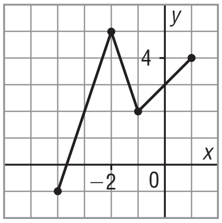
The graph of $y = f(x)$ has a local maximum at (2, 9).
This graph has a local maximum at (5, 11).
So, the graph of $y = f(x)$ was translated 3 units right and 2 units up.
The equation of the image graph has the form: $y - k = f(x - h)$, where $h = 3$ and $k = 2$
So, an equation of the image graph is: $y - 2 = f(x - 3)$

3.2

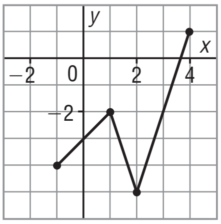
4. Multiple Choice The graph of $y = f(x)$ was reflected in the x-axis. Which graph below is its reflection image?



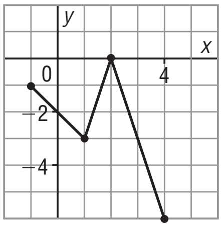
A.



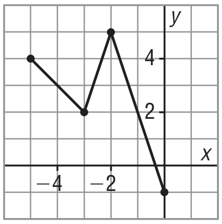
[✓ **B.**]



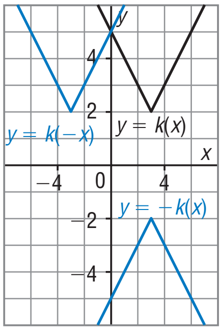
C.



D.



5. Here is the graph of $y = k(x)$. On the same grid, sketch and label the graph of each function below, then state its domain and range.



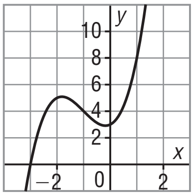
a) $y = -k(x)$

The graph of $y = -k(x)$ is the image of the graph of $y = k(x)$ after a reflection in the x-axis.
Mark some lattice points on $y = k(x)$, then reflect them in the x-axis. Mark these image points, then join them.
Domain: $x \in \mathbb{R}$
Range: $y \leq -2$

b) $y = k(-x)$

The graph of $y = k(-x)$ is the image of the graph of $y = k(x)$ after a reflection in the y-axis.
Mark some lattice points on $y = k(x)$, then reflect them in the y-axis. Mark these image points, then join them.
Domain: $x \in \mathbb{R}$
Range: $y \geq 2$

6. The graph of $y = -x^3 + 3x^2 - x + 3$ was reflected in the y-axis and its image is shown. What is an equation of the image?



When the graph of $y = f(x)$ is reflected in the y-axis, the equation of its image is $y = f(-x)$.

So, an equation of the image is:

$y = f(-x)$

$y = -(-x)^3 + 3(-x)^2 - (-x) + 3$

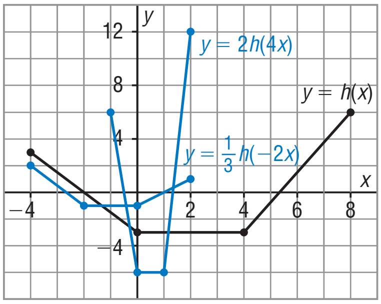
$y = x^3 + 3x^2 + x + 3$

3.3

7. Multiple Choice The point $(-6, 2)$ lies on the graph of $y = f(x)$. After vertical and horizontal stretches or compressions of the graph, the equation of the image is $y = 3f(2x)$. Which point is the image of $(-6, 2)$?

[✓ **A.**] $(-3, 6)$ **B.** $(-12, 6)$ **C.** $(-2, 4)$ **D.** $(-18, 1)$

8. Here is the graph of $y = h(x)$. On the same grid, sketch the graph of each function below, then state its domain and range.



a) $y = \frac{1}{3}h(-2x)$

Compare $y = ah(bx)$ to $y = \frac{1}{3}h(-2x)$: $a = \frac{1}{3}$ and $b = -2$

So, the graph of $y = h(x)$ is vertically compressed by a factor of $\frac{1}{3}$, horizontally compressed by a factor of $\frac{1}{2}$, then reflected in the y-axis. Use mental math and the transformation: (x, y) on $y = h(x)$ corresponds to $\left(\frac{x}{-2}, \frac{1}{3}y\right)$ on $y = \frac{1}{3}h(-2x)$, to mark some image points, then join them.

Domain: $-4 \leq x \leq 2$; range: $-1 \leq y \leq 2$

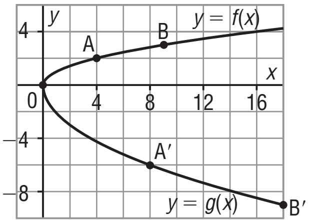
b) $y = 2h(4x)$

Compare $y = ah(bx)$ to $y = 2h(4x)$: $a = 2$ and $b = 4$

So, the graph of $y = h(x)$ is vertically stretched by a factor of 2, and horizontally compressed by a factor of $\frac{1}{4}$. Use mental math and the transformation: (x, y) on $y = h(x)$ corresponds to $\left(\frac{x}{4}, 2y\right)$ on $y = 2h(4x)$, to mark some image points, then join them.

Domain: $-1 \leq x \leq 2$; range: $-6 \leq y \leq 12$

9. The graph of $y = g(x)$ is the image of the graph of $y = f(x)$ after a vertical and/or horizontal stretch and/or reflection. Corresponding points are labelled. Write an equation of the image graph in terms of the function f.



Point A(4, 2) on $y = f(x)$ corresponds to point A′(8, −6) on $y = g(x)$.
An equation for the image graph after a vertical or horizontal stretch or compression can be written in the form $y = af(bx)$.

A point (x, y) on $y = f(x)$ corresponds to the point $\left(\frac{x}{b}, ay\right)$ on $y = af(bx)$.

The image of A(4, 2) is $\left(\frac{4}{b}, a(2)\right)$, which is A′(8, −6).

Equate the x-coordinates: $b = \frac{1}{2}$

Equate the y-coordinates: $a = -3$

So, an equation of $y = g(x)$ is: $y = -3f\left(\frac{1}{2}x\right)$

I used the coordinates of B and B′, and mental math to verify the equation.