PRACTICE TEST, pages 261–264

1. Multiple Choice The graph of $y + 2 = -3f\left(\frac{1}{2}(x + 5)\right)$ is the image of the graph of $y = f(x)$ after several transformations. Which statement about how the graph of $y = f(x)$ was transformed is false?

A. The graph was reflected in the x-axis.

B. The graph was horizontally stretched by a factor of 2.

C. The graph was translated 2 units down.

D. The graph was reflected in the y-axis.

2. Multiple Choice Which statement about a function and its inverse is not always true?

A. The inverse of a function is a function.

B. The domain of a function is the range of its inverse, and the range of a function is the domain of its inverse.

C. The graph of the inverse of a function can be sketched by reflecting the graph of the function in the line $y = x$.

D. Each point (x, y) on the graph of a function corresponds to the point (y, x) on the graph of its inverse.

3. Write an equation of the function $y = \sqrt{x - 3}$ after each transformation below.

a) a translation of 2 units right and 5 units down

The equation of the image graph has the form $y - k = \sqrt{(x - h) - 3}$, where $k = -5$ and $h = 2$.
So, an equation of the image graph is: $y + 5 = \sqrt{x - 5}$

b) a reflection in the x-axis

The y-coordinates of points on the graph of $y = \sqrt{x - 3}$ change sign.
So, an equation of the image graph is: $y = -\sqrt{x - 3}$

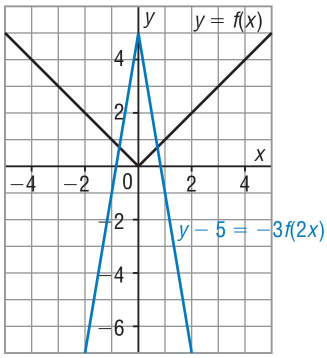
c) a reflection in the y-axis

The x-coordinates of points on the graph of $y = \sqrt{x - 3}$ change sign.
So, an equation of the image graph is: $y = \sqrt{-x - 3}$

d) a vertical stretch by a factor of 2 and a horizontal compression by a factor of $\frac{1}{3}$

The equation of the image graph has the form $y = a\sqrt{bx - 3}$, where $a = 2$ and $b = 3$.
So, an equation of the image graph is: $y = 2\sqrt{3x - 3}$

4. Here is the graph of $y = f(x)$. On the same grid, use transformations to sketch the graph of $y - 5 = -3f(2x)$. Describe the transformations.



Compare: $y - k = af(b(x - h))$ to $y - 5 = -3f(2x)$
$k = 5$, $a = -3$, $b = 2$, and $h = 0$
A point (x, y) on $y = f(x)$ corresponds to the point $\left(\frac{x}{b} + h, ay + k\right)$ on $y - k = af(b(x - h))$.
Substitute the values above.
A point on $y - 5 = -3f(2x)$ has coordinates $\left(\frac{x}{2}, -3y + 5\right)$.

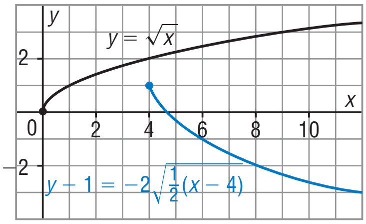
Transform some points on the lines.

Point on $y = f(x)$	Point on $y - 5 = -3f(2x)$
$(-4, 4)$	$(-2, -7)$
$(0, 0)$	$(0, 5)$
$(4, 4)$	$(2, -7)$

Draw 2 lines through the points for the graph of $y - 5 = -3f(2x)$.
The graph of $y = f(x)$ was compressed horizontally by a factor of $\frac{1}{2}$, stretched vertically by a factor of 3, reflected in the x-axis, then translated 5 units up.

5. Here is the graph of $y = \sqrt{x}$.

a) Use this graph to sketch a graph of $y - 1 = -2\sqrt{\frac{1}{2}(x - 4)}$.



Compare: $y - k = af(b(x - h))$ to

$y - 1 = -2\sqrt{\frac{1}{2}(x - 4)}$

$k = 1$, $a = -2$, $b = \frac{1}{2}$, and $h = 4$

A point (x, y) on $y = f(x)$ corresponds to the point $\left(\frac{x}{b} + h, ay + k\right)$ on $y - k = af(b(x - h))$.

Substitute the values above.

A point on $y - 1 = -2\sqrt{\frac{1}{2}(x - 4)}$ has coordinates: $(2x + 4, -2y + 1)$

Transform some points on $y = \sqrt{x}$.

Point on $y = \sqrt{x}$	Point on $y - 1 = -2\sqrt{\frac{1}{2}(x - 4)}$
(0, 0)	(4, 1)
(1, 1)	(6, −1)
(4, 2)	(12, −3)

Join the points with a smooth curve for the graph of $y - 1 = -2\sqrt{\frac{1}{2}(x - 4)}$.

b) Write the domain and range of the function in part a.

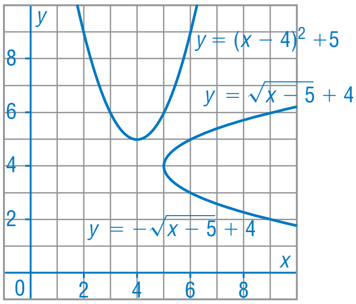
From the graph, the domain is $x \geq 4$; and the range is $y \leq 1$.

6. **a)** Determine an equation of the inverse of $y = (x - 4)^2 + 5$.

Interchange x and y in the equation.

$$x = (y - 4)^2 + 5 \quad \text{Solve for } y.$$
$$(y - 4)^2 = x - 5$$
$$y - 4 = \pm\sqrt{x - 5}$$
$$y = \pm\sqrt{x - 5} + 4$$

b) Sketch the graph of the inverse. Is the inverse a function? Explain.



Graph $y = (x - 4)^2 + 5$.
This is the graph of $y = x^2$ after a translation of 4 units right and 5 units up.
Interchange the coordinates of points on this graph, then plot the new points to get the graph of $y = \pm\sqrt{x - 5} + 4$.
No, the inverse is not a function because its graph does not pass the vertical line test.

c) If your answer to part b is yes, explain how you know. If your answer to part b is no, determine a possible restriction on the domain of $y = (x - 4)^2 + 5$ so its inverse is a function, then write the equation of the inverse function.

The domain can be restricted by considering each part of the graph of $y = (x - 4)^2 + 5$ to the right and left of the vertex.
So, one restriction is $x \geq 4$ and the equation of the inverse function is: $y = \sqrt{x - 5} + 4$
Another restriction is $x \leq 4$ and the equation of the inverse function is: $y = -\sqrt{x - 5} + 4$