Lesson 4.1 Math Lab: Assess Your Understanding, pages 268–271

1. Use the graphs of $y = f(x)$ and $y = g(x)$ on page 269 to sketch the graph of each function below, then identify its domain and range. Estimate the range, where necessary.

a) $y = f(x) + g(x)$

b) $y = f(x) - g(x)$

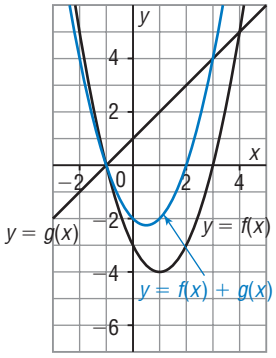
c) $y = f(x) \cdot g(x)$

d) $y = \dfrac{f(x)}{g(x)}$

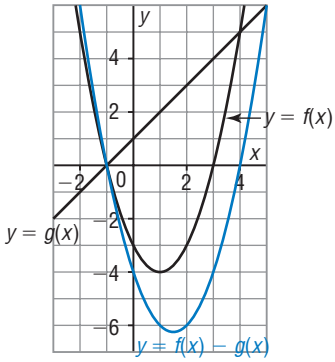
From the graphs:

x	$f(x)$	$g(x)$	$f(x) + g(x)$	$f(x) - g(x)$	$f(x) \cdot g(x)$	$\frac{f(x)}{g(x)}$
−2	5	−1	4	6	−5	−5
−1	0	0	0	0	0	undefined
0	−3	1	−2	−4	−3	−3
1	−4	2	−2	−6	−8	−2
2	−3	3	0	−6	−9	−1
3	0	4	4	−4	0	0
4	5	5	10	0	25	1

a)



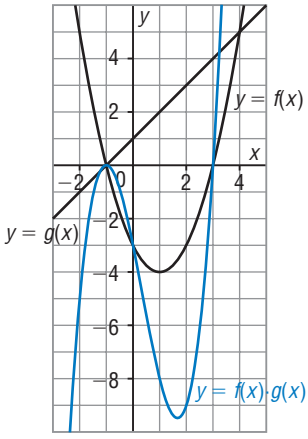
b)



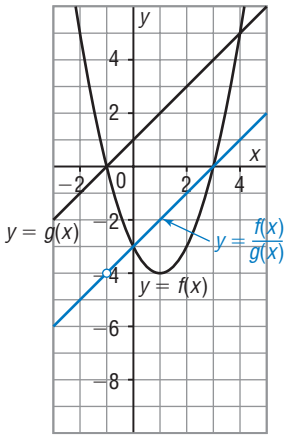
Plot points at: (−2, 4), (−1, 0), (0, −2), (1, −2), (2, 0), (3, 4)
Join the points with a smooth curve.
Domain: $x \in \mathbb{R}$
Range: $y \geq -2.25$

Plot points at: (−2, 6), (−1, 0), (0, −4), (1, −6), (2, −6), (3, −4), (4, 0)
Join the points with a smooth curve.
Domain: $x \in \mathbb{R}$
Range: $y \geq -6.25$

c)



d)



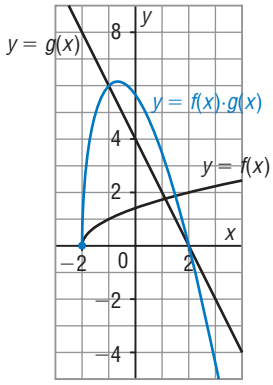
Plot points at: (−2, −5), (−1, 0), (0, −3), (1, −8), (2, −9), (3, 0)
Join the points with a smooth curve.
Domain: $x \in \mathbb{R}$
Range: $y \in \mathbb{R}$

Plot points at: (−2, −5), (0, −3), (1, −2), (2, −1), (3, 0), (4, 1)
Since these points lie on a line, draw an open circle at (−1, −4), then draw a line through the points.
Domain: $x \neq -1$
Range: $y \neq -4$

2. Use the graphs of $y = f(x)$ and $y = g(x)$.

a) State the domain and range of $y = f(x)$.

From the graph:
domain: $x \geq -2$; range: $y \geq 0$



b) State the domain and range of $y = g(x)$.

From the graph:
domain: $x \in \mathbb{R}$; range: $y \in \mathbb{R}$

c) Sketch the graph of $y = f(x) \cdot g(x)$.

From the graphs:

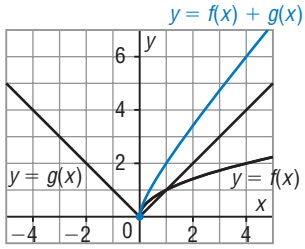
x	$f(x)$	$g(x)$	$f(x) \cdot g(x)$
-2	0	8	0
-1	1	6	6
0	$\doteq 1.4$	4	$\doteq 5.6$
1	$\doteq 1.7$	2	$\doteq 3.4$
2	2	0	0
3	$\doteq 2.2$	-2	$\doteq -4.4$

Plot points at: $(-2, 0)$, $(-1, 6)$, $(0, 5.6)$, $(1, 3.4)$, $(2, 0)$, $(3, -4.4)$
Join the points with a smooth curve.

d) What is the domain of $y = f(x) \cdot g(x)$? How is it related to the domains of $y = f(x)$ and $y = g(x)$?

The domain of $y = f(x) \cdot g(x)$ is: $x \geq -2$
This domain is the same as the domain of $y = f(x)$. It is not equal to the domain of $y = g(x)$, which is all real numbers, because the graph of $y = f(x) \cdot g(x)$ does not extend to the left of $x = -2$.

3. Use the graphs of $y = f(x)$ and $y = g(x)$.



a) What are the domain and range of $y = f(x)$?

The domain is: $x \geq 0$
The range is: $y \geq 0$

b) What are the domain and range of $y = g(x)$?

The domain is: $x \in \mathbb{R}$
The range is: $y \geq 0$

c) Consider the function $h(x) = f(x) + g(x)$. Without graphing, determine the domain and the range of this function and justify your answer.

The domain of $h(x)$ is the same as the domain of $f(x)$, because x cannot be negative; that is, $x \geq 0$.
The range of $h(x)$ is the same as the ranges of $f(x)$ and $g(x)$; that is, all real numbers greater than or equal to 0: $y \geq 0$.

d) Use the graphs in parts a and b to sketch the graph of $h(x)$. Use the graph to verify the domain and range.

From the graphs:

x	$f(x)$	$g(x)$	$f(x) + g(x)$
0	0	0	0
1	1	1	2
2	$\doteq 1.4$	2	$\doteq 3.4$
3	$\doteq 1.7$	3	$\doteq 4.7$
4	2	4	6

Plot points at: (0, 0), (1, 2), (2, 3.4), (3, 4.7), (4, 6)
Join the points with a smooth curve.
From the graphs, the domain and range are the same as given in part c.