

PRACTICE TEST, pages 335–338

1. Multiple Choice Given $f(x) = \sqrt{x}$ and $g(x) = 3x - 6$, which function is a composition of f and g ?

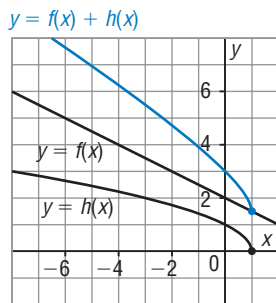
A. $y = \sqrt{x} + 3x - 6$ **B.** $y = 3\sqrt{x} - 6$

C. $y = \frac{\sqrt{x}}{3x - 6}$ D. $y = \sqrt{x} - 3x + 6$

2. Multiple Choice For $f(x) = 3x - 5$ and $g(x) = 4x^2 - 7$, which value is greatest?

A. $f(g(1))$ B. $g(f(1))$ C. $f(f(1))$ **D.** $g(g(1))$

3. Use the graphs of $y = f(x)$ and $y = h(x)$ to graph $y = f(x) + h(x)$.



From the graphs:

x	$f(x)$	$h(x)$	$f(x) + h(x)$
1	1.5	0	1.5
0	2	1	3
-2	3	$\doteq 1.7$	$\doteq 4.7$
-3	3.5	2	5.5
-4	4	$\doteq 2.2$	$\doteq 6.2$
-6	5	$\doteq 2.6$	$\doteq 7.6$
-8	6	3	9

Plot the points with coordinates $(x, f(x) + h(x))$, then join the points with a smooth curve.

Use these functions for questions 4 to 9:

$$f(x) = 2 - 0.5x \quad g(x) = x^2 + 5x \quad h(x) = \sqrt{1 - x} \quad k(x) = \frac{1}{4 - x}$$

4. Write an explicit equation for each combination of functions, then determine the domain and range of the function. Approximate the range where necessary.

a) $y = g(x) \cdot f(x)$

$$\begin{aligned}y &= (x^2 + 5x)(2 - 0.5x) \\y &= 2x^2 - 0.5x^3 + 10x - 2.5x^2 \\y &= -0.5x^3 - 0.5x^2 + 10x\end{aligned}$$

This is a cubic function; its domain is: $x \in \mathbb{R}$; and its range is: $y \in \mathbb{R}$

b) $y = \frac{f(x)}{h(x)}$

$$y = \frac{2 - 0.5x}{\sqrt{1 - x}}$$

Since $1 - x \geq 0$, then $x \leq 1$
The domain is: $x \leq 1$
Use graphing technology. From the graph, the approximate range is: $y \geq 1.7$

5. Write explicit equations in each case.

a) two functions $a(x)$ and $b(x)$ so that $g(x) = a(x) \cdot b(x)$

Sample response:

Factor: $g(x) = x^2 + 5x$

$$g(x) = x(x + 5)$$

So, $a(x) = x$ and $b(x) = x + 5$

b) three functions $a(x)$, $b(x)$, and $c(x)$ so that $f(x) = a(x) + b(x) + c(x)$

Sample response:

$$f(x) = 2 - 0.5x$$

Write $-0.5x$ as the sum of two terms.

$$f(x) = 2 - x + 0.5x$$

$$f(x) = 2 + (-x) + 0.5x$$

So, $a(x) = 2$, $b(x) = -x$, and $c(x) = 0.5x$

c) i) two functions $a(x)$ and $b(x)$ so that $k(x) = a(b(x))$

Sample response:

$$k(x) = \frac{1}{4 - x}$$

$$b(x) = 4 - x \text{ and } a(x) = \frac{1}{x}$$

ii) three functions $a(x)$, $b(x)$, and $c(x)$ so that $k(x) = a(b(c(x)))$

Sample response:

$$k(x) = \frac{1}{4 - x}$$

$$c(x) = -x, b(x) = 4 + x, \text{ and } a(x) = \frac{1}{x}$$

6. Determine each value.

a) $f(g(2))$

$$\begin{aligned} g(x) &= x^2 + 5x \text{ and} \\ f(x) &= 2 - 0.5x \\ g(2) &= 2^2 + 5(2) \\ g(2) &= 14 \\ \text{So, } f(g(2)) &= f(14) \\ &= 2 - 0.5(14) \\ &= -5 \end{aligned}$$

b) $g(h(-4))$

$$\begin{aligned} h(x) &= \sqrt{1-x} \\ \text{and } g(x) &= x^2 + 5x \\ h(-4) &= \sqrt{1-(-4)} \\ h(-4) &= \sqrt{5} \\ \text{So, } g(h(-4)) &= g(\sqrt{5}) \\ &= (\sqrt{5})^2 + 5\sqrt{5} \\ &= 5 + 5\sqrt{5} \end{aligned}$$

7. Determine an explicit equation for each composite function and explain any restrictions on x .

a) $g(h(x))$

$$\begin{aligned} h(x) &= \sqrt{1-x} \text{ and} \\ g(x) &= x^2 + 5x \\ \text{In } g(x) &= x^2 + 5x, \text{ replace} \\ x &\text{ with } \sqrt{1-x}. \\ g(h(x)) &= (\sqrt{1-x})^2 + 5\sqrt{1-x} \\ \text{Since the square root of a real} \\ \text{number cannot be negative,} \\ 1-x &\geq 0, \text{ so } x \leq 1 \\ \text{So, } g(h(x)) &= 1-x + 5\sqrt{1-x} \end{aligned}$$

b) $h(f(x))$

$$\begin{aligned} f(x) &= 2 - 0.5x \text{ and} \\ h(x) &= \sqrt{1-x} \\ \text{In } h(x) &= \sqrt{1-x}, \text{ replace } x \text{ with} \\ 2 - 0.5x. \\ h(f(x)) &= \sqrt{1-(2-0.5x)} \\ h(f(x)) &= \sqrt{-1+0.5x} \\ \text{Since the square root of a real} \\ \text{number cannot be negative,} \\ -1+0.5x &\geq 0, \text{ so } x \geq 2 \end{aligned}$$

8. Sketch a graph of the composite function $y = k(f(x))$, then state the domain of the composite function.

$$f(x) = 2 - 0.5x \text{ and } k(x) = \frac{1}{4-x}$$

The domain of $f(x)$ is $x \in \mathbb{R}$, and the domain of $k(x)$ is $x \neq 4$.

For $k(f(x))$, replace x in $k(x) = \frac{1}{4-x}$ with $2 - 0.5x$.

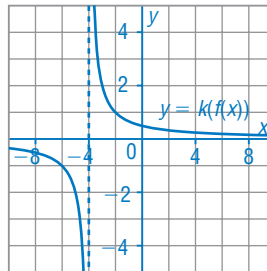
$$k(f(x)) = \frac{1}{4-(2-0.5x)}$$

$$k(f(x)) = \frac{1}{2+0.5x}$$

For $k(f(x))$: $2 + 0.5x \neq 0$, or $x \neq -4$

So, the domain of $k(f(x))$ is: $x \neq -4$

$k(f(x)) = \frac{1}{2+0.5x}$ is a reciprocal function; it has a vertical asymptote with equation $x = -4$, and a horizontal asymptote with equation $y = 0$. Make a table of values, then join the plotted points with 2 smooth curves.



x	-6	-5	-3	-2	0	2	6
$k(f(x))$	-1	-2	2	1	0.5	$0.\bar{3}$	0.2

9. Write an explicit equation for each combination.

a) $y = k(x) + h(g(x))$

$$g(x) = x^2 + 5x \text{ and}$$

$$h(x) = \sqrt{1 - x}$$

$$h(g(x)) = \sqrt{1 - (x^2 + 5x)}$$

$$h(g(x)) = \sqrt{1 - x^2 - 5x}$$

$$\text{So, } y = \frac{1}{4 - x} + \sqrt{1 - x^2 - 5x}$$

b) $y = f(f(x)) - g(g(x))$

$$f(x) = 2 - 0.5x$$

$$f(f(x)) = 2 - 0.5(2 - 0.5x)$$

$$f(f(x)) = 1 + 0.25x$$

$$g(g(x)) = (x^2 + 5x)^2 + 5(x^2 + 5x)$$

$$g(g(x)) = x^4 + 10x^3 + 25x^2 + 5x^2 + 25x$$

$$g(g(x)) = x^4 + 10x^3 + 30x^2 + 25x$$

$$\text{So, } y = 1 + 0.25x - (x^4 + 10x^3 + 30x^2 + 25x)$$

$$y = 1 - 24.75x - x^4 - 10x^3 - 30x^2$$