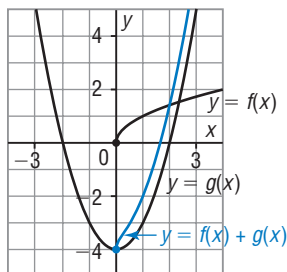


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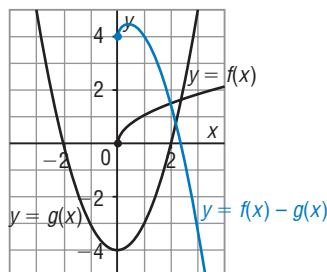
4.1

1. Use the graphs of $y = f(x)$ and $y = g(x)$ to sketch the graph of each function below, then identify its domain and range. Estimate the range if necessary.

a) $y = f(x) + g(x)$



b) $y = f(x) - g(x)$



From the graphs:

x	$f(x)$	$g(x)$	$f(x) + g(x)$	$f(x) - g(x)$
-3	—	5	—	—
-2	—	0	—	—
-1	—	-3	—	—
0	0	-4	-4	4
1	1	-3	-2	4
2	$\doteq 1.4$	0	$\doteq 1.4$	$\doteq 1.4$
3	$\doteq 1.7$	5	$\doteq 6.7$	$\doteq -3.3$

Plot points at: $(0, -4)$, $(1, -2)$, $(2, 1.4)$

Join the points with a smooth curve.

Domain: $x \geq 0$

Range: $y \geq 4$

Plot points at: $(0, 4)$, $(1, 4)$, $(2, 1.4)$, $(3, -3.3)$

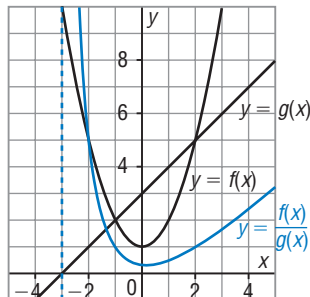
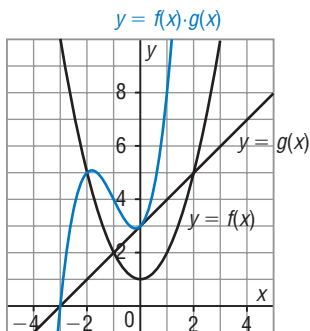
Join the points with a smooth curve.

Domain: $x \geq 0$

From the graph, the approximate range is: $y \leq 4.5$

2. Use the graphs of $y = f(x)$ and $y = g(x)$ to sketch the graph of each function below.

a) $y = f(x) \cdot g(x)$ b) $y = \frac{f(x)}{g(x)}$



From the graphs:

x	$f(x)$	$g(x)$	$f(x) \cdot g(x)$	$\frac{f(x)}{g(x)}$
-3	10	0	0	undefined
-2	5	1	5	5
-1	2	2	4	1
0	1	3	3	$0.\bar{3}$
1	2	4	8	0.5
2	5	5	25	1
3	10	6	60	$1.\bar{6}$

Plot points at: $(-3, 0)$, $(-2, 5)$, $(-1, 4)$, $(0, 3)$, $(1, 8)$

Join the points with a smooth curve.

Plot points at: $(-2, 5)$, $(-1, 1)$, $(0, 0.\bar{3})$, $(1, 0.5)$, $(2, 1)$, $(3, 1.\bar{6})$

Draw a vertical asymptote through $x = -3$.

Join the points with a smooth curve.

4.2

3. Given $f(x) = 3x - 4$ and $g(x) = |2 - 4x|$, write an explicit equation for each function below then determine its domain and range.

a) $h(x) = f(x) + g(x)$ b) $d(x) = f(x) - g(x)$

$h(x) = 3x - 4 + |2 - 4x|$

The domain is: $x \in \mathbb{R}$

Use graphing technology.

From the graph of the function, the range is: $y \geq -2.5$

$d(x) = 3x - 4 - |2 - 4x|$

The domain is: $x \in \mathbb{R}$

Use graphing technology.

From the graph of the function, the range is: $y \leq -2.5$

c) $p(x) = f(x) \cdot g(x)$

$$p(x) = (3x - 4) \cdot |2 - 4x|$$

The domain is: $x \in \mathbb{R}$

Use graphing technology.

From the graph of the function, the range is: $y \in \mathbb{R}$

d) $q(x) = \frac{f(x)}{g(x)}$

$$q(x) = \frac{3x - 4}{|2 - 4x|}$$

For the domain:

$$2 - 4x \neq 0$$

$$x \neq 0.5$$

The domain is: $x \neq 0.5$

Use graphing technology.

From the graph and table, the range is: $y < 0.75$

4. Given the function $h(x) = 3x^2 - 7x + 4$, write explicit equations for:

- a) two functions $f(x)$ and $g(x)$ so that $h(x) = f(x) \cdot g(x)$

Sample response:

Factor: $h(x) = 3x^2 - 7x + 4$

$$h(x) = (3x - 4)(x - 1)$$

So, $f(x) = 3x - 4$ and $g(x) = x - 1$

- b) three functions $f(x)$, $g(x)$, and $k(x)$ so that

$$h(x) = f(x) + g(x) + k(x)$$

Sample response:

Write the given function as: $h(x) = 3x^2 + (-7x) + 4$

So, $f(x) = 3x^2$, $g(x) = -7x$, and $k(x) = 4$

- c) three functions $f(x)$, $g(x)$, and $k(x)$ so that

$$h(x) = f(x) - g(x) - k(x)$$

Sample response:

Write the given function as: $h(x) = 3x^2 - (7x) - (-4)$

So, $f(x) = 3x^2$, $g(x) = 7x$, and $k(x) = -4$

- d) two functions $f(x)$ and $g(x)$ so that $h(x) = \frac{f(x)}{g(x)}$

Sample response:

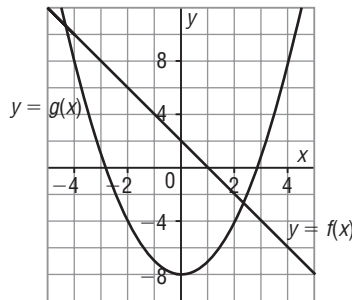
Multiply and divide $h(x)$ by an expression that is never 0, such as $x^2 + 3$.

$$h(x) = \frac{(x^2 + 3)(3x^2 - 7x + 4)}{x^2 + 3}$$

So, $f(x) = (x^2 + 3)(3x^2 - 7x + 4)$ and $g(x) = x^2 + 3$

4.3

5. Given the graphs of $y = f(x)$ and $y = g(x)$, determine each value below.



a) $f(g(2))$

From the graph of $y = g(x)$, $g(2) = -4$
 From the graph of $y = f(x)$, $f(-4) = 10$
 So, $f(g(2)) = 10$

b) $g(f(2))$

From the graph of $y = f(x)$, $f(2) = -2$
 From the graph of $y = g(x)$, $g(-2) = -4$
 So, $g(f(2)) = -4$

c) $f(f(1))$

From the graph of $y = f(x)$, $f(1) = 0$
 From the graph of $y = f(x)$, $f(0) = 2$
 So, $f(f(1)) = 2$

d) $g(g(-2))$

From the graph of $y = g(x)$, $g(-2) = -4$
 From the graph of $y = g(x)$, $g(-4) = 8$
 So, $g(g(-2)) = 8$

6. Given the functions $f(x) = -2x + 3$, $g(x) = x^2 - 3x$, and $h(x) = \sqrt{x - 4}$, determine each value.

a) $f(g(1))$

$$\begin{aligned} g(1) &= 1^2 - 3(1) \\ &= -2 \\ f(g(1)) &= f(-2) \\ &= -2(-2) + 3 \\ &= 7 \end{aligned}$$

b) $h(f(-3))$

$$\begin{aligned} f(-3) &= -2(-3) + 3 \\ &= 9 \\ h(f(-3)) &= h(9) \\ &= \sqrt{9 - 4} \\ &= \sqrt{5} \end{aligned}$$

c) $f(g(h(9)))$

$$\begin{aligned} h(9) &= \sqrt{9 - 4} \\ &= \sqrt{5} \\ g(\sqrt{5}) &= (\sqrt{5})^2 - 3\sqrt{5} \\ &= 5 - 3\sqrt{5} \\ f(g(h(9))) &= f(5 - 3\sqrt{5}) \\ &= -2(5 - 3\sqrt{5}) + 3 \\ &= -7 + 6\sqrt{5} \end{aligned}$$

d) $h(g(f(-0.5)))$

$$\begin{aligned} f(-0.5) &= -2(-0.5) + 3 \\ &= 4 \\ g(4) &= 4^2 - 3(4) \\ &= 4 \\ h(g(f(-0.5))) &= h(4) \\ &= 0 \end{aligned}$$

7. Given $f(x) = 2x^2 - x$ and $g(x) = x + 4$, determine an explicit equation for each composite function then state its domain and range.

a) $f(g(x))$

$$f(g(x)) = f(x + 4)$$

$$f(g(x)) = 2(x + 4)^2 - (x + 4)$$

$$f(g(x)) = 2x^2 + 16x + 32 - x - 4$$

$$f(g(x)) = 2x^2 + 15x + 28$$

This is a quadratic function; its

domain is: $x \in \mathbb{R}$

Use graphing technology to graph the function; its range is: $y \geq -0.125$

b) $g(f(x))$

$$g(f(x)) = g(2x^2 - x)$$

$$g(f(x)) = 2x^2 - x + 4$$

This is a quadratic function; its

domain is: $x \in \mathbb{R}$

Use graphing technology to graph the function; its range is: $y \geq 3.875$

c) $g(g(x))$

$$g(g(x)) = g(x + 4)$$

$$g(g(x)) = x + 4 + 4$$

$$g(g(x)) = x + 8$$

This is a linear function;

its domain is: $x \in \mathbb{R}$; and its

range is: $y \in \mathbb{R}$

d) $f(f(x))$

$$f(f(x)) = f(2x^2 - x)$$

$$f(f(x)) = 2(2x^2 - x)^2 - (2x^2 - x)$$

$$f(f(x)) = 8x^4 - 8x^3 + 2x^2 - 2x^2 + x$$

$$f(f(x)) = 8x^4 - 8x^3 + x$$

This is a quartic function; its domain

is: $x \in \mathbb{R}$

Use graphing technology to graph the function; its range is: $y \geq -0.125$

8. Use composition of functions to determine whether $f(x) = \frac{1}{2}x - 3$ and $g(x) = 2x + 6$ are inverse functions.

$$\begin{aligned} \text{Determine: } f(g(x)) &= \frac{1}{2}(2x + 6) - 3 \\ &= x \end{aligned}$$

$$\begin{aligned} \text{Determine: } g(f(x)) &= 2\left(\frac{1}{2}x - 3\right) + 6 \\ &= x \end{aligned}$$

Since $f(g(x)) = g(f(x)) = x$, the functions are inverses.

4.4

9. Determine possible functions f and g so that $f(g(x)) = 2x^2 + 10x - 6$. Do this in two ways.

Sample response: Complete the square:

$$f(g(x)) = 2x^2 + 10x - 6$$

$$f(g(x)) = 2(x^2 + 5x) - 6$$

$$f(g(x)) = 2(x^2 + 5x + 6.25 - 6.25) - 6$$

$$f(g(x)) = 2(x + 2.5)^2 - 18.5$$

One way:

Replace $x + 2.5$ with x .

Let $g(x) = x + 2.5$, then

$$f(x) = 2x^2 - 18.5$$

A second way:

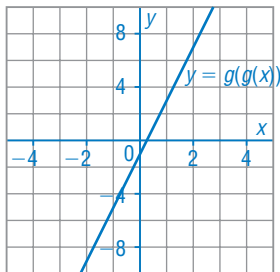
Replace $(x + 2.5)^2$ with x .

Let $g(x) = (x + 2.5)^2$, then

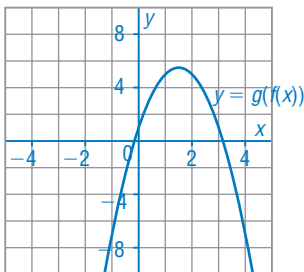
$$f(x) = 2x - 18.5$$

10. Given the functions $f(x) = x^2 - 3x$ and $g(x) = -2x + 1$, sketch a graph of each composite function below. Why are there no restrictions on x ?

a) $y = g(g(x))$



b) $y = g(f(x))$



Make a table of values for the functions.

x	$f(x)$	$g(x)$	$g(g(x))$	$g(f(x))$
-1	4	3	-5	-7
0	0	1	-1	1
1	-2	-1	3	5
2	-2	-3	7	5
3	0	-5	11	1
4	4	-7	15	-7

- a) Graph the points with coordinates $(x, g(g(x)))$ that fit on the grid.
Draw a line through the points for the graph of $y = g(g(x))$.
- b) Graph the points with coordinates $(x, g(f(x)))$ that fit on the grid.
Draw a smooth curve through the points for the graph of $y = g(f(x))$.
- There are no restrictions on x because both $f(x)$ and $g(x)$ are polynomial functions, which have domain $x \in \mathbb{R}$.

11. Given the functions $f(x) = \sqrt{3 + x}$, $g(x) = 1 - 2x^2$, and $k(x) = \frac{1}{x - 3}$, write an explicit equation for each combination.

a) $h(x) = f(g(x)) - k(x)$

$$f(g(x)) = \sqrt{3 + 1 - 2x^2}$$

$$f(g(x)) = \sqrt{4 - 2x^2},$$

$$-\sqrt{2} \leq x \leq \sqrt{2}$$

$$\text{So, } h(x) = \sqrt{4 - 2x^2} - \frac{1}{x - 3},$$

$$-\sqrt{2} \leq x \leq \sqrt{2}$$

b) $h(x) = g(f(x)) - f(g(x))$

$$g(f(x)) = 1 - 2(\sqrt{3 + x})^2$$

$$g(f(x)) = 1 - 2(3 + x), x \geq -3$$

$$g(f(x)) = -5 - 2x, x \geq -3$$

$$\text{So, } h(x) = -5 - 2x - \sqrt{4 - 2x^2},$$

$$-\sqrt{2} \leq x \leq \sqrt{2}$$

c) $h(x) = k(g(x)) + k(f(x))$ d) $h(x) = g(g(x)) \cdot k(x)$

$$k(g(x)) = \frac{1}{1 - 2x^2 - 3}$$

$$k(g(x)) = \frac{1}{-2x^2 - 2}$$

$$k(f(x)) = \frac{1}{\sqrt{3+x}-3}$$

$$\text{So, } h(x) = \frac{1}{-2x^2 - 2} + \frac{1}{\sqrt{3+x}-3},$$

$$x \geq -3, x \neq 6$$

$$g(g(x)) = 1 - 2(1 - 2x^2)^2$$

$$g(g(x)) = 1 - 2(1 - 4x^2 + 4x^4)$$

$$g(g(x)) = -1 + 8x^2 - 8x^4$$

$$\text{So, } h(x) = (-1 + 8x^2 - 8x^4) \left(\frac{1}{x-3} \right), x \neq 3$$

12. Given the functions $f(x) = \sqrt{x}$ and $g(x) = x^2 - 3x - 18$, determine an explicit equation for each composite function then state its domain.

a) $f(g(x))$

In $f(x) = \sqrt{x}$, replace x with $x^2 - 3x - 18$.

$$f(g(x)) = \sqrt{x^2 - 3x - 18}$$

The domain of

$$g(x) = x^2 - 3x - 18 \text{ is: } x \in \mathbb{R}$$

The domain of $f(x) = \sqrt{x}$ is:
 $x \geq 0$.

$$\text{So, } g(x) \geq 0$$

$$\text{And } x^2 - 3x - 18 \geq 0$$

Solve the corresponding quadratic equation.

$$x^2 - 3x - 18 = 0$$

$$(x - 6)(x + 3) = 0$$

$$x = 6 \text{ or } x = -3$$

Choose a value of $x < -3$, such as $x = -4$.

Use mental math to substitute

$$x = -4 \text{ in } x^2 - 3x - 18 \geq 0:$$

$$\text{L.S.} = 10 \quad \text{R.S.} = 0$$

So, the interval $x \leq -3$

is the correct interval. And the interval $x \geq 6$ will also be correct.

So, the domain of $f(g(x))$ is $x \leq -3$ or $x \geq 6$.

b) $g(f(x))$

In $g(x) = x^2 - 3x - 18$, replace x with \sqrt{x} .

$$g(f(x)) = (\sqrt{x})^2 - 3\sqrt{x} - 18$$

$$g(f(x)) = x - 3\sqrt{x} - 18$$

The domain of $f(x)$ is: $x \geq 0$

The domain of $g(x)$ is: $x \in \mathbb{R}$

So, the domain of $g(f(x))$ is: $x \geq 0$

13. For each function below

i) Determine possible functions f and g so that $y = f(g(x))$.

ii) Determine possible functions f , g , and h so that $y = f(g(h(x)))$.

a) $y = -3\sqrt{x+4}$

b) $y = (2 - 5x)^3$

Sample response:

i) $g(x) = x + 4$ and

$f(x) = -3\sqrt{x}$

ii) $h(x) = x + 4$, $g(x) = \sqrt{x}$,

and $f(x) = -3x$

Sample response:

i) $g(x) = 2 - 5x$ and $f(x) = x^3$

ii) $h(x) = 5x$, $g(x) = 2 - x$,

and $f(x) = x^3$