Lesson 5.2 Exercises, pages 349–355

A

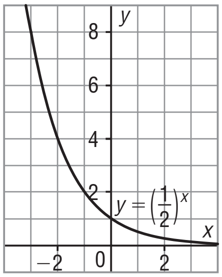
3. Identify each exponential function, then determine whether it is increasing or decreasing.

a) $y = 2x$

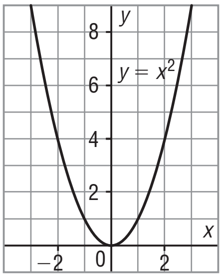
This function is not exponential. It has the form $y = mx$, so it is a linear function.

b) $y = 2^x$

This function is exponential, and it is increasing.

c) 

This function is exponential, and it is decreasing.

d) 

This function is not exponential. It is quadratic.

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4. Each table of values represents an exponential function. Complete each table. Describe your strategy.

a)

x	y
-2	**16**
-1	4
0	1
1	**0.25**
2	**0.0625**

In the 3rd row, y is 4 times its value in the 4th row. So, move up the table and multiply by 4. Move down the table and divide by 4.

b)

x	y
-1	**0.4**
0	**1**
1	2.5
2	6.25
3	**15.625**

In the 5th row, y is 2.5 times its value in the 4th row. So, move down the table and multiply by 2.5. Move up the table and divide by 2.5.

B

5. a) Write the equations of two exponential functions that are increasing. Explain your strategy. Use a graphing calculator to verify the equations.

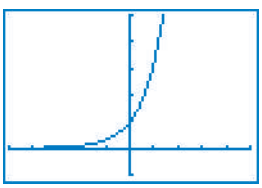
For an increasing function, the base of the power is greater than 1. Sample response: $y = 2.8^x$ and $y = 1.3^x$

b) Write the equations of two exponential functions that are decreasing. Explain your strategy. Use a graphing calculator to verify the equations.

For a decreasing function, the base of the power is between 0 and 1. Sample response: $y = 0.8^x$ and $y = 0.3^x$

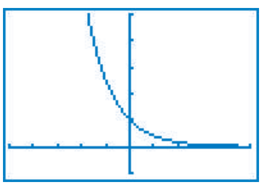
6. Use technology to graph each function below. Sketch the graph, then identify its intercepts and the equation of its asymptote.

a) $y = 3.2^x$



There is no x-intercept. The y-intercept is 1. The asymptote has equation $y = 0$.

b) $y = 0.4^x$



There is no x-intercept. The y-intercept is 1. The asymptote has equation $y = 0$.

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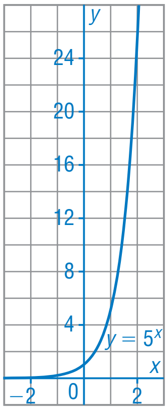
7. Graph each exponential function below. Determine:

- **i)** whether the function is increasing or decreasing
- **ii)** the intercepts
- **iii)** the equation of the asymptote and explain its significance
- **iv)** the domain of the function
- **v)** the range of the function

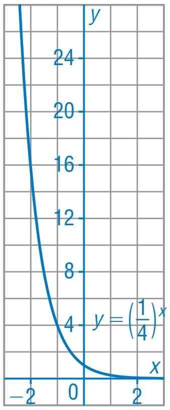
a) $y = 5^x$



x	y
−2	$\frac{1}{25}$
−1	$\frac{1}{5}$
0	1
1	5
2	25

i) Since y increases as x increases, the function is increasing.
ii) From the graph, there is no x-intercept. The y-intercept is 1.
iii) The x-axis is a horizontal asymptote; its equation is $y = 0$. This means that the graph approaches $y = 0$.
iv) The domain is $x \in \mathbb{R}$.
v) The range is $y > 0$.

b) $y = \left(\frac{1}{4}\right)^x$



x	y
−2	16
−1	4
0	1
1	$\frac{1}{4}$
2	$\frac{1}{16}$

i) Since y decreases as x increases, the function is decreasing.
ii) From the graph, there is no x-intercept. The y-intercept is 1.
iii) The x-axis is a horizontal asymptote; its equation is $y = 0$. This means that the graph approaches $y = 0$.
iv) The domain is $x \in \mathbb{R}$.
v) The range is $y > 0$.

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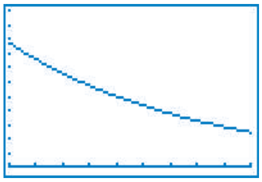
8. When a person drinks coffee, tea, cola, or chocolate, the caffeine in these drinks stays in the body for some time. The percent, P, that remains after t hours can be modelled by the function $P = 100(0.87)^t$.

a) Use technology to graph this function for $0 < t < 10$. Sketch the graph.



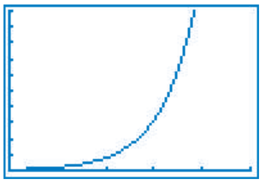
b) Is the function increasing or decreasing? Include 2 reasons in your solution.

The function is decreasing because the graph goes down to the right, the base of the power in the function is between 0 and 1, and the graph was not reflected in the y-axis.

9. Bacteria increase in number by each bacterium splitting in two. Suppose there are 50 bacteria and each bacterium splits in two every 30 min. The number of bacteria, N, after t hours, is modelled by the exponential function $N = 50(2)^{2t}$.

When you input the exponent, insert brackets around $2t$.

a) Use technology to graph this function for $0 < t < 5$. Sketch the graph.



b) Is the function increasing or decreasing? Include 2 reasons in your solution.

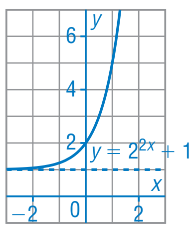
The function is increasing because the graph goes up to the right, the base of the power in the function is greater than 1, and the graph was not reflected in the y-axis.

10. Use transformations to sketch the graph of each function below. Determine:

i) whether the function is increasing or decreasing
ii) the intercepts
iii) the equation of the asymptote
iv) the domain of the function
v) the range of the function

a) $y = 2^{2x} + 1$

Compare $y - 1 = 2^{2x}$ with $y - k = c2^{d(x-h)}$:
$k = 1$, $c = 1$, $d = 2$, and $h = 0$
The graph of $y = 2^{2x} + 1$ is the image of the graph of $y = 2^x$ after a horizontal compression by a factor of $\frac{1}{2}$, then a translation of 1 unit up.
The point (x, y) on $y = 2^x$ corresponds to the point $\left(\frac{x}{2}, y + 1\right)$ on $y - 1 = 2^{2x}$.

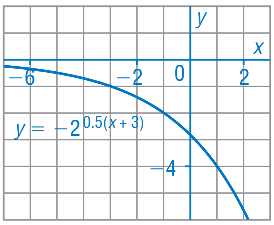


(x, y)	$\left(\frac{x}{2}, y + 1\right)$
$(-2, 0.25)$	$(-1, 1.25)$
$(-1, 0.5)$	$(-0.5, 1.5)$
$(0, 1)$	$(0, 2)$
$(1, 2)$	$(0.5, 3)$
$(2, 4)$	$(1, 5)$

i) The function is increasing.
ii) The graph has y-intercept 2 and no x-intercept.
iii) Since the translation is 1 unit up, the horizontal asymptote has equation $y = 1$.
iv) The domain is: $x \in \mathbb{R}$
v) The range is: $y > 1$

b) $y = -2^{0.5(x + 3)}$

Compare $y = -2^{0.5(x+3)}$ with $y - k = c2^{d(x-h)}$:
$k = 0$, $c = -1$, $d = 0.5$, and $h = -3$
The graph of $y = -2^{0.5(x+3)}$ is the image of the graph of $y = 2^x$
after a horizontal stretch by a factor of 2, a reflection in the x-axis, then a translation of 3 units left.
The point (x, y) on $y = 2^x$ corresponds to the point $(2x - 3, -y)$ on $y = -2^{0.5(x+3)}$.



(x, y)	$(2x - 3, -y)$
$(-2, 0.25)$	$(-7, -0.25)$
$(-1, 0.5)$	$(-5, -0.5)$
$(0, 1)$	$(-3, -1)$
$(1, 2)$	$(-1, -2)$
$(2, 4)$	$(1, -4)$

i) The function is decreasing.
ii) There is no x-intercept. For the y-intercept, substitute $x = 0$ in
$y = -2^{0.5(x+3)}$
$y = -2^{1.5}$
$y = -2.8284\ldots$
iii) Since there is no vertical translation, the horizontal asymptote has equation $y = 0$.
iv) The domain is: $x \in \mathbb{R}$
v) The range is: $y < 0$

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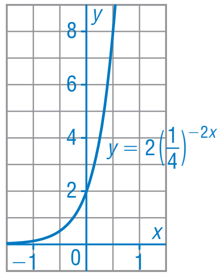
c) $y = 2\left(\frac{1}{4}\right)^{-2x}$

Compare $y = 2\left(\frac{1}{4}\right)^{-2x}$ with $y - k = c\left(\frac{1}{4}\right)^{d(x-h)}$:
$k = 0$, $c = 2$, $d = -2$, and $h = 0$
The graph of $y = 2\left(\frac{1}{4}\right)^{-2x}$ is the image of the graph of $y = \left(\frac{1}{4}\right)^{x}$ after a vertical stretch by a factor of 2, a horizontal compression by a factor of $\frac{1}{2}$, and a reflection in the y-axis.
The point (x, y) on $y = \left(\frac{1}{4}\right)^{x}$ corresponds to the point $\left(-\frac{x}{2}, 2y\right)$ on $y = 2\left(\frac{1}{4}\right)^{-2x}$.



(x, y)	$\left(-\frac{x}{2}, 2y\right)$
$(-1, 4)$	$(0.5, 8)$
$(-0.5, 2)$	$(0.25, 4)$
$(0, 1)$	$(0, 2)$
$(0.5, 0.5)$	$(-0.25, 1)$
$(1, 0.25)$	$(-0.5, 0.5)$

i) The function is increasing.
ii) There is no x-intercept. The y-intercept is 2.
iii) Since there is no vertical translation, the horizontal asymptote has equation $y = 0$.
iv) The domain is: $x \in \mathbb{R}$
v) The range is: $y > 0$

11. Describe two different strategies to graph the function $y = 4(2^x)$.

One strategy is to graph $y = 2^x$, then stretch it vertically by a factor of 4 to get $y = 4(2^x)$.
Write $4(2^x)$ as $2^2(2^x) = 2^{2+x}$, or $2^{x-(-2)}$. Another strategy is to graph $y = 2^x$, then translate this graph 2 units left to get $y = 2^{2+x}$.

C

12. The definition of the exponential function $y = a^x$ includes the restriction $a > 0$. Show that if $a < 0$, the function is not defined for all values of x.

When $a < 0$, such as $a = -2$, then $y = (-2)^x$
When $x = 0.5$, then $y = (-2)^{0.5}$, or $y = \sqrt{-2}$, which is undefined
So, when $a < 0$, the function is undefined for values of x that produce radicals with even indices.

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13. Use transformations to graph this function:

$$y = -2\left(\frac{1}{4}\right)^{-0.5x+4} - 4$$

Write $y = -2\left(\frac{1}{4}\right)^{-0.5x+4} - 4$ as $y + 4 = -2\left(\frac{1}{4}\right)^{-0.5(x-8)}$.

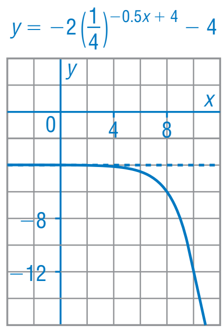
Compare $y + 4 = -2\left(\frac{1}{4}\right)^{-0.5(x-8)}$ with $y - k = c\left(\frac{1}{4}\right)^{d(x-h)}$:

$k = -4$, $c = -2$, $d = -0.5$, and $h = 8$

The graph of this function is the image of the graph of $y = \left(\frac{1}{4}\right)^x$ after vertical and horizontal stretches by a factor of 2, reflections in the x- and y-axes, then translations 8 units right and 4 units down.

The point (x, y) on $y = \left(\frac{1}{4}\right)^x$ corresponds to the point $(-2x + 8, -2y - 4)$ on $y = -2\left(\frac{1}{4}\right)^{-0.5x+4} - 4$.

(x, y)	$(-2x + 8, -2y - 4)$
$(-1, 4)$	$(10, -12)$
$(-0.5, 2)$	$(9, -8)$
$(0, 1)$	$(8, -6)$
$(0.5, 0.5)$	$(7, -5)$
$(1, 0.25)$	$(6, -4.5)$



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