

## Lesson 5.5 Exercises, pages 393–398

**A**

4. Simplify each expression. Use a calculator to verify the answer.

a)  $\log 6 + \log 5$

$$= \log (6 \cdot 5)$$

$$= \log 30$$

Verify:  $\log 6 + \log 5$

$$= 1.4771\dots$$

$$\log 30 = 1.4771\dots$$

b)  $3 \log 2$

$$= \log 2^3$$

$$= \log 8$$

Verify:  $3 \log 2 = 0.9030\dots$

$$\log 8 = 0.9030\dots$$

c)  $\log 48 - \log 6$

$$= \log \left( \frac{48}{6} \right)$$

$$= \log 8$$

Verify:  $\log 48 - \log 6$

$$= 0.9030\dots$$

$$\log 8 = 0.9030\dots$$

d)  $\log 8 + \log 5$

$$= \log (8 \cdot 5)$$

$$= \log 40$$

Verify:  $\log 8 + \log 5 = 1.6020\dots$

$$\log 40 = 1.6020\dots$$

5. Write each expression as a single logarithm.

a)  $\log a - \log b$   
 $= \log \left( \frac{a}{b} \right)$

b)  $\log x + \log y$   
 $= \log xy$

c)  $5 \log m$   
 $= \log m^5$

d)  $\log x - \log y + \log z$   
 $= \log \left( \frac{x}{y} \right) + \log z$   
 $= \log \left( \frac{xz}{y} \right)$

**B**

6. Use each law of logarithms to write an expression that is equal to  $\log 16$ . Use a calculator to verify each expression.

a) the product law

Sample response: Determine 2 numbers whose product is 16: 2 and 8  
So,  $\log 16 = \log 2 + \log 8$   
Verify:  $\log 16 = 1.2041\dots$       $\log 2 + \log 8 = 1.2041\dots$

b) the quotient law

Determine 2 numbers whose quotient is 16: 80 and 5  
So,  $\log 16 = \log 80 - \log 5$   
Verify:  $\log 16 = 1.2041\dots$       $\log 80 - \log 5 = 1.2041\dots$

c) the power law

Write 16 as a power:  $4^2$   
So,  $\log 16 = 2 \log 4$   
Verify:  $\log 16 = 1.2041\dots$       $2 \log 4 = 1.2041\dots$

7. Substitute values of  $a$  and  $b$  to verify each statement.

a)  $\frac{\log a}{\log b} \neq \log \left( \frac{a}{b} \right)$

Substitute:  $a = 3$  and  $b = 5$   
 $\frac{\log a}{\log b} = \frac{\log 3}{\log 5}$   
 $= 0.6826\dots$

$\log \left( \frac{a}{b} \right) = \log \left( \frac{3}{5} \right)$   
 $= -0.2218\dots$

Since the left side is not equal to the right side, the statement is verified.

b)  $\log (a + b) \neq \log ab$

Substitute:  $a = 3$  and  $b = 5$   
 $\log (a + b) = \log 8$   
 $= 0.9030\dots$   
 $\log ab = \log 15$   
 $= 1.1760\dots$

Since the left side is not equal to the right side, the statement is verified.

8. Write each expression as a single logarithm.

a)  $\log x - 5 \log y$

$$= \log x - \log y^5$$

$$= \log \left( \frac{x}{y^5} \right)$$

b)  $\frac{1}{2} \log x + 3 \log y$

$$= \log x^{\frac{1}{2}} + \log y^3$$

$$= \log (x^{\frac{1}{2}} y^3)$$

c)  $\frac{2}{3} \log_5 x - 4 \log_5 y - 3 \log_5 z$

$$= \log_5 x^{\frac{2}{3}} - \log_5 y^4 - \log_5 z^3$$

$$= \log_5 x^{\frac{2}{3}} - (\log_5 y^4 + \log_5 z^3)$$

$$= \log_5 x^{\frac{2}{3}} - \log_5 y^4 z^3$$

$$= \log_5 \left( \frac{x^{\frac{2}{3}}}{y^4 z^3} \right)$$

d)  $5 + \log_2 x$

$$= \log_2 2^5 + \log_2 x$$

$$= \log_2 32 + \log_2 x$$

$$= \log_2 32x$$

9. Explain each step in this proof of the power law for logarithms.

To prove that  $\log_b x^k = k \log_b x$ :

Let:  $\log_b x = n$

Write the logarithm as a power.

Then  $x = b^n$

Raise each side to the power  $k$ .

$$x^k = (b^n)^k$$

Simplify.

$$x^k = b^{kn}$$

Take the logarithm base  $b$  of each side.

$$\log_b x^k = \log_b b^{kn}$$

Simplify the right side.

$$\log_b x^k = kn$$

Substitute:  $n = \log_b x$

$$\log_b x^k = k \log_b x$$

10. Use the strategy from the proof of the product law for logarithms to

prove the quotient law:  $\log_b \left( \frac{x}{y} \right) = \log_b x - \log_b y$

Let  $\log_b x = m$  and  $\log_b y = n$

Apply the definition of a logarithm.

Then  $x = b^m$

$y = b^n$

So,  $\frac{x}{y} = \frac{b^m}{b^n}$

Use the quotient law for exponents.

$$\frac{x}{y} = b^{m-n}$$

Write this exponential statement as a logarithmic statement.

$$\log_b \left( \frac{x}{y} \right) = m - n$$

Substitute for  $m$  and  $n$ .

$$\log_b \left( \frac{x}{y} \right) = \log_b x - \log_b y$$

**11.** Use two different strategies to write  $2(\log x + \log y)$  as a single logarithm.

**Strategy 1**

$$\begin{aligned} &2(\log x + \log y) \\ &= 2 \log x + 2 \log y \\ &= \log x^2 + \log y^2 \\ &= \log x^2 y^2 \end{aligned}$$

**Strategy 2**

$$\begin{aligned} &2(\log x + \log y) \\ &= 2 \log xy \\ &= \log (xy)^2 \\ &= \log x^2 y^2 \end{aligned}$$

**12.** Write each expression as a single logarithm.

a)  $3 \log 2 + \log 6$

$$\begin{aligned} &= \log 2^3 + \log 6 \\ &= \log 8 + \log 6 \\ &= \log (8 \cdot 6) \\ &= \log 48 \end{aligned}$$

b)  $\frac{1}{2} \log 9 + 2 \log 5$

$$\begin{aligned} &= \log 9^{\frac{1}{2}} + \log 5^2 \\ &= \log 3 + \log 25 \\ &= \log 75 \end{aligned}$$

c)  $3 \log_2 6 - 2$

$$\begin{aligned} &= \log_2 6^3 - \log_2 2^2 \\ &= \log_2 216 - \log_2 4 \\ &= \log_2 \left( \frac{216}{4} \right) \\ &= \log_2 54 \end{aligned}$$

d)  $5 \log_5 2 - \log_5 4 + 2$

$$\begin{aligned} &= \log_5 2^5 - \log_5 4 + \log_5 5^2 \\ &= \log_5 32 - \log_5 4 + \log_5 25 \\ &= \log_5 \left( \frac{32 \cdot 25}{4} \right) \\ &= \log_5 200 \end{aligned}$$

**13.** Evaluate each expression.

a)  $2 \log_3 6 - 3 \log_3 2 + \log_3 18$

$$\begin{aligned} &= \log_3 6^2 - \log_3 2^3 + \log_3 18 \\ &= \log_3 36 - \log_3 8 + \log_3 18 \\ &= \log_3 \left( \frac{36 \cdot 18}{8} \right) \\ &= \log_3 81 \\ &= \log_3 3^4 \\ &= 4 \end{aligned}$$

b)  $\frac{1}{2} \log_2 36 + \log_2 12 - 2 \log_2 3$

$$\begin{aligned} &= \log_2 36^{\frac{1}{2}} + \log_2 12 - \log_2 3^2 \\ &= \log_2 6 + \log_2 12 - \log_2 9 \\ &= \log_2 \left( \frac{6 \cdot 12}{9} \right) \\ &= \log_2 8 \\ &= \log_2 2^3 \\ &= 3 \end{aligned}$$

c)  $9 \log_9 3 - \log_9 75 + 2 \log_9 5$

$$\begin{aligned} &= \log_9 3^9 - \log_9 75 + \log_9 5^2 \\ &= \log_9 19\,683 - \log_9 75 + \log_9 25 \\ &= \log_9 \left( \frac{19\,683 \cdot 25}{75} \right) \\ &= \log_9 6561 \\ &= \log_9 9^4 \\ &= 4 \end{aligned}$$

d)  $\log_4 98 - 2 \log_4 7 - 2$

$$\begin{aligned} &= \log_4 98 - \log_4 7^2 - 2 \\ &= \log_4 98 - \log_4 49 - 2 \\ &= \log_4 \left( \frac{98}{49} \right) - 2 \\ &= \log_4 2 - 2 \\ &= \log_4 4^{\frac{1}{2}} - 2 \\ &= \frac{1}{2} - 2 \\ &= -\frac{3}{2} \end{aligned}$$

14. Given  $\log a \doteq 1.301$ , determine an approximate value for each logarithm.

a)  $\log a^3$

$$\begin{aligned} &= 3 \log a \\ &\doteq 3(1.301) \\ &\doteq 3.903 \end{aligned}$$

b)  $\log 10a$

$$\begin{aligned} &= \log 10 + \log a \\ &\doteq 1 + 1.301 \\ &\doteq 2.301 \end{aligned}$$

c)  $\log \left( \frac{a^2}{100} \right)$

$$\begin{aligned} &= \log a^2 - \log 100 \\ &= 2 \log a - 2 \\ &\doteq 2(1.301) - 2 \\ &\doteq 0.602 \end{aligned}$$

15. Identify the errors in the solution to the question below.

Write the correct solution.

Write  $\log \left( \frac{a^{\frac{1}{2}}}{c^3 b^2} \right)$  in terms of  $\log a$ ,  $\log b$ , and  $\log c$ .

$$\log \left( \frac{a^{\frac{1}{2}}}{c^3 b^2} \right)$$

$$= \log a^{\frac{1}{2}} - \log c^3 b^2$$

$$= \log a^{\frac{1}{2}} - \log c^3 + \log b^2$$

$$= \frac{1}{2} \log a - \log 3c + 2 \log b$$

$$\log \left( \frac{a^{\frac{1}{2}}}{c^3 b^2} \right)$$

$$= \log a^{\frac{1}{2}} - \log c^3 b^2$$

$$= \log a^{\frac{1}{2}} - (\log c^3 + \log b^2)$$

$$= \log a^{\frac{1}{2}} - \log c^3 - \log b^2$$

$$= \frac{1}{2} \log a - 3 \log c - 2 \log b$$

In the second line of the solution, when  $\log c^3 b^2$  is written as a sum of logarithms, both logarithms should be negative. In the third line of the solution,  $\log c^3$  should be  $3 \log c$ .

16. Write each expression in terms of  $\log a$ ,  $\log b$ , and/or  $\log c$ .

a)  $\log a^3 b^{\frac{1}{2}}$

$$\begin{aligned} &= \log a^3 + \log b^{\frac{1}{2}} \\ &= 3 \log a + \frac{1}{2} \log b \end{aligned}$$

b)  $\log ab^2 c^{\frac{2}{3}}$

$$\begin{aligned} &= \log a + \log b^2 + \log c^{\frac{2}{3}} \\ &= \log a + 2 \log b + \frac{2}{3} \log c \end{aligned}$$

c)  $\log \left( \frac{a^3}{b^2} \right)$

$$\begin{aligned} &= \log a^3 - \log b^2 \\ &= 3 \log a - 2 \log b \end{aligned}$$

d)  $\log \left( \frac{a^4 b^{\frac{3}{5}}}{c} \right)$

$$\begin{aligned} &= \log a^4 + \log b^{\frac{3}{5}} - \log c \\ &= 4 \log a + \frac{3}{5} \log b - \log c \end{aligned}$$

17. Given  $\log 3 \doteq 0.477$  and  $\log 7 \doteq 0.845$ , determine the approximate value of  $\log (132\,300)$  without using a calculator.

Write 132 300 as a product of factors that involve powers of 3 and 7.

$$132\,300 \div 9 = 14\,700$$

$$14\,700 \div 3 = 4900$$

$$4900 \div 49 = 100$$

$$\text{So, } 132\,300 = 3^3 \cdot 7^2 \cdot 10^2$$

$$\begin{aligned} \log (132\,300) &= \log (3^3 \cdot 7^2 \cdot 10^2) \\ &= \log 3^3 + \log 7^2 + \log 10^2 \\ &= 3 \log 3 + 2 \log 7 + 2 \\ &\doteq 3(0.477) + 2(0.845) + 2 \\ &\doteq 5.121 \end{aligned}$$

### C

18. Write each expression as a single logarithm.

a)  $3 \log x + \log (2x - 3)$       b)  $\log (x + 1) + \log (2x - 1)$

$$\begin{aligned} &= \log x^3 + \log (2x - 3) && = \log (x + 1)(2x - 1) \\ &= \log x^3(2x - 3) \end{aligned}$$

c)  $\log (x^2 - 1) - \log (x - 1)$       d)  $\log (2x^2 + x - 3) - \log (x^2 - 1)$

$$\begin{aligned} &= \log \left( \frac{x^2 - 1}{x - 1} \right) && = \log \left( \frac{2x^2 + x - 3}{x^2 - 1} \right) \\ &= \log \frac{(x - 1)(x + 1)}{x - 1} && = \log \frac{(2x + 3)(x - 1)}{(x - 1)(x + 1)} \\ &= \log (x + 1) && = \log \left( \frac{2x + 3}{x + 1} \right) \end{aligned}$$

19. Without using the power law, prove the law of logarithms for radicals:

$$\log_b \sqrt[k]{x} = \frac{1}{k} \log_b x, \quad b > 0, b \neq 1, k \in \mathbb{N}, x > 0$$

Sample response:

Let:  $\log_b x = n$

Then  $x = b^n$

$$\sqrt[k]{x} = b^{\frac{n}{k}}$$

$$\log_b \sqrt[k]{x} = \log_b b^{\frac{n}{k}}$$

$$\log_b \sqrt[k]{x} = \frac{n}{k}$$

$$\log_b \sqrt[k]{x} = \frac{1}{k} \log_b x$$

Or,  $\log_b \sqrt[k]{x} = \frac{1}{k} \log_b x$

Write the logarithm as a power.

Raise each side to the power  $\frac{1}{k}$ .

Take the logarithm base  $b$  of each side.

Simplify the right side.

Substitute:  $n = \log_b x$