Lesson 5.7 Exercises, pages 422–427

A

3. Write each expression as a single logarithm.

a) $\log 24 - \log 3$

Use the quotient law.

$\log 24 - \log 3 = \log \left(\dfrac{24}{3}\right)$

$= \log 8$

b) $\log 2 + \log 4$

Use the product law.

$\log 2 + \log 4 = \log (2 \cdot 4)$

$= \log 8$

c) $2 \log 4 - \log 4$

Subtract.

$2 \log 4 - \log 4 = \log 4$

d) $3 \log 2 + 2 \log 3$

Use the power law then the product law.

$3 \log 2 + 2 \log 3 = \log 2^3 + \log 3^2$

$= \log (8 \cdot 9)$

$= \log 72$

4. Write each expression as a single logarithm.

a) $\log_5(x + 2) + \log_5 x$

Use the product law.

$\log_5(x + 2) + \log_5 x$

$= \log_5(x + 2)x$

$= \log_5(x^2 + 2x)$

b) $\log_3(x - 5) + \log_3(x + 3)$

Use the product law.

$\log_3(x - 5) + \log_3(x + 3)$

$= \log_3(x - 5)(x + 3)$

$= \log_3(x^2 - 2x - 15)$

c) $\log (x - 7) + \log (x + 7)$

Use the product law.

$\log (x - 7) + \log (x + 7)$

$= \log (x - 7)(x + 7)$

$= \log (x^2 - 49)$

d) $\log_6(3x - 2) - 2 \log_6 x$

Use the power law then the quotient law.

$\log_6(3x - 2) - 2 \log_6 x$

$= \log_6(3x - 2) - \log_6 x^2$

$= \log_6\left(\dfrac{3x - 2}{x^2}\right)$

5. Use substitution or logical reasoning to determine whether $x = 2$ is a root of each equation.

a) $\log_2 x + \log_2 2x = 3$

Use substitution.

L.S. $= \log_2 2 + \log_2 4$

$= 1 + 2$

$= 3$

$= $ R.S.

So, $x = 2$ is a root.

b) $\log (x - 4) + \log (x - 7) = 1$

Use logical reasoning.

When $x = 2$ is substituted in $\log (x - 4)$, it becomes $\log (-2)$, which is not defined.

So, $x = 2$ is not a root.

6. Explain why $\dfrac{\log 80}{\log 4} \neq \log 20$. Use a calculator to verify.

I cannot divide the logarithms of 2 numbers by determining the logarithm of the quotient of the numbers.

$$\text{L.S.} = \dfrac{\log 80}{\log 4} \qquad\qquad \text{R.S.} = \log 20$$
$$= 3.1609... \qquad\qquad\qquad = 1.3010...$$

Since the left side is not equal to the right side, $\dfrac{\log 80}{\log 4} \neq \log 20$

7. Solve each exponential equation. Give the solution to the nearest hundredth.

a) $60 = 3^{x+1}$

$$\log_3 60 = \log_3 3^{x+1}$$
$$\log_3 60 = x + 1$$
$$x = \log_3 60 - 1$$
$$x = \dfrac{\log 60}{\log 3} - 1$$
$$x \doteq 2.73$$

b) $5^{x-3} = 200$

$$\log_5 5^{x-3} = \log_5 200$$
$$x - 3 = \log_5 200$$
$$x = \log_5 200 + 3$$
$$x = \dfrac{\log 200}{\log 5} + 3$$
$$x \doteq 6.29$$

8. Consider the exponential equation $3^x = 30$.

a) Solve this equation algebraically. Give the root to the nearest hundredth.

$$3^x = 30$$
$$\log_3 3^x = \log_3 30$$
$$x = \dfrac{\log 30}{\log 3}$$
$$x \doteq 3.10$$

b) Verify the solution by graphing.

In a graphing calculator, input: $Y_1 = 3^x$ and $Y_2 = 30$

Use 5: intersect from the CALC menu to determine the approximate x-coordinate of the point of intersection of the graphs: 3.0959033 The solution is verified.

9. Solve, then verify each logarithmic equation.

a) $4 = \log_2 x + \log_2(x + 6)$

$x > 0$ and $x > -6$; so $x > 0$
$$4 = \log_2 x(x + 6)$$
$$2^4 = x(x + 6)$$
$$x^2 + 6x - 16 = 0$$
$$(x - 2)(x + 8) = 0$$
$$x = 2 \text{ or } x = -8$$
$x = -8$ is extraneous.
Verify $x = 2$:
$$\text{R.S.} = \log_2 2 + \log_2 8$$
$$= 1 + 3$$
$$= 4$$
$$= \text{L.S.}$$
The solution is verified.

b) $\log_6 x + \log_6(x - 16) = 2$

$x > 0$ and $x > 16$; so $x > 16$
$$\log_6 x(x - 16) = 2$$
$$x(x - 16) = 6^2$$
$$x^2 - 16x - 36 = 0$$
$$(x - 18)(x + 2) = 0$$
$$x = 18 \text{ or } x = -2$$
$x = -2$ is extraneous.
Verify $x = 18$:
$$\text{L.S.} = \log_6 18 + \log_6 2$$
$$= \log_6 36$$
$$= 2$$
$$= \text{R.S.}$$
The solution is verified.

10. Consider the equation $4^{x+2} = 8^{x-1}$.

a) Solve this equation algebraically using logarithms.

$$\log_2 4^{x+2} = \log_2 8^{x-1}$$
$$\log_2 2^{2(x+2)} = \log_2 2^{3(x-1)}$$
$$2(x + 2) = 3(x - 1)$$
$$2x + 4 = 3x - 3$$
$$x = 7$$

b) Solve this equation using a different algebraic strategy.

$$4^{x+2} = 8^{x-1} \qquad \text{Write each side with base 2.}$$
$$2^{2(x+2)} = 2^{3(x-1)} \qquad \text{Equate exponents.}$$
$$2x + 4 = 3x - 3$$
$$x = 7$$

c) Which strategy in parts a and b is more efficient? Justify your answer.

The strategy in part b is more efficient because there are fewer steps in the solution. This strategy only works because both bases can be changed to the same base to allow the exponents to be equated.

11. Solve, then verify each logarithmic equation.

a) $\log_6 48 = \log_6(x + 7) + \log_6(x - 1)$

$x > -7$ and $x > 1$; so $x > 1$
$$\log_6 48 = \log_6(x + 7)(x - 1)$$
$$48 = (x + 7)(x - 1)$$
$$48 = x^2 + 6x - 7$$
$x^2 + 6x - 55 = 0$
$(x - 5)(x + 11) = 0$
$x = 5$ or $x = -11$
$x = -11$ is extraneous.
Verify $x = 5$:
R.S. $= \log_6 12 + \log_6 4$
$= \log_6 48$
$=$ L.S.
The solution is verified.

b) $\log(2x + 4) - \log(x + 2) = \log(x + 1)$

$x > -2$, $x > -2$, and $x > -1$; so $x > -1$
$\log\left(\dfrac{2x + 4}{x + 2}\right) = \log(x + 1)$ Factor the left side, then divide.
$$\log 2 = \log(x + 1)$$
$$2 = x + 1$$
$$x = 1$$
Verify $x = 1$:
L.S. $= \log 6 - \log 3$ R.S. $= \log 2$
$= \log 2$
The solution is verified.

12. Solve each exponential equation algebraically. Write the solution to the nearest hundredth.

a) $200 = 5(2^{x-1})$

$40 = 2^{x-1}$
$\log_2 40 = \log_2 2^{x-1}$
$\log_2 40 = x - 1$
$x = \log_2 40 + 1$
$x = \dfrac{\log 40}{\log 2} + 1$
$x \doteq 6.32$

b) $4^x = 6^{x-2}$

$\log 4^x = \log 6^{x-2}$
$x \log 4 = (x - 2) \log 6$
$x \log 4 = x \log 6 - 2 \log 6$
$x \log 4 - x \log 6 = -2 \log 6$
$x(\log 4 - \log 6) = -2 \log 6$
$x = \dfrac{-2 \log 6}{\log 4 - \log 6}$
$x \doteq 8.84$

c) $5^{x+2} = 10^{x-1}$

$\log 5^{x+2} = \log 10^{x-1}$
$(x + 2) \log 5 = x - 1$
$x \log 5 + 2 \log 5 = x - 1$
$x \log 5 - x = -2 \log 5 - 1$
$x(\log 5 - 1) = -2 \log 5 - 1$
$x = \dfrac{-2 \log 5 - 1}{\log 5 - 1}$
$x \doteq 7.97$

d) $3(2^x) = 6^{x-1}$

$\log 3(2^x) = \log 6^{x-1}$
$\log 3 + \log 2^x = (x - 1) \log 6$
$\log 3 + x \log 2 = x \log 6 - \log 6$
$x \log 2 - x \log 6 = -\log 6 - \log 3$
$x(\log 2 - \log 6) = -\log 6 - \log 3$
$x = \dfrac{-\log 6 - \log 3}{\log 2 - \log 6}$
$x \doteq 2.63$

13. Solve, then verify each logarithmic equation.

a) $\log (2x - 7) + \log (x - 1) = \log (x + 1) + \log (x - 3)$

$x > 3.5, x > 1, x > -1$, and $x > 3$; so $x > 3.5$
$\log (2x - 7)(x - 1) = \log (x + 1)(x - 3)$
$\quad (2x - 7)(x - 1) = (x + 1)(x - 3)$
$\quad\quad 2x^2 - 9x + 7 = x^2 - 2x - 3$
$\quad\quad\quad x^2 - 7x + 10 = 0$
$\quad\quad\quad (x - 5)(x - 2) = 0$
$x = 5$ or $x = 2$
$x = 2$ is extraneous, because $x > 3.5$
Verify $x = 5$:
L.S. $= \log 3 + \log 4 \quad\quad$ R.S. $= \log 6 + \log 2$
$\quad\quad = \log 12 \quad\quad\quad\quad\quad\quad = \log 12$
The solution is verified.

b) $\log_2(x + 4) - \log_2(x - 2) = 1 + \log_2(2x - 1) - \log_2(x + 1)$

$x > -4, x > 2, x > 0.5$, and $x > -1$; so $x > 2$
$\log_2(x + 4) - \log_2(x - 2) = \log_2 2 + \log_2(2x - 1) - \log_2(x + 1)$
$$\log_2\left(\frac{x + 4}{x - 2}\right) = \log_2\left(\frac{2(2x - 1)}{x + 1}\right)$$
$$\frac{x + 4}{x - 2} = \frac{2(2x - 1)}{x + 1}$$
$$(x + 4)(x + 1) = (4x - 2)(x - 2)$$
$$x^2 + 5x + 4 = 4x^2 - 10x + 4$$
$$3x^2 - 15x = 0$$
$$3x(x - 5) = 0$$
$x = 0$ or $x = 5$
$x = 0$ is extraneous, because $x > 2$
Verify $x = 5$:
L.S. $= \log_2 9 - \log_2 3 \quad\quad$ R.S. $= \log_2 2 + \log_2 9 - \log_2 6$
$\quad\quad = \log_2 3 \quad\quad\quad\quad\quad\quad\quad = \log_2 18 - \log_2 6$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad = \log_2 3$
The solution is verified.

14. a) Solve the equation $10^x = k$, where $k > 0$.

$\quad\quad 10^x = k$
$\quad \log 10^x = \log k$
$x \log 10 = \log k \quad$ Substitute: $\log 10 = 1$
$\quad\quad\quad x = \log k$

b) Explain the result.

x is the power to which 10 is raised to get k.
And, by definition, the power to which 10 is raised to get k is $\log k$.

C

15. Consider the equation $\log_2(x - 1) + \log_2(x - 3) = 1$.

a) Use graphing technology to solve the equation. Write the solution to the nearest hundredth.

Write each logarithm to base 10 by dividing by log 2.

Input: $Y_1 = \dfrac{\log(x - 1)}{\log 2} + \dfrac{\log(x - 3)}{\log 2}$, and $Y_2 = 1$

Use 5: intersect from the CALC menu to determine the approximate x-coordinate of the point of intersection: 3.7320508
The solution is: $x \doteq 3.73$

b) Solve the equation algebraically to determine the exact value of x.

$x > 1$ and $x > 3$; so $x > 3$
$\log_2(x - 1) + \log_2(x - 3) = 1$
$\qquad \log_2(x - 1)(x - 3) = \log_2 2$
$\qquad\qquad (x - 1)(x - 3) = 2$
$\qquad\qquad\qquad x^2 - 4x + 1 = 0$
Use the quadratic formula.
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Substitute: $a = 1, b = -4, c = 1$
$x = \dfrac{4 \pm \sqrt{16 - 4}}{2}$
$x = \dfrac{4 \pm 2\sqrt{3}}{2}$
$x = 2 \pm \sqrt{3}$
Since $x > 3$, $x = 2 - \sqrt{3}$ is extraneous.
The solution is: $x = 2 + \sqrt{3}$