CUMULATIVE REVIEW Chapters 1-5, pages 457–464

1

1. When $bx^5 + x^4 - 5x^3 + x^2 + 2x - 4b$ is divided by $x + 1$, the remainder is -10. Determine the value of b.

Let $P(x) = bx^5 + x^4 - 5x^3 + x^2 + 2x - 4b$
Write $x + 1$ as $x - (-1)$
$P(-1) = b(-1)^5 + (-1)^4 - 5(-1)^3 + (-1)^2 + 2(-1) - 4b$
$= -b + 1 + 5 + 1 - 2 - 4b$
$= -5b + 5$
The remainder is -10.
So, $-5b + 5 = -10$ Solve for b.
$-5b = -15$
$b = 3$
The value of b is 3.

2

2. For the graph of each rational function below, determine without technology:
i) the equations of any asymptotes and the coordinates of any hole
ii) the domain of the function

Use graphing technology to verify the characteristics.

a) $y = \dfrac{x^2 - 6x + 8}{x - 3}$

i) The function is undefined when $x - 3 = 0$; that is, when $x = 3$.
There are no common factors, so there is a vertical asymptote with equation $x = 3$.
The degree of the numerator is 1 more than that of the denominator, so there is an oblique asymptote.
Determine:
$(x^2 - 6x + 8) \div (x - 3)$

$$\begin{array}{r|rrr} 3 & 1 & -6 & 8 \\ & & 3 & -9 \\ \hline & 1 & -3 & -1 \end{array}$$

The quotient is $x - 3$, so the oblique asymptote has equation $y = x - 3$.
ii) The domain is: $x \neq 3$

b) $y = \dfrac{-3x + 12}{x^2 + 3}$

i) Since $x^2 + 3$ is always positive, the function is defined for all real values of x.
The degree of the numerator is less than the degree of the denominator, so there is a horizontal asymptote with equation $y = 0$.
ii) The domain is: $x \in \mathbb{R}$

3

3. Describe how the graph of $y = \sqrt{x}$ has been translated to create the graph of each function below. Use graphing technology to check.

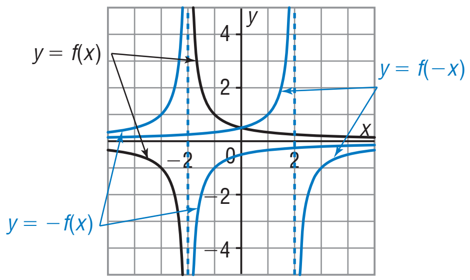
a) $y = \sqrt{x + 3}$

Write $y = \sqrt{x + 3}$ as $y = \sqrt{x - (-3)}$, then compare to $y - k = \sqrt{x - h}$: $h = -3$ and $k = 0$
So, the graph of $y = \sqrt{x + 3}$ is the graph of $y = \sqrt{x}$ after a translation of 3 units left.

b) $y = \sqrt{x + 1} + 1$

Write $y = \sqrt{x + 1} + 1$ as $y - 1 = \sqrt{x - (-1)}$, then compare to $y - k = \sqrt{x - h}$: $h = -1$ and $k = 1$. So, the graph of $y = \sqrt{x + 1} + 1$ is the graph of $y = \sqrt{x}$ after a translation of 1 unit left and 1 unit up.

4. Here is the graph of $y = f(x)$. On the same grid, sketch and label the graph of each function.



a) $y = f(-x)$

The graph of $y = f(-x)$ is the image of the graph of $y = f(x)$ after a reflection in the y-axis.
Choose points on $y = f(x)$, then reflect them in the y-axis. The line $x = -2$ is an asymptote and its reflection has equation $x = 2$. Join the points with 2 smooth curves to form the graph of $y = f(-x)$.

b) $y = -f(x)$

The graph of $y = -f(x)$ is the image of the graph of $y = f(x)$ after a reflection in the x-axis. Choose points on $y = f(x)$, then reflect them in the x-axis. The line $x = -2$ is not changed by the reflection. Join the points with 2 smooth curves to form the graph of $y = -f(x)$.

5. Graph $y = \sqrt{x}$, then apply transformations to sketch the graph of $y - 2 = -\frac{1}{2}\sqrt{3x}$. What are the domain and range of this function?

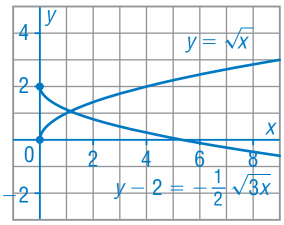
Compare $y - 2 = -\frac{1}{2}\sqrt{3x}$ to

$y - k = a\sqrt{b(x - h)}$:

$k = 2$, $a = -\frac{1}{2}$, $b = 3$, and $h = 0$

(x, y) on $y = \sqrt{x}$ corresponds to

$\left(\frac{x}{3}, -\frac{1}{2}y + 2\right)$ on $y - 2 = -\frac{1}{2}\sqrt{3x}$.

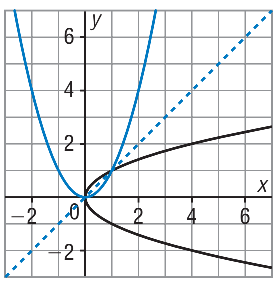


Point on $y = \sqrt{x}$	Point on $y - 2 = -\frac{1}{2}\sqrt{3x}$
(0, 0)	(0, 2)
(1, 1)	$\left(\frac{1}{3}, \frac{3}{2}\right)$
(9, 3)	$\left(3, \frac{1}{2}\right)$

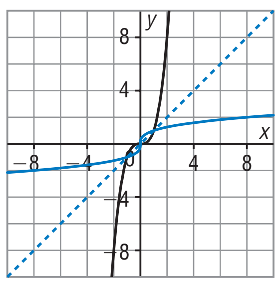
The domain is: $x \geq 0$; the range is: $y \leq 2$

6. For each graph of a relation below, sketch the graph of its inverse. Is the inverse a function? How do you know?

a)



b)



Reflect each graph in the line $y = x$.

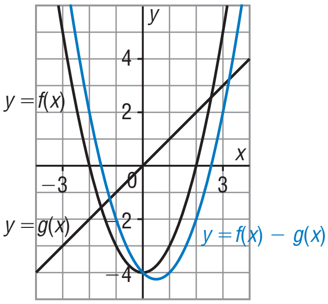
a) The inverse is a function because its graph passes the vertical line test.

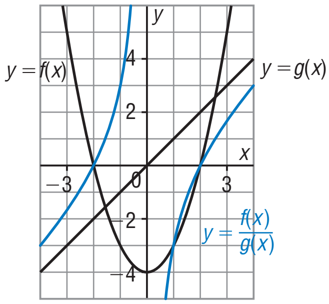
b) The inverse is a function because its graph passes the vertical line test.

4

7. Use the graphs of $y = f(x)$ and $y = g(x)$ to sketch the graph of each function below, then identify its domain and range.

a) $y = f(x) - g(x)$ **b)** $y = \frac{f(x)}{g(x)}$





From the graphs:

x	$f(x)$	$g(x)$	$f(x) - g(x)$	$\frac{f(x)}{g(x)}$
−3	5	−3	8	$-1.\overline{6}$
−2	0	−2	2	0
−1	−3	−1	−2	3
0	−4	0	−4	undefined
1	−3	1	−4	−3
2	0	2	−2	0
3	5	3	2	$1.\overline{6}$

Plot points with coordinates $(x, f(x) - g(x))$, which fit on the grid. Join the points with a smooth curve. From the graph, the domain is: $x \in \mathbb{R}$; and the range appears to be $y \geq -4.25$

Plot points with coordinates $\left(x, \frac{f(x)}{g(x)}\right)$. Since $x \neq 0$, the y-axis is an asymptote, so join the points with 2 smooth curves. From the graph, the domain is: $x \neq 0$; and the range is: $y \in \mathbb{R}$

8. Given that $f(x) = 2 + x^2$, $g(x) = -3x + 5$, and $h(x) = \sqrt{2 - 4x}$, write an explicit equation for $k(x)$ then state its domain.

a) $k(x) = f(x) + g(x) + h(x)$

$k(x) = 2 + x^2 - 3x + 5 + \sqrt{2 - 4x}$
$k(x) = x^2 - 3x + 7 + \sqrt{2 - 4x}$
For the domain, $2 - 4x \geq 0$, so $x \leq 0.5$

b) $k(x) = f(x) \cdot g(x) - h(x)$

$k(x) = (2 + x^2)(-3x + 5) - \sqrt{2 - 4x}$
$k(x) = -3x^3 + 5x^2 - 6x + 10 - \sqrt{2 - 4x}$
For the domain, $2 - 4x \geq 0$, so $x \leq 0.5$

9. Use the functions $f(x) = x^2 - x$, $g(x) = \frac{1}{3 - x}$, and $h(x) = |x + 1|$.

a) Determine each value.

i) $f(g(4))$

$g(4) = \frac{1}{3 - 4}$, or -1
$f(g(4)) = (-1)^2 - (-1)$
$= 2$

ii) $g(h(-3))$

$h(-3) = |-3 + 1|$, or 2
$g(h(-3)) = \frac{1}{3 - 2}$
$= 1$

b) Write an explicit equation for each composite function, then state its domain.

i) $h(g(x))$

$h(g(x)) = \left|\frac{1}{3 - x} + 1\right|$
$h(g(x)) = \left|\frac{4 - x}{3 - x}\right|$
For the domain, $3 - x \neq 0$, so $x \neq 3$

ii) $f(f(x))$

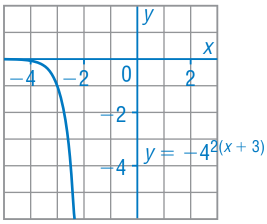
$f(f(x)) = (x^2 - x)^2 - (x^2 - x)$
$f(f(x)) = x^4 - 2x^3 + x^2 - x^2 + x$
$f(f(x)) = x^4 - 2x^3 + x$
This is a quartic function; its domain is: $x \in \mathbb{R}$

10. For the function $y = 3x^2 - 12x + 5$, determine possible functions f and g so that $y = f(g(x))$.

Sample response:
Complete the square.
$y = 3(x^2 - 4x + 4 - 4) + 5$
$y = 3(x^2 - 4x + 4) - 7$
$y = 3(x - 2)^2 - 7$
Let $f(g(x)) = 3(x - 2)^2 - 7$
Then $g(x) = x - 2$ and $f(x) = 3x^2 - 7$

11. Use transformations to graph $y = -4^{2(x+3)}$. Describe the transformations, then state the characteristics of the graph; include the intercepts, equation of the asymptote, domain, and range.

Compare $y = -4^{2(x+3)}$ with $y - k = c4^{d(x-h)}$:
$k = 0$, $c = -1$, $d = 2$, and $h = -3$
The graph of $y = -4^{2(x+3)}$ is the image of the graph of $y = 4^x$ after a horizontal compression by a factor of $\frac{1}{2}$, a reflection in the x-axis, then a translation of 3 units left.
A point (x, y) on $y = 4^x$ corresponds to the point $\left(\frac{x}{2} - 3, -y\right)$ on $y = -4^{2(x+3)}$.



(x, y)	$\left(\frac{x}{2} - 3, -y\right)$
$(-2, 0.0625)$	$(-4, -0.0625)$
$(-1, 0.25)$	$(-3.5, -0.25)$
$(0, 1)$	$(-3, -1)$
$(1, 4)$	$(-2.5, -4)$
$(2, 16)$	$(-2, -16)$

The function is decreasing. There is no x-intercept.
For the y-intercept, substitute $x = 0$ in $y = -4^{2(x+3)}$.
$y = -4^{2(0+3)}$
$y = -4^6$ or -4096
The horizontal asymptote has equation $y = 0$.
The domain is: $x \in \mathbb{R}$; and the range is: $y < 0$

12. A cup of coffee contains 150 mg of caffeine. In a healthy adult, the mass, m grams, that remains after t hours is modelled by the function $m = 150(0.5)^{0.2t}$. Use technology.

a) To the nearest half hour, how long will it take until only 10 mg of caffeine remain?

Use $m = 150(0.5)^{0.2t}$ Substitute: $m = 10$
$10 = 150(0.5)^{0.2t}$
Graph the related function: $y = 150(0.5)^{0.2t} - 10$
Determine the approximate zero of the function: 19.534453
It will take approximately 19.5 h until only 10 mg of caffeine remain.

b) To the nearest tenth of a milligram, how much caffeine remains after 24 h?

Use $m = 150(0.5)^{0.2t}$ Substitute: $t = 24$

$m = 150(0.5)^{0.2(24)}$

$m = 5.3845...$

After 24 h, approximately 5.4 mg of caffeine remain.

13. Solve each equation.

a) $9^{2x+1} = \dfrac{\sqrt[4]{27}}{81}$

$3^{2(2x+1)} = \dfrac{3^{\frac{3}{4}}}{3^4}$

$3^{2(2x+1)} = 3^{-\frac{13}{4}}$

$2(2x + 1) = -\dfrac{13}{4}$

$2x + 1 = -\dfrac{13}{8}$

$2x = -\dfrac{21}{8}$

$x = -\dfrac{21}{16}$

Use a calculator to verify.

b) $\log_3(x + 5) + \log_3(x - 1) = 2 + \log_3(x + 1)$

$x > -5$, $x > 1$, and $x > -1$, so $x > 1$

$\log_3(x + 5)(x - 1) = \log_3 9 + \log_3(x + 1)$

$(x + 5)(x - 1) = 9(x + 1)$

$x^2 + 4x - 5 = 9x + 9$

$x^2 - 5x - 14 = 0$

$(x + 2)(x - 7) = 0$

$x = -2$ or $x = 7$

$x = -2$ is extraneous, because $x > 1$

Verify $x = 7$:

L.S. $= \log_3 12 + \log_3 6$, or $\log_3 72$

R.S. $= \log_3 9 + \log_3 8$, or $\log_3 72$

The solution is verified.

14. Write each expression in terms of log a, log b, and log c.

a) $\log\left(\dfrac{a^3\sqrt{b}}{c}\right)$

$= \log a^3 b^{\frac{1}{2}} - \log c$

$= \log a^3 + \log b^{\frac{1}{2}} - \log c$

$= 3 \log a + \dfrac{1}{2} \log b - \log c$

b) $\log\left(\dfrac{a^{\frac{2}{3}}}{b\sqrt[4]{c^3}}\right)$

$= \log a^{\frac{2}{3}} - \log b\sqrt[4]{c^3}$

$= \log a^{\frac{2}{3}} - (\log b + \log \sqrt[4]{c^3})$

$= \dfrac{2}{3} \log a - \log b - \log c^{\frac{3}{4}}$

$= \dfrac{2}{3} \log a - \log b - \dfrac{3}{4} \log c$

15. Write each expression as a single logarithm.

a) $4 \log 3 - 2 \log 6 + 2 \log 2$

$= \log 3^4 - \log 6^2 + \log 2^2$

$= \log 81 - \log 36 + \log 4$

$= \log\left(\dfrac{81 \cdot 4}{36}\right)$

$= \log 9$

b) $2 \log_7 x - \log_7 y - \dfrac{1}{4} \log_7 z$

$= \log_7 x^2 - \log_7 y - \log_7 z^{\frac{1}{4}}$

$= \log_7\left(\dfrac{x^2}{yz^{\frac{1}{4}}}\right)$

16. On March 11, 2011, an earthquake off the coast of Japan had a magnitude of 9.0 and it caused a tsunami. To the nearest tenth, determine the magnitude of an earthquake that is one-half as intense as this earthquake.

Use: $M = \log\left(\frac{I}{S}\right)$ Substitute: $M = 9.0$

$$9.0 = \log\left(\frac{I}{S}\right)$$
$$\frac{I}{S} = 10^{9.0}$$
$$I = 10^{9.0}S$$

An earthquake that is one-half as intense has intensity: $I = 0.5(10^{9.0}S)$

Work backward.

$$\frac{I}{S} = 0.5(10^{9.0})$$ Take the common logarithm of each side.

$$\log\left(\frac{I}{S}\right) = \log\left[0.5(10^{9.0})\right]$$
$$M = 8.6989...$$

The magnitude is approximately 8.7.