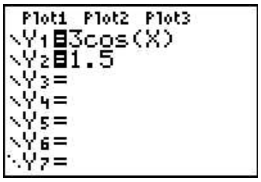
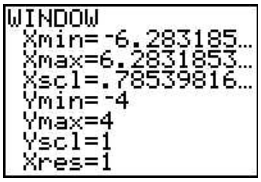
Lesson 7.1 Exercises, pages 577–581

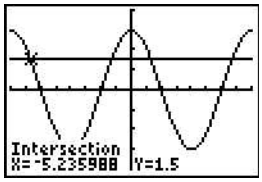
Use graphing technology to solve each equation. Where necessary, round the roots to the nearest hundredth.

A

4. Use a graphing calculator and enter the settings shown below to solve the equation $3 \cos x = 1.5$. State the restricted domain indicated by the WINDOW screen, then determine the roots of the equation over this domain.







The domain is: $-2\pi \leq x \leq 2\pi$
Graph the function, then determine the approximate x-coordinate of each point of intersection.
X = −5.235988; X = −1.047198; X = 1.047198; X = 5.235988
To the nearest hundredth, the roots are: $x = \pm 5.24$, $x = \pm 1.05$

5. Solve each equation for $0 \leq x < 2\pi$.

a) $\sin x = \frac{2}{5}$

Graph $y = \sin x$ and $y = \frac{2}{5}$.
The approximate x-coordinates of the points of intersection are:
X = 0.41151685 and
X = 2.7300758
To the nearest hundredth, the roots are: $x = 0.41$ and $x = 2.73$

b) $\cos x = -\frac{1}{3}$

Graph $y = \cos x$ and $y = -\frac{1}{3}$.
The approximate x-coordinates of the points of intersection are:
X = 1.9106332 and
X = 4.3725521
To the nearest hundredth, the roots are: $x = 1.91$ and $x = 4.37$

B

6. Solve the equation $\sin x = -\frac{4}{7}$ over the domain $0 \leq x < 2\pi$. Assume x is an angle in standard position. In which quadrants do the terminal arms of the angles lie? How do you know?

Graph $y = \sin x$ and $y = -\frac{4}{7}$.
To the nearest hundredth, the roots are: $x = 3.75$ and $x = 5.67$
The terminal arms lie in Quadrants 3 and 4 because the sine of an angle is negative when its terminal arm lies in those quadrants.

7. Solve each equation for $-2\pi \leq x < 0$.

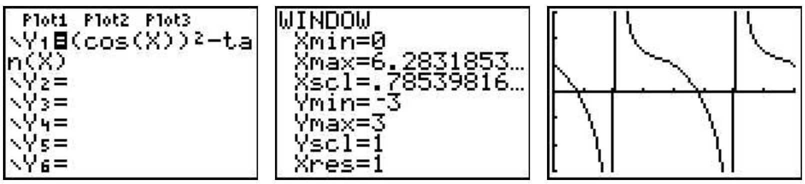
a) $\tan x - 3 = \cos x + 2$

Graph $y = \tan x - 3$ and $y = \cos x + 2$.
To the nearest hundredth, the roots are: $x = -4.90$ and $x = -1.78$

b) $2 = 4 \sin x - 3 \cos x$

Graph $y = 2$ and $y = 4 \sin x - 3 \cos x$.
To the nearest hundredth, the roots are: $x = -5.23$ and $x = -2.91$

8. Use a graphing calculator and enter the settings below to solve a trigonometric equation. State the restricted domain indicated by the WINDOW screen, then determine the roots of the equation over this domain.



The domain is: $0 \leq x \leq 2\pi$
To the nearest hundredth, the roots are: $x = 0.60$ and $x = 3.74$

9. Solve each equation for $0 \leq x < 2\pi$, then write the general solution.

a) $5 \sin^2 x - \sin x = 2$

Graph $y = 5 \sin^2 x - \sin x - 2$.
To the nearest hundredth, the roots are: $x = 0.83$, $x = 2.31$, $x = 3.71$, and $x = 5.71$
The period is 2π, so the general solution is approximately:
$x = 0.83 + 2\pi k, k \in \mathbb{Z}$ or
$x = 2.31 + 2\pi k, k \in \mathbb{Z}$ or
$x = 3.71 + 2\pi k, k \in \mathbb{Z}$ or
$x = 5.71 + 2\pi k, k \in \mathbb{Z}$

b) $3 \tan x - 1 = \tan^2 x$

Graph $y = \tan^2 x - 3 \tan x + 1$.
To the nearest hundredth, the roots are: $x = 0.36$, $x = 1.21$, $x = 3.51$, and $x = 4.35$
The period is π, so the general solution is approximately:
$x = 0.36 + \pi k, k \in \mathbb{Z}$ or
$x = 1.21 + \pi k, k \in \mathbb{Z}$

10. Solve each equation for $0 \leq x < 2\pi$, then write the general solution.

a) $\cos 3x = \frac{1}{2}$

Graph $y = \cos 3x - \frac{1}{2}$.
To the nearest hundredth, the roots are: $x = 0.35$, $x = 1.75$, $x = 2.44$, $x = 3.84$, $x = 4.54$, $x = 5.93$

The period is $\frac{2\pi}{3}$, so the general solution is approximately:

$x = 0.35 + \frac{2\pi k}{3}, k \in \mathbb{Z}$ or

$x = 1.75 + \frac{2\pi k}{3}, k \in \mathbb{Z}$

b) $1 - 4 \tan 3x = -7$

Graph $y = 8 - 4 \tan 3x$.
To the nearest hundredth, the roots are: $x = 0.37$, $x = 1.42$, $x = 2.46$, $x = 3.51$, $x = 4.56$, $x = 5.61$

The period is $\frac{\pi}{3}$, so the general solution is approximately:

$x = 0.37 + \frac{\pi k}{3}, k \in \mathbb{Z}$

11. The first two positive roots of the equation $\sin 5x = \frac{1}{3}$ are $x \doteq 0.07$ and $x \doteq 0.56$. Determine the general solution of this equation. Explain how this solution is determined.

The period of the function is: $\frac{2\pi}{5} \doteq 1.26$

The graph of $y = \sin 5x - \frac{1}{3}$ indicates that the two given roots are the only zeros of the function in the domain $0 \leq x \leq \frac{2\pi}{5}$.

So, the general solution is approximately: $x = 0.07 + \frac{2\pi k}{5}, k \in \mathbb{Z}$ or $x = 0.56 + \frac{2\pi k}{5}, k \in \mathbb{Z}$

12. Solve each equation over the given domain, then write the general solution.

a) $\cos \pi x = 0$ for $-3 \leq x \leq 3$

Graph $y = \cos \pi x$.
The graph is symmetrical about the y-axis.
The roots are: $x = \pm 2.5$, $x = \pm 1.5$, $x = \pm 0.5$
The general solution is: $x = 0.5 + k, k \in \mathbb{Z}$

b) $-1 = 2 \sin 3\pi x$ for $-1 \leq x \leq 1$

Graph $y = 2 \sin 3\pi x + 1$.
To the nearest hundredth, the roots are: $x = -0.94$, $x = -0.72$, $x = -0.28$, $x = -0.06$, $x = 0.39$, $x = 0.61$

The period is $\frac{2\pi}{3\pi} = \frac{2}{3}$, so the general solution is approximately:

$x = -0.72 + \frac{2k}{3}, k \in \mathbb{Z}$ or $x = -0.94 + \frac{2k}{3}, k \in \mathbb{Z}$

13. Solve each equation over the set of real numbers.

a) $3 \cos x = x^2 + 1$

Graph $y = 3 \cos x - x^2 - 1$.
The solution is: $x \doteq \pm 0.91$

b) $x^3 - 2 = 2 \sin x$

Graph $y = 2 \sin x - x^3 + 2$
The solution is: $x \doteq 1.59$

14. a) Solve $\frac{\cos x}{1 - \sin x} = \frac{1 + \sin x}{\cos x}$ over the set of real numbers by graphing the two functions $y = \frac{\cos x}{1 - \sin x}$ and $y = \frac{1 + \sin x}{\cos x}$.
What do you notice about the solution?

The graphs coincide so the solution is all real values of x, except for those values for which the denominators are 0.

b) The equation in part a is called an *identity*. Why is that an appropriate name?

One definition of identity is "exact likeness." This is appropriate because one side of the equation is exactly the same as the other side.

C

15. Solve each equation over the set of real numbers.

a) $\sec x = \sqrt{4 - x^2}$

Graph $y = \frac{1}{\cos x} - \sqrt{4 - x^2}$
The solution is: $x \doteq \pm 0.96$

b) $\sin x + 2 = 2x$

Graph $y = \sin x + 2 - 2x$
The solution is: $x \doteq 1.50$

16. a) Solve each equation, and explain the results.

i) $\frac{\sin x}{x} = 1$

Graph $y = \frac{\sin x}{x} - 1$
There is no real solution.
When $x = 0$, the left side is undefined

ii) $\sin x = x$

Graph $y = \sin x - x$
The solution is $x = 0$.

b) Why are the solutions in part a different?

The solutions are different because in part i, $x = 0$ is non-permissible; while in part ii, $x = 0$ is permissible.

17. **a)** Solve each equation, and explain the results.

i) $\frac{\cos x}{x} = 1$

Graph $y = \frac{\cos x}{x} - 1$

The solution is: $x \doteq 0.74$

ii) $\cos x = x$

Graph $y = \cos x - x$

The solution is $x \doteq 0.74$

b) Why are the solutions in part a the same?

The solutions are the same because the equations are equivalent for $x \neq 0$.